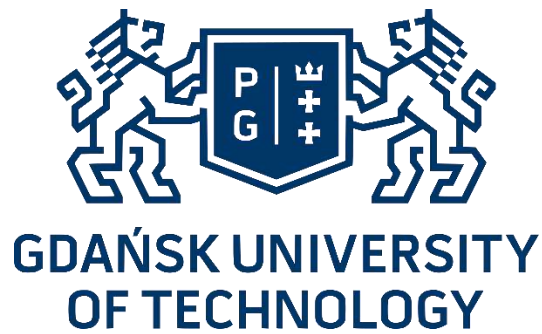


Gdańsk University of Technology
Faculty of Civil and Environmental Engineering



**Shape optimization, stress control, and
effective design of flexible structures**
(Doctoral Thesis)

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Gdańsk, 2026



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Scientific discipline: Civil Engineering, Geodesy and Transport

DOCTORAL DISSERTATION

Title of doctoral dissertation: Shape optimization, stress control, and effective design of flexible structures

Title of doctoral dissertation (in Polish): Optymalizacja kształtu, kontrola naprężeń i efektywne projektowanie konstrukcji podatnych

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Gdańsk, 2026



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DESCRIPTION OF DOCTORAL DISSERTATION

The Author of the doctoral dissertation: Ahmed Aulla Hamed Manguri

Title of doctoral dissertation: Shape optimization, stress control, and effective design of flexible structures.

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Summary of doctoral dissertation in English:

Flexible structures, such as trusses, frames, and scissor structures, may undergo significant deformation and excessive stress in certain members under loading. In this PhD study, several innovative solutions have been proposed, evaluated, and verified to optimize shape and control stress in such structures. This was achieved by employing the minimum necessary resources using optimization algorithms. The shape and stress of flexible systems were constrained, while the total change in bar length or the total structural weight was minimized. The shape and stress of loaded flexible systems were maintained within desired limits by adjusting the length of certain active members. The optimization algorithms were used to minimize the total change in bar length and eliminate inactive members, thereby reducing the number of required actuators. On the other hand, the total weight of the structures was minimized by selecting the smallest possible cross-sectional areas for the structural members. This was achieved by



implementing optimization algorithms and analytically differentiating the constraint functions to enhance the optimization process and improve convergence. In addition to minimizing total weight, shape, stress, and structural stability were considered. The study also examined the effect of adding extra members on structural optimization and the mitigation of deformation and stresses. The results of the PhD thesis demonstrate that the flexible structures can be effectively reshaped after loading, and that stresses can be redistributed while remaining within safe limits, with minimal changes to the total bar length or structural weight.

Summary of doctoral dissertation in Polish:

Konstrukcje podatne, takie jak kratownice, ramy i konstrukcje nożycowe, mogą ulegać znacznym odkształceniom i nadmiernym naprężeniom pod wpływem obciążenia. W niniejszej pracy doktorskiej zaproponowano, oceniono i zweryfikowano kilka innowacyjnych rozwiązań mających na celu optymalizację kształtu i kontrolę naprężeń w takich konstrukcjach. Osiągnięto to dzięki zastosowaniu minimalnych niezbędnych zasobów wykorzystując algorytmy optymalizacyjne. Kształt i naprężenia w konstrukcji zostały ograniczone, a całkowita zmiana długości prętów lub całkowita masa konstrukcji została zminimalizowana. Wymagany efekt uzyskano poprzez modyfikację długości wybranych elementów aktywnych. Algorytmy optymalizacyjne zastosowano w celu zminimalizowania koniecznej zmiany długości prętów i wyeliminowania elementów nieaktywnych, zmniejszając w ten sposób liczbę koniecznych do użycia siłowników. Z drugiej strony, całkowita masa konstrukcji została zminimalizowana poprzez dobór elementów konstrukcyjnych o najmniejszych możliwych przekrojach poprzecznych. Osiągnięto to wykorzystując algorytmy optymalizacyjne i dokonując różnicowania funkcji ograniczeń w celu usprawnienia procesu optymalizacji i poprawy zbieżności. Oprócz minimalizacji masy całkowitej, wzięto także pod uwagę kształt, poziom naprężeń i stateczność konstrukcji. W badaniach przeanalizowano również wpływ zastosowania dodatkowych elementów na optymalizację konstrukcji oraz zmniejszenie wartości odkształceń i naprężeń. Wyniki rozprawy doktorskiej pokazują, że kształt konstrukcji podatnych po ich obciążeniu można efektywnie zoptymalizować, a naprężenia redystrybuować zachowując jednocześnie wymagany poziom bezpieczeństwa, przy minimalnych zmianach całkowitej długości prętów lub masy konstrukcji.

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Summary

Flexible structures, such as trusses, frames, and scissor structures, may undergo significant deformation and excessive stress in certain members under loading. In this PhD study, several innovative solutions have been proposed, evaluated, and verified to optimize shape and control stress in such structures. This was achieved by employing the minimum necessary resources using optimization algorithms. The shape and stress of flexible systems were constrained, while the total change in bar length or the total structural weight was minimized. The shape and stress of loaded flexible systems were maintained within desired limits by adjusting the length of certain active members. The optimization algorithms were used to minimize the total change in bar length and eliminate inactive members, thereby reducing the number of required actuators. On the other hand, the total weight of the structures was minimized by selecting the smallest possible cross-sectional areas for the structural members. This was achieved by implementing optimization algorithms and analytically differentiating the constraint functions to enhance the optimization process and improve convergence. In addition to minimizing total weight, shape, stress, and structural stability were considered. The study also examined the effect of adding extra members on structural optimization and the mitigation of deformation and stresses. The results of the PhD thesis demonstrate that the flexible structures can be effectively reshaped after loading, and that stresses can be redistributed while remaining within safe limits, with minimal changes to the total bar length or structural weight.

Publications within the PhD thesis

The following published papers are used in the PhD thesis:

Paper I: Manguri, A., N. Saeed, and R. Jankowski, *A review: structural shape and stress control techniques and their applications*. Archives of Computational Methods in Engineering, 2025. **32**(2): p. 885-898. <https://doi.org/10.1007/s11831-024-10149-9>

Paper II: Manguri, A., H. Hassan, N. Saeed, and R. Jankowski, *Topology, size, and shape optimization in civil engineering structures: a review*. Computer Modeling in Engineering & Sciences, 2025. **142**(2). <https://doi.org/10.32604/cmescs.2025.059249>

Paper III: Manguri, A., N. Saeed, and R. Jankowski, *Deformation mitigation and twisting moment control in space frames*. Structures, 2024. **68**: p. 107227. <https://doi.org/10.1016/j.istruc.2024.107227>

Paper IV: Manguri, A., N. Saeed, and R. Jankowski, *Controlling nodal displacement of pantographic structures using matrix condensation and interior-point optimization: A numerical and experimental study*. Engineering Structures, 2024. **304**: p. 117603. <https://doi.org/10.1016/j.engstruct.2024.117603>

Paper V: Manguri, A., N. Saeed, M. Szczepanski, and R. Jankowski, *Deformation and bending moment mitigation through quasi-static adjustment in scissor structures*. Acta Mechanica, 2026. <https://doi.org/10.1007/s00707-026-04723-4>

Paper VI: Manguri, A., D. Magisano, and R. Jankowski, *Optimal shape and stress control of geometrically nonlinear structures with exact gradient with respect to the actuation inputs*. Structures, 2024. **70**: p. 107738. <https://doi.org/10.1016/j.istruc.2024.107738>

Paper VII: Manguri, A., D. Magisano, and R. Jankowski, *Gradient-based weight minimization of nonlinear truss structures with displacement, stress, and stability constraints*. International Journal for Numerical Methods in Engineering, 2025. **126**(14): p. e70096. <https://doi.org/10.1002/nme.70096>

Paper VIII: Manguri, A., N. Saeed, M. Szczepanski, and R. Jankowski, *Bending moment control and weight optimization in space structures by adding extra members in the optimal locations*. Advances in Science and Technology Research Journal, 2023. **17**(4): p. 313-324. <https://doi.org/10.12913/22998624/169573>

The journal metrics (based on the released data in 2024)

Paper	Journal	Polish Ministry points	Impact factor (WoS)	Cite score (Scopus)
I	Archives of Computational Methods in Engineering	200	12.1	27.4
II	CMES-Computer Modeling in Engineering & Sciences	40	2.5	4.4
III	Structures	100	4.3	6.5
IV	Engineering Structures	140	6.4	11.2
V	Acta Mechanica	100	2.9	4.3
VI	Structures	100	4.3	6.5
VII	International Journal for Numerical Methods in Engineering	200	2.9	6.1
VIII	Advances in Science and Technology Research Journal	100	1.3	2.1

The author's contribution to the published papers

Paper I

Ahmed Manguri (the first and corresponding author) planned the structure of the review paper. He conceptualized the research and organized the work. He conducted research on structural shape control, stress control, and actuators. He wrote the first version of the manuscript and prepared the revised version.

Paper II

Ahmed Maguri (the first and corresponding author) planned the structure of this review paper. He implemented the idea of reviewing published papers in the field and categorizing the types of literature research. He wrote the first draft of the manuscript and prepared the revised version.

Paper III

Ahmed Maguri (the first and corresponding author) planned and conducted the research in space frames. He modeled the single-layer space frame and wrote the computer code for the analysis. He performed a formal analysis and validation of the model and the results. He interpreted the findings, wrote the first draft of the manuscript and prepared the revised version.

Paper IV

Ahmed Maguri (the first and corresponding author) planned both numerical and experimental tests on scissor structures. He first conducted the numerical analysis, then prepared and assembled the physical models and tested them in the laboratory. He performed a formal analysis and validation. He compared the numerical results to those available in the literature and to the experimental results. He wrote the first draft of the manuscript and prepared the revised version.

Paper V

Ahmed Maguri (the first and corresponding author) conceptualized and conducted the research on scissor structures and the layout of extra members. He also worked on formulating equations and analyzing examples. He wrote the computer code and performed formal analysis. He carried out the validation using numerical software. He wrote the first version of the manuscript and prepared the revised version.

Paper VI

Ahmed Maguri (the first author) conceptualized the research and conducted the study on geometrically nonlinear flexible structures. He worked on the methodological part, contributed to formulating the hypotheses, and prepared examples from the literature. He performed formal analysis and validation. He interpreted the results, wrote the first draft of the manuscript, and contributed to the revised version.

Paper VII

Ahmed Maguri (the first author) planned the study and conducted the research on structural optimization in relation to geometrically nonlinear flexible structures. He prepared examples from the literature to validate the proposed method. He worked on numerical modeling, conducted formal analysis, and performed data curation. He wrote the first draft of the manuscript. He also prepared the revised manuscript.

Paper VIII

Ahmed Maguri (the first and corresponding author) conceptualized the research, prepared the formulation, and wrote the computer code. He performed the formal analysis, data curation, and interpreted the results. He also wrote the first draft of the manuscript and prepared the revised manuscript.

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1. Introduction

Flexible structures, such as trusses, frames, and scissor structures are prone to deformation under loading or in severe environments, which is particularly significant for structures that support sensitive communication or scientific equipment [1, 2]. In this context, restoring structural shapes is necessary, and this can be accomplished by adjusting the positions of relocated joints, since nodal positions define structural configurations. Active shape control is vital for delicate structures used in space missions, such as antennas and solar sails, to achieve specific mission objectives, including maintaining shape precision and adapting to various functional conditions [3]. On the other hand, some components may undergo excessive stress, especially in systems composed of cables, like tensegrity structures and cable bridges, where cables may become slack [4]. In such cases, stress redistribution becomes essential. In some cases, it is also impractical to control shape or stress independently since adjusting certain nodal coordinates requires length changes in other members, which may result in excessive stress. Therefore, in specific cases, it is necessary to control both shape and stress simultaneously.

The process of small changes in the length of certain members to reshape or redistribute stress in a structure's members is called adjustment [5]. The adjustment technique was first introduced by Weeks [6] and then analytically formulated by Haftka and Adelman [7]. Generally, shape control through nodal relocation and stress redistribution is achieved by adjusting the length of certain members through devices called actuators [8, 9]. There are several types of actuators, including Shape Memory Alloy (SMA) actuators [10], mechanical actuators (such as turnbuckles) [11-13], and piezoelectric actuators [14-16]. To date, two review studies concerning shape control have been solely focused on the use of piezoelectric actuators [17, 18]. The correlation between bar length alterations and axial forces within space trusses was examined using the Force Method based on Singular Decomposition of the equilibrium matrix (SVD-FM) [19]. The equivalence of the Integrated Force Method (IFM) with the more computationally efficient SVD-FM was established [20]. Subsequently, the Extended Integrated Force Method (EIFM) was proposed to analyze prestressable structures [21], thereby broadening the applicability of the IFM to kinematically indeterminate systems. Moreover, optimization techniques were also applied to minimize the cost of structural shape and stress control [22-26].

The structural layout and stress can also be controlled while minimizing the total structural weight. Structural optimization aimed at reducing structural weight is widely used in

various types of structures [27]. The main branches of structural weight optimization concern topology, shape, and size optimization. These optimization techniques are applied to several types of structures, including trusses [28], steel frames [29], space frame systems [30], and tensegrity structures [31].

In recent decades, the design of large space frames has become a trademark of modern cityscapes, successfully combining structural strength with aesthetic appeal [32]. These systems, which are both strong and lightweight, minimize the need for interior supports [33], for example, in arenas and airports [34]. However, they face challenges in ensuring the safety and functionality of their components under external loads. This has driven ongoing research into methods aimed at reducing displacement responses by varying the length of active members. In the context of shape control studies in space frames, efforts have been focused on reshaping a single-layer egg-shaped dome space frame [35]. Regarding the sources of inducing twisting moment in structures, studies have focused on the dynamic effects of earthquakes on twisting of columns [36] including the effect resulting from the soil-structure interaction [37]. The effect of different devices, such as triple-friction pendulum systems [38] multiple magnetorheological dampers [39], and multiple-tuned liquid dampers [40, 41] has been investigated to reduce the twisting moment in buildings during earthquakes. However, dynamic loading is not the only source of twisting moment; it might also come from eccentric static loadings [42] and structural irregularity [43].

In addition to trusses and frames, other flexible systems, such as scissor structures [44] or lazy tongs [45], used in engineering applications, have configurations that affecting their performance. Due to their flexibility, the nodal positions can change after loading; therefore, their distorted shape should be reshaped to match the desired configuration. Scissor structures are applied in various engineering fields, including bridges [46, 47], foldable stairs [48, 49], robots [50], tensegrity structures [51], and solar roof panels [52]. In 1961, Emilio Pérez Piñero was the first one to build a three-dimensional deployable structure [53]. Various fundamental unit types for scissor structures can be generated through alterations in the placement of the intermediate hinge or the configuration of the bars. These include translational, polar, and angulated units [54]. An extensive review of scissor structures was carried out [55]. The behavior of such structures, for example, geometric configurations, was also studied by various researchers [56]. Pellegrino *et al.* [57] established a method for analyzing complex systems, such as scissor structures, based on condensing the matrices of FM, which was then further developed [58]. However, these techniques still require a large amount of bar length change

and many turnbuckles. This is due to the complex relationship between bar length change and nodal positions.

When scissor structures are loaded, a significant bending moment is developed in macro elements. Overall, researchers have predominantly focused their investigations on the mechanisms for deploying and retracting scissor structures to achieve various configurations. Incorporating additional members (such as struts or cables) in structures significantly enhances structural performance by augmenting stiffness and energy absorption, particularly against seismic forces, thereby reducing stress and deformation [59]. Furthermore, this technique maintains structural integrity in systems vulnerable to component loss, as evidenced by studies demonstrating deformation mitigation [35].

Geometric nonlinearity is another significant issue in flexible structures, where large deformations have a substantial impact on their stiffness and strength [60]. Studies have been conducted for shape and stress control based on nonlinear FM [61] but they may lack robustness. Reduced-order modeling was employed for the gradient-based optimization of buckling-prone shells, thereby improving computational efficiency [62, 63]. Additionally, the computational demands of gradient calculation in nonlinear design optimization can be mitigated through parallel computing, while metaheuristic approaches, such as Genetic Algorithms (GA) [64] and the Mine Blast Algorithm (MBA) [65], provide alternative optimization strategies.

There are three types of optimization algorithms used in structural optimization: gradient-based [66, 67], non-gradient-based [68] and machine learning techniques [69]. Gradient-based optimizations converge quickly, as compared to gradient-free methods, to find an optimal solution when the function is convex [70]. For non-convex problems, non-gradient algorithms, such as GA and Particle Swarm Optimization (PSO), are employed, as they are efficient in finding global optimality; however, they are slower than gradient-based algorithms [70]. The use of machine learning techniques in structural optimization depends on the accuracy of the results, which, in turn, relies on the quality and quantity of available training data [69]. When numerical differentiation is applied to find the gradients, it faces a bottleneck in nonlinear problems [71] which may require parallel computing to tackle this challenge. A reduction in computational cost for large-scale optimization of linear trusses was reported by leveraging sparsity in the gradient and Hessian matrices [72].

Shape and stress can also be controlled while minimizing structural weight using optimization techniques. Missoum *et al.* [73] demonstrated a weight optimization technique

based on gradients for both geometrically linear and nonlinear trusses, emphasizing displacement variables. While the above methods are focused on general optimization approaches, more particular methodologies have been established for unusual structural behavior which, for example, concern systems which may undergo snap-through phenomena. A comprehensive methodology for sizing and geometric optimization of shell structures was elucidated (see [74]), where the nonlinear critical load is maximized using the null vector method [75, 76]. This technique requires the iterative resolution of the nonlinear equilibrium of the generalized discretized structure alongside the singularity of the tangent stiffness matrix. Implicit differentiation is used to effectively determine the gradient of the objective function; however, numerical differentiation still plays a role in the context of the residual equations. Lindgaard and Lund [77] introduced a recursive strategy designed to address the most detrimental imperfection shape in the structural optimization of buckling-prone plates.

Besides truss structures, optimization techniques have also been applied to frame structures under different constraints and loading conditions. Structural optimization was performed for seismic design in frames [78], and single and built-up cold-formed steel beam-column members [79, 80]. Furthermore, the optimum location of different types of shear walls for various buildings (see [81]) on several soil types (see [82]) was also investigated. The optimal design was conducted for I- and H-shaped crane bridge girders [83], reinforced concrete sections [84], and steel moment frames [85].

Although different researchers have proposed techniques to control shape and/or stress in trusses, other flexible structures, such as frames and scissor structures, have received less attention due to their complexity. Also, the previous studies on trusses are computationally costly and less precise, as they employ numerical differentiation to reach optimality. Therefore, this PhD thesis proposes innovative methods to control shape and/or stress in nonlinear trusses, frames, such as single-layer egg-shaped space structures and complex systems such as scissor structures, minimizing actuation (member length change) and total structural weight through the application of optimization techniques. Moreover, the PhD thesis considers geometric nonlinearity to minimize the structural weight of large-scale flexible systems, including those that undergo instability when the critical load is reached. Additionally, the PhD thesis proposes a novel approach that utilizes analytical gradients of the constraints to enhance accuracy and computational efficiency.

2. Objective and scope

The overall aim of this PhD thesis is to analyze methods for controlling the shape and stress of flexible structures by designing optimal changes in bar length or weight. The goal is to determine the minimum change in bar length required to control structural configuration and/or stress, or to find the minimum structural weight while maintaining displacement and/or stress control and ensuring structural stability. In this thesis, **Papers I** and **II** investigate the previous studies conducted on structural control and optimization, respectively. **Papers III-VI** are dedicated to structural shape and/or stress control through the alteration of bar length using mechanical actuators (turnbuckles). **Papers VII** and **VIII** focus on minimizing structural weight while considering shape, stress, and structural stability.

The aim of the two review papers (**Papers I** and **II**) is to highlight and categorize prior research on structural control and optimization. **Paper I** covers the studies conducted on the shape and/or stress control of structures. It also includes studies on various types of actuators and research on optimizing the number and optimal locations of actuators. The system control was categorized into three primary classifications: control of nodal movement, control of axial forces, and simultaneous control of both parameters. In addition, the review explores strategies to minimize the number of devices required to change the length of members (actuators) for modulation and to optimize their positioning to achieve superior structural control. Furthermore, various types of actuators were elaborated, highlighting their respective merits and drawbacks.

A comprehensive review of structural weight optimization, considering shape, stress, and stability, is detailed in **Paper II**. The study discussed optimization techniques for topology, sizing, and shaping, as well as multi-objective optimization. Trends and innovations that promote the development of more efficient, cost-effective, and reliable structural designs were identified. Furthermore, the review discussed emerging innovative techniques, particularly the use of efficient algorithms and machine learning in conjunction with various optimization methods. The investigation also demonstrated how the optimization algorithms minimize the structural weight over several iterations. The investigation of optimization in trusses, frames, tensegrity, reinforced concrete, origami, scissors, and adaptive structures was presented and analyzed.

The main purpose of **Paper III** is to simultaneously control deformation and twisting moment in a single-layer egg-shaped space frame. Two methodologies were proposed: (i) to

mitigate deformation and twisting moment through bar length change, and (ii) to mitigate deformation by adding extra members, followed by shape and torsional control by bar length change. A computational algorithm was designed to minimize the induced torsional moments by restoring the shape within a predetermined limit, adjusting the lengths of the active bars incorporated into the space frames. The objective function (total bar length change) and the constraint functions (displacement and torsion) were optimized using different techniques, such as linear programming and Sequential Quadratic Programming (SQP), to minimize the total bar length change.

Since the relationship between bar length change and nodal displacements in scissor structures is complicated, and controlling several nodes simultaneously is challenging, it requires a large amount of bar length change and many turnbuckles. Therefore, the primary objective of **Paper IV** is to propose an efficient technique that effectively controls nodal displacements of scissor structures within the desired limit, significantly reduces the amount of bar length change, and eliminates ineffective members that require turnbuckles. A technique was developed and coupled with optimization algorithms that can control all nodes of the structure in a single step by assigning domains to the nodal positions after a change in bar length. The efficiency of the approach was verified by comparing it with the standard method described in the literature and by experimental test results.

In loaded scissor structures, the threat of significant bending moment in macro elements is the dominant factor and has a significant impact. For this purpose, the study in **Paper V** is established, with two main objectives. First, a sensitivity analysis was conducted to assess the effectiveness of mitigating deformations and bending moments in scissor structures by adding cables at specific locations. Second, a novel formulation was proposed to mitigate bending moments within a desired domain across the entire structure by adjusting the lengths of certain active members. Geometric nonlinearity was also considered by applying load and bar-length changes in iterations and updating the structural configuration at each iteration. Additionally, the techniques considered in **Papers IV** and **V** were used, based on condensed matrices of FM, coupled with interior-point optimization to minimize the total change in bar length.

For large deformations, linear elastic methods may be inadequate for convergence; therefore, it's essential to consider geometric nonlinearity. Optimizing nonlinear structures is also computationally intensive. Although gradient-based methods offer faster convergence, they often rely on costly and error-prone numerical differentiation. That is why the aim of **Paper VI** is to introduce a robust gradient-based optimization technique integrated with the

nonlinear finite element method to minimize total actuation while controlling deformed shape and stress in geometrically nonlinear large-scale flexible trusses. The analytical gradients of the constraint functions (displacement and stress) with respect to the actuation (design variables) were derived to enhance the computational efficiency of the technique. The objective includes satisfying constraints on actuation bounds and target nodal displacements.

A new and robust computational technique that employs analytical gradients of the functions to minimize the total structural weight of large 3D trusses is the aim of **Paper VII**. The optimization technique accommodated: (i) cross-sectional area as a design variable at each element of the structure, (ii) predefined limits for nodal displacements and stress in elements, and (iii) nonlinear stability for configurations susceptible to snap-through phenomenon. These constraints were systematically and concurrently assessed by integrating path-following finite element analysis with the null vector method. A comprehensive analytical evaluation of the gradient for nonlinear deformation, stress, and stability constraints was presented. The evaluation was effectively incorporated within the finite element solver, thereby enhancing the robustness and efficiency of the optimization process. Examples of validation range from straightforward benchmarks to grand-scale structures were presented.

Regarding structural optimization in frames, the analysis in **Paper VIII** aimed to reduce the structural weight of a single-layer egg-shaped frame system by adding extra members. The paper aims to reduce total structural weight by analyzing the mitigation of bending moments in structural members through the strategic inclusion of extra members at their optimal locations. Instead of enlarging the sizes of the initial members to resist the moment, eight extra members were placed at optimally determined locations to reduce the bending moments in the structural members.

3. Methodology and Modeling

3.1. Review on structural control and actuators (Paper I)

3.1.1 Research classification

The structural control techniques were classified into three general categories: shape control, stress control, and simultaneous control of both parameters. This enables the detailed thematic exploration of each control technique. In each category, research was analyzed with regard to its basic approaches, ranging from analytical through numerical simulations to experimental validation. This classification enables better comparisons of methodologies, efficiency, and fields of application across various disciplines.

3.1.2 Actuator analysis and optimization

An essential aspect of the methodology in this review paper was the survey and study of actuator optimization, which involves devices used for controlling structures. Mechanical, piezoelectric, thermal-based, and SMA actuators were considered. Each of them was analyzed in terms of its practicality, efficiency, and limitations. This comparison concerned practical compromises in actuator selection for various structural systems. The study focused on methods to minimize the number of actuators by exploring how the number and placement of actuators affect system performance and costs. The investigation considered various optimization algorithms employed in the selected studies, including the GA, Simulated Annealing (SA), and Interior-point methods. The efficiency of these algorithms, as measured by their ability to reduce the cost of structural control in terms of actuation and actuator placement, was discussed.

3.2. Review on structural weight optimization (Paper II)

The latest advancements in structural weight optimization were reviewed. The overview covered studies from the past twenty years, highlighting techniques, developments, and their applications to various structures.

3.2.1 Scope and classification

The study focused on four types of optimizations, namely, topology, size, shape, and multi-objective optimization. Topology optimization determines the optimal material distribution throughout a specified domain, whereas size optimization seeks the optimal cross-sectional areas. Shape optimization seeks to determine the optimal member shape that achieves

a stable stress distribution. While multi-objective optimization involves one or more objective functions, the Pareto front is used to achieve optimality [86].

Another classification was made in the level of structural types and their behavior. The study encompassed various popular structural types, including trusses, frames, and tensegrity systems. It also classified their behavior in terms of geometric nonlinearity and plasticity. The review focused on the powerful optimizations and evaluated them, discussing their applications to structural types.

3.3. Deformation and twisting moment mitigation of space frames (Paper III)

3.3.1 *Method formulation*

Using the SVD-FM, internal forces and deformations were analyzed. Then, the **Y** and **Z** matrices, which relate changes in bar length to deformation and internal forces, were established.

3.3.2 *Numerical model*

A single-layer, egg-shaped frame structure was considered in the analysis (see Figure 1). It was modeled with 90 nodes and 176 members, including eight additional members. The modulus of elasticity of the members was 200 GPa, and the diameter was 6 mm. The structure was constrained against translation at 9 bottom nodes, with a 60 N downward load applied to nodes 1-9 and 18-25. Two cases were studied. The first concerned mitigating deformation by varying the bar length, then examining its effect on the twisting moment, and finally controlling both nodal displacement and the twisting moment by varying the bar length. The second case was focused on adding 8 extra members (see Figure 1) to mitigate deformation and monitor the twisting moment, then control both nodal displacement and twisting moment simultaneously through the bar length change.

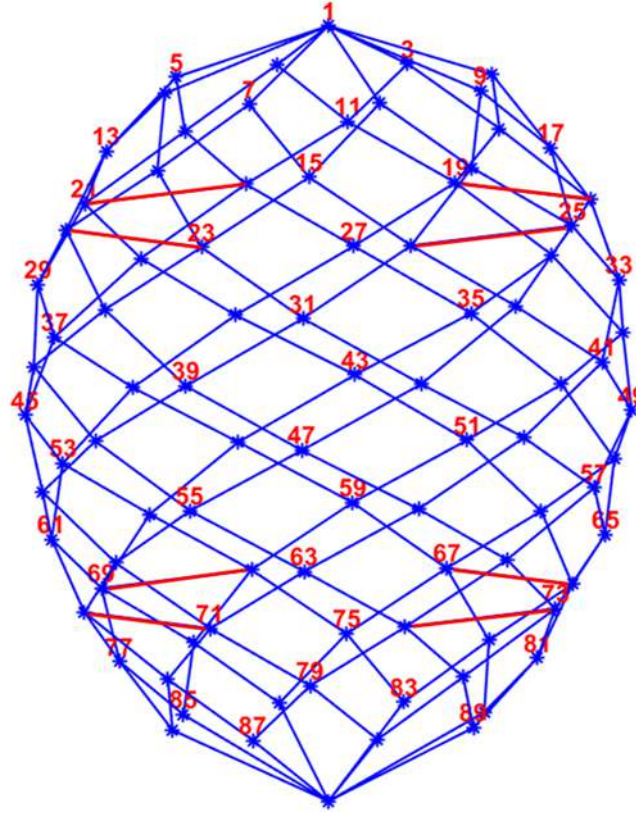


Figure 1 The single-layer egg-shaped structure and its odd joint numbers (**Paper III**).

3.3.3 Optimization of actuation

The aim of employing optimization algorithms was to minimize the total change in bar length and eliminate members equipped with turnbuckles that require actuation of less than 0.1 mm, thereby reducing the number of turnbuckles. Two MATLAB functions were used to determine the optimal actuation for controlling deformation in Case 1. Interior-point algorithm and SQP were considered and compared.

3.4. Optimal displacement control in scissor systems (**Paper IV**)

3.4.1 Scissor structures

The optimal displacement control for scissor structures was investigated through numerical analysis and experimental tests. A morphing system with 8 scissors elements was used in the study. The model consisted of 15 macro elements connected with a shear connector, each of them formed by two members, as illustrated in Figure 2. The location of the shear connector varies based on the structural configuration stages of the system. Each macro element contained two connected, intact bars, with their ends pinned and linked to another scissor or a constraint or free at the end.

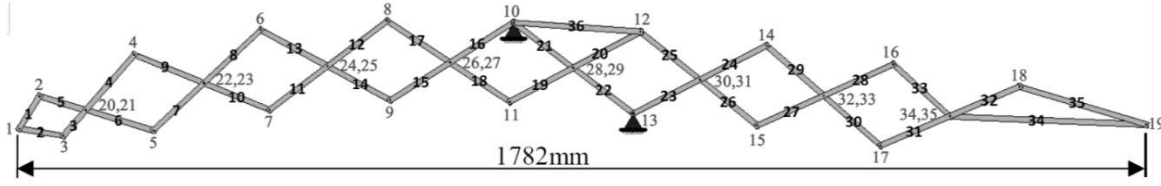


Figure 2 The morphing (aerofoil) structure (*Paper IV*).

3.4.2 Method formulation

The research was based on the SVD-FM, while the equilibrium, compatibility, and flexibility matrices were condensed to reduce complexity and degrees of freedom. The shear connectors were assumed to be unloaded, meaning that only pin joints were loadable. The relationship between bar length change and nodal displacements was established using the condensed matrices. The objective function, representing total actuation, was minimized using an interior-point optimization algorithm. The displacements were constrained to be kept within a predefined limit. The results of the numerical analysis were verified by comparing them with the referenced literature and the experimental test results.

3.4.3 Experimental setup

The physical model shown in Figure 3 was prepared for testing. For each test, the new configuration was used, accounting for slight changes in loading and unloading, bar-length changes, and reassembly of the system. The structure was composed of 9 pin joints, 3 shear connectors, and 15 individual members. The members at the joints were connected with a 4 mm stainless steel (the members had no friction at the joints). The members featured a consistent cross-section of 6×9 mm and exhibited a modulus of elasticity measuring 70 GPa and a yield strength of 276 MPa. The mechanical actuators were equipped with the members that needed to be actuated. To measure the nodal displacements, a high-speed camera and Auto CAD software were used. In three stages (before loading, after loading, and after actuation), photos of the structure were taken and imported into the software to measure differences in nodal positions. A right-angle ruler was fixed close to the two reference points (hinged joints) of the structure (see Figure 3) to scale the photo after its model was transferred to the software.

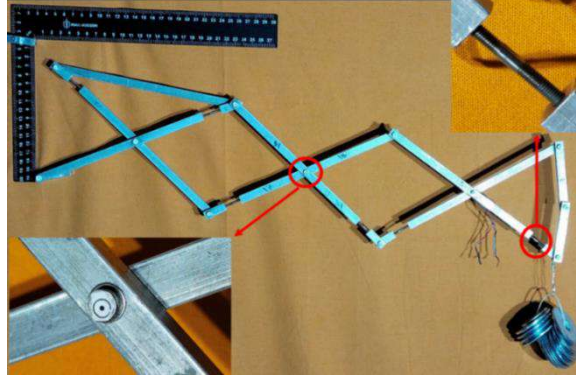


Figure 3 The cantilever pantographic structure (**Paper IV**).

3.4.4 Interior point algorithm

The interior-point optimization algorithm was used to minimize the total change in bar length and to eliminate inactive members that require only minor actuation. Furthermore, the algorithm allowed the domain of the targeted displacements to be specified as inequality constraints.

3.5. Deformation and moment mitigation in scissor structures (**Paper V**)

Two novel solutions were presented to mitigate deformations and bending moments in scissor structures. It should be noted that, due to the structural flexibility, loading and bar length changes were applied in a quasistatic manner to account for geometric nonlinearity.

3.5.1 Adding cables

A sensitivity analysis was conducted to determine the optimal cable placement in tension zones. Cables were used to reduce nodal displacements and, therefore, deformations and bending moments.

3.5.2 Novel formulation

A novel formulation based on the condensed matrices of the FM was proposed to modify bending moments in scissor structures by changing the lengths of certain active members. The formulation is defined as: $\mathbf{bm}_{\min} \leq \mathbf{bm}_c + \mathbf{Z}_c \mathbf{e}_{oc} \leq \mathbf{bm}_{\max}$, where, $\mathbf{bm}_{\min}$ and $\mathbf{bm}_{\max}$ are the minimum and maximum allowable bending moments for all members, $\mathbf{Z}_c$ is the matrix that relates the bending moment in macro elements to the axial bar length change ($\mathbf{e}_{oc}$); the details of deriving $\mathbf{Z}_c$ are given in **Paper V**. Finally, $\mathbf{bm}_c$ is the bending moment due to loading before applying the member length change.

3.6. Optimal Shape and stress control of geometrically nonlinear trusses (Paper VI)

3.6.1 The relation between the force and displacement methods

In the previous studies (**Papers III-V**), the techniques based on the FM were considered. In **Papers VI - VII**, a methodology was developed based on the DM. FM solves internal forces from flexibility and compatibility matrices, while DM solves deformations from the stiffness matrix and equilibrium conditions.

3.6.2 Structural modeling and formulations

A gradient-based optimization method for designing actuation systems that can achieve desired shape and stress control in geometrically nonlinear structures was presented. The Green-Lagrange strain, ε_d , with geometric nonlinearity was considered, defined as:

$$\varepsilon_d = \frac{1}{2} \left(\frac{l_e^2 - L_e^2}{L_e^2} \right), \text{ where, } l_e \text{ and } L_e \text{ are the deformed and initial element lengths, respectively. } \varepsilon_d$$

can also be defined as: $\varepsilon_d = \varepsilon + \varepsilon_o + \varepsilon_a$, where ε is the elastic strain, ε_o is the initial strain and ε_a is the actuation-induced strain (the optimization variable). The axial force, $\mathbf{N}$, in the element level is obtained as: $\mathbf{N} = EA(\varepsilon_d - \varepsilon_o - \varepsilon_a)$, where E is the Young's modulus, and A is the cross-sectional area of the element. Internal force vector ($\mathbf{s}_e$) and stiffness matrix, $\mathbf{K}_e$, in the element level are obtained from the strain energy, and the global nonlinear equilibrium condition is: $r(\mathbf{d}, \lambda) = \mathbf{s}(\mathbf{d}) - \lambda \mathbf{p}$, where r is the nonlinear vectorial function of the vector $(\mathbf{d}, \lambda)$ collecting $\mathbf{d}$ and λ which is the increment of applied load, $\mathbf{P}$. The incremental iterative solution is obtained through $\mathbf{K}(\mathbf{d}_j) \Delta \mathbf{d} = -r(\mathbf{d}_j, \lambda)$, where $\mathbf{K}$ is the tangent stiffness matrix that is factorized at each iteration.

3.6.3 Problem optimization

The objective function is described as: $\min \sum_{i \in v} |a_i|$, where a_i is the amount of actuation in each element, and v is the subset of the actuated elements. The constraints include nodal displacements and axial forces in structural members, as well as the actuation bounds.

3.6.4 Exact gradient computation via implicit differentiation

An innovative method for analytically differentiating the nonlinear finite element

solution, considering both displacement and stress with respect to the actuation input parameters, was introduced. Numerical differentiation involved many expensive Finite Element (FE) analyses, which are also sensitive to round-off errors and are inefficient for large-scale problems. A closed-form analytical gradient was derived using implicit differentiation of the equilibrium condition $\mathbf{K}(\mathbf{d}(a))\nabla_a \mathbf{d} = -\frac{\partial \mathbf{s}(\mathbf{d}, a)}{\partial a}$. It should be underlined that $\mathbf{K}$ is already

available and $\frac{\partial \mathbf{s}(\mathbf{d}, a)}{\partial a}$ is computationally cheap, where $\nabla_a \mathbf{d}$ is the gradient of the global

displacement vector. The gradient of stress was computed as:

$$\nabla_a \mathbf{N}_e = \frac{EA}{L_e} \mathbf{q}_e(\mathbf{d}_e(a))^T \nabla_a \mathbf{d}_e + \frac{\partial \mathbf{N}_e(\mathbf{d}_e, a_e)}{\partial a_e}.$$

3.6.5 Optimization algorithm

SQP was employed in the study. It solves a sequence of quadratic problems that use the exact gradients of the constraints, thereby achieving substantially faster convergence than gradient methods, which rely on numerical differentiation, and gradient-free methods, such as GA. They are robust to nonlinearities due to accurate gradients.

3.7. Structural weight optimization of geometrically nonlinear trusses (Paper VII)

Similar to **Paper VI**, the methodology of the research described in **Paper VII** was based on DM. The work considered a gradient-based optimization framework for weight minimization in geometrically nonlinear truss structures, incorporating displacement, stress, and stability constraints.

3.7.1 Optimization problem formulation

The objective function concerned the minimization of the total structural weight through the minimization of the cross-sectional area, A_e , of the members through $\min W(A) = \sum_{e \in \mathcal{U}} W_e \equiv \rho A_e L_e$ subjected to:

$$\begin{aligned} \text{-area bound} & \quad A_e^{(\min)} \leq A_e \leq A_e^{(\max)} \quad e \in \mathcal{V} \\ \text{-stress constraints} & \quad \sigma_{(i)}^{(\min)} \leq \sigma_i(A) \leq \sigma_{(i)}^{(\max)} \quad i \in I \\ \text{-displacement constraints} & \quad \mathbf{d}_k^{(\min)} \leq \mathbf{d}_k(A) \leq \mathbf{d}_k^{(\max)} \quad k \in K \\ \text{-stability constraints} & \quad \lambda_c(A) \geq 1 \end{aligned}$$

where, ρ is the material density, σ is the stress in the members, subsets $\mathcal{V}$, I , and K represent,

respectively, the design elements, the elements where we aim to control the axial stress σ_i , and the controlled nodal displacement components $\mathbf{d}_k$ selected from those within the global DOFs vector $\mathbf{d}$.

3.7.2 Exact gradient computation via implicit differentiation

Gradients of displacement and stress, which are functions of the design variable, were obtained from the implicit differentiation of the nonlinear equilibrium condition. For the displacement gradient $\mathbf{K}(\mathbf{d}(A))\mathbf{d}_{,A} = -\frac{\partial \mathbf{s}(\mathbf{d}, A)}{\partial A}$, here $\mathbf{K}(\mathbf{d}(A))$ is the tangent stiffness matrix at the final iteration of the structural equilibrium Newton solver, which is the Jacobian of the internal force vector with respect to the displacements of the currently assigned A ($\frac{\partial \mathbf{s}(\mathbf{d}, A)}{\partial \mathbf{d}}$).

Subsequent to determining the gradient of displacements, the gradient of the axial stress σ_i in the generic controlled bar i could then be evaluated as $\sigma_{i,A} = E\mathbf{q}_i(\mathbf{d}_i)^T \mathbf{d}_{i,A}$, where

$$E\mathbf{q}_i(\mathbf{d}_i)^T = \left(\frac{\sigma_i(\mathbf{d}_i, A)}{\partial d_i} \bigg|_{\mathbf{d}_i(A)} \right)^T. \text{ The details of the gradient of the nonlinear stability constraint}$$

were discussed in **Paper VI**.

3.7.3 Optimization algorithm

SQP was used to solve nonlinear problems by utilizing the exact gradient to achieve accurate results.

3.8. Optimizing structural weight in space frames (Paper VIII)

3.8.1 Model description

The optimization of the structural weight of a 3D egg-shaped, single-layer space frame was conducted (see Figure 4). The structure consisted of 176 steel bars and 90 joints, with a total height of 5000 mm. The system was restrained against translation at 9 bottom joints, and the members had a modulus of elasticity of 200 GPa. Five different load scenarios were tested: two horizontal, two vertical, and a combination of horizontal and vertical loading. The details are given in **Paper VIII**. Ten cases with different locations of extra bars were investigated for each load scenario to determine the optimal configuration that yields the smallest bending moment, thereby achieving the lightest structural weight. The extra members were attached as circular ties at various locations within the egg-shaped system. MATLAB and SAP2000

software were used in the analysis.

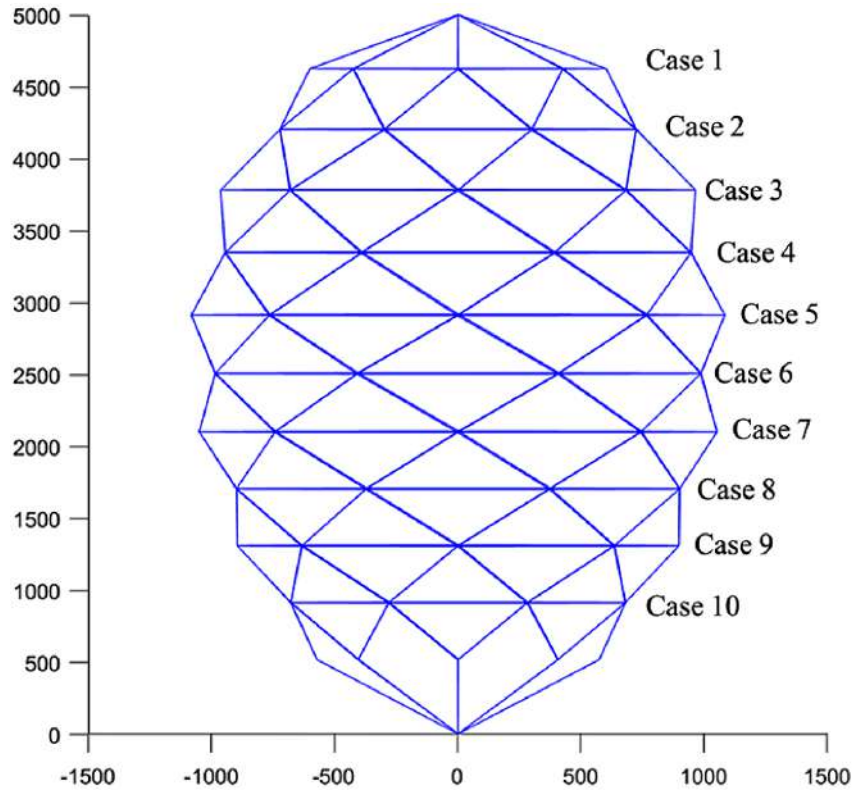


Figure 4 Egg-shaped space frame with 10 different cases of extra bars (**Paper VIII**).

3.8.2 Optimal structural weight

The initial structural design was established based on the maximum bending moment (M_{maxi}) in each i -th case for the egg-shaped space frame with no extra members as presented in Figure 4. The section modules (sx_i) were determined from the relation: $sx_i = M_{maxi} / f_y$, where f_y is the material's yield strength. The diameter of the bars, D_i , for each case was determined based on the relation: $D_i = 2Iy_i / sx_i$, where, Iy_i is the moment of inertia. Then, the total structural weight for each case was obtained from $W = A_i * \gamma * \sum L$, where, A is the designed cross-sectional area, γ is the material's density, and L is the length of the members.

3.8.3 Loading scenarios

- Horizontal loading of 60 N at joints:
 - 18-21
 - 26-29
- Vertical downward loading of 180 N at joints:
 - 1, 10-17

- 1, 18-25
- Vertical loading of 180 N at joints 1-9 and horizontal loading of 60 N at joints 18-21 simultaneously.

4. Results and discussion

4.1. Deformation and twisting moment mitigation of space frames (Paper III)

The simultaneous control of deformation and twisting moment in space structures was analyzed in **Paper III**. The single-layer egg-shaped frame structure was considered as an example. Two cases were investigated. The first case involved controlling the displacement and twisting moment without regard to each other through bar length change. It was found that controlling deformation only increases the twisting moment. Therefore, in the second case, the displacement was reduced by adding eight extra cables, and both deformations and twisting moments were controlled simultaneously.

4.1.1 Case 1

In this case, the structure underwent significant deformation under the given loading, and an attempt was made to reduce the deformation by adjusting the length of the active members, so that the deformation did not exceed 10 mm. After applying actuation, the deformation reduced to 10 mm, but a noticeable increase in torsion was observed in some members, specifically from 500 N·mm to approximately 2000 N·mm, i.e., by 75%. Another attempt was made to control only the twisting moment, aiming for a peak value of 400 N · mm, but this increased the deformation.

4.1.2 Case 2

In this scenario, the deformation was mitigated by adding eight extra members at the quarter bottom and top of the structure, which caused a significant reduction in nodal displacement from 13.41 mm to 8.7 mm, i.e. by 35%. On the other hand, the twisting moment in some members reached 576 N · mm. Similar to Case 1, an attempt was made to keep the nodal displacement and twisting moment within the ranges of 10 mm and 400 N · mm, respectively.

4.1.3 Optimization performance

Linear and nonlinear programming, using the interior-point and SQP methods, were performed to analyze Case 1. All three trials converged on the same actuation (40.27 mm) but required different numbers of iterations and computation times. Linear programming using the interior-point approach required 10 iterations and 0.05 seconds to converge, while nonlinear programming using the interior-point and SQP algorithms took 28 iterations and 0.15 seconds

and 15 iterations and 0.03 seconds, respectively.

4.2. Optimal displacement control in scissor systems (Paper IV)

4.2.1 Comparative studies

The structure shown in Figure 2 was analyzed in **paper IV** using the proposed approach, and the results were compared to the methods described in the literature [87]. Comparison of four case studies was conducted, and the results of the proposed approach were by far better than the method in the literature, as illustrated in Figure 5. Considering Case 3, the method in the literature utilized 6 actuators and 13.96 mm of actuation, whereas the method proposed in this thesis employed only 4 actuators and 4.53 mm of actuation to achieve the target.

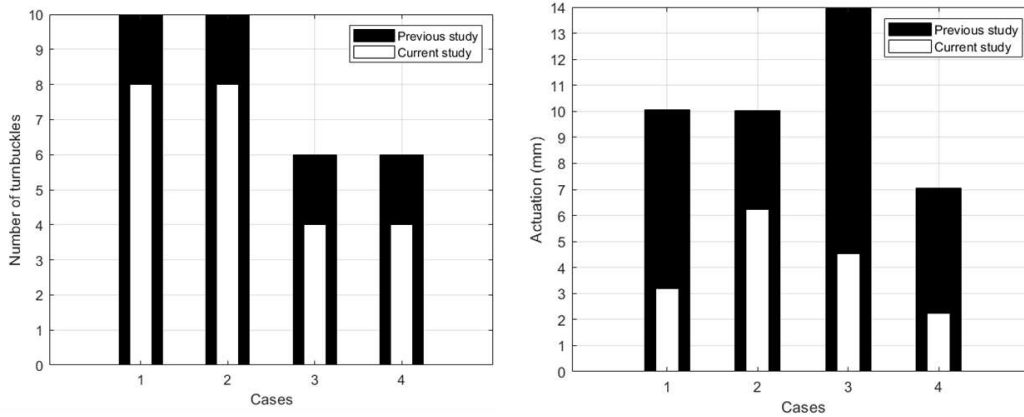


Figure 5 Comparison of the proposed method and the technique in the literature in the reduction in the amount of actuation and the number of actuators (**Paper IV**).

4.2.2 Experimental tests

As a result of loading the structure shown in Figure 3 at joint no. 8 with the downward force of 10 N, the vertical displacement (d_{pc}) of the loaded joint reached 6.05 mm. The goal is to nullify ($d_c=0$) the vertical displacement of joints 7-9. It can be seen that the displacement after applying actuation (d_a) at joints 7-9 became zero by shortening member no. 4 by 0.51 mm and lengthening member no. 7 by 1.56 mm. The results of the experimental and numerical tests are presented in Table 1 and the structural configuration in three different stages is illustrated in Figure 6.

Table 1 Controlling the vertical displacement of three nodes

Joints	Direction	$\mathbf{d}_{pc}$ (mm)			$\mathbf{d}_c$ (mm)	$\mathbf{d}_a$ (mm)			Members	Actuation (mm)
		Numerical				Numerical				
		Exp.	Lin.	NL.		Exp.	Lin.	NL.		
7	Y	-3.95	-3.88	-4.25	0.00	-0.00	-0.03	-0.33	4	-0.51
8	Y	-6.05	-5.92	-6.85	0.00	-0.00	-0.05	-0.48	7	1.56
9	Y	-5.52	-5.38	-5.90	0.00	-0.00	-0.06	-0.43	-	-
Total actuation (mm)										2.07

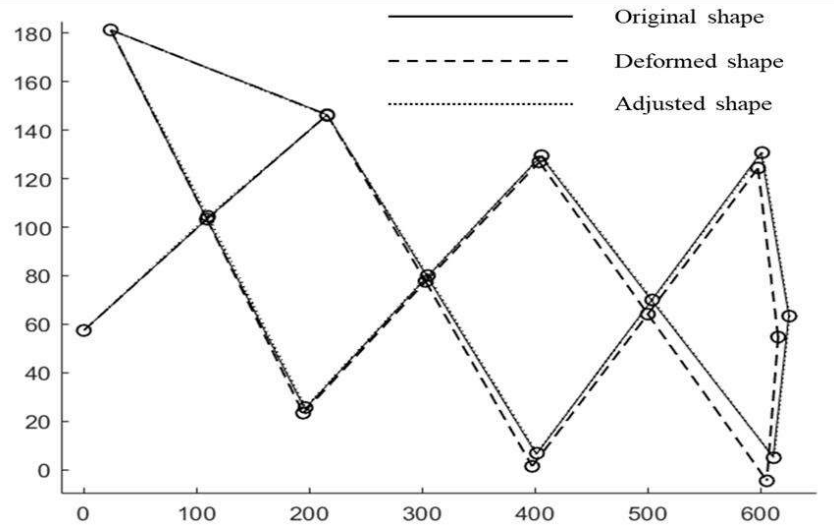


Figure 6 Configuration of the cantilever scissor system in three stages (Paper IV).

4.3. Deformation and moment mitigation in scissor structures (Paper V)

In this paper, two strategies were considered to reduce deformation and bending moment. First, cables were attached in tension zones. Second, a novel formulation was applied to further reduce and control the bending moment within the specified range by adjusting the length of some active members.

4.3.1 Moment and deformation mitigation by attaching cables

The results of a one-bay cantilever (Ex. 1), three-bay cantilever (Ex. 2), three-bay simply supported scissor structure (Ex. 3), and a large-scale morphing system (Ex. 4) were presented in **Paper V**. The maximum bending moments, nodal displacements and axial forces in cables of the analyzed scissor structures before and after attaching cables are presented in Table 2. The results show a substantial reduction in bending moment and deformation when the cables are attached at the optimal places. For Ex. 1, the reductions in both the maximum bending moment and the deformation are approximately 90% when cables 1 and 2 are attached simultaneously. The reductions in maximum bending moment and displacement reach 31.2% and 43.9% for Ex. 2; 85.6% and 89% for Ex. 3; 38.5% and 59.4% for Ex. 4, respectively. The

axial force in the cables is maximum in the case when the maximum reduction in the bending moments occurred. It can be seen that placing cables in the opposite direction of the displacement is effective for mitigating the bending moment and deformation.

Table 2 Maximum bending moments, cable axial force, and nodal displacements of scissor structures before and after attaching cables, and the percentage of change

Example	Case	Max M before (N.mm)	Max M after (N.mm)	Change %	Max disp before (mm)	Max disp after (mm)	Change %	Max cable force (N)
Ex1	Cable1	-2241	-1151	-48.6	-0.73	-0.37	49.3	0
	Cable2	-2241	-2241	0	-0.73	-0.73	0	12
	Cable1,2	-2241	-225	-90	-0.73	-0.073	90	22.5
Ex2	Cable1	-6622	-5586	-15.6	-13.55	-12.77	-5.7	11.8
	Cable2	-6622	-6622.3	0.0	-13.55	-10.72	-20.9	47.4
	Cable1,2	-6622	-4557.1	-31.2	-13.55	-7.60	-43.9	69.8
	Cable1,3	-6622	-6665.9	0.7	-13.55	-11.66	-14.0	23.9
	Cable1,4	-6622	-5789.2	-12.6	-13.55	-12.55	-7.4	13
	Cable1,2, 3	-6622	-4644.1	-29.9	-13.55	-7.59	-43.9	68.8
Ex3	Cable1	2253.1	3148	39.7	-1.46	-1.17	-19.9	29.9
	Cable2	2253.1	3148	39.7	-1.46	-1.17	-19.9	29.9
	Cable1,2	2253.1	1025	-54.5	-1.46	-0.26	-82.2	40.8
	Cable1,2, 3	2253.1	325	-85.6	-1.46	-0.16	-89.0	44.5
Ex4	Cable 1	-4975	-3060.1	-38.5	-15.55	-6.32	-59.4	14.3

4.3.2 Moment control by bar length change (actuation)

The results of applying the presented novel formula for controlling the bending moment within the specified value were presented in **Paper V** for a one bay scissor structure (Ex.5), and a large-scale morphing system (Ex. 6). Ex. 5 had the same geometry and properties as Ex. 1, and it was loaded by the force of 100 N. In addition, the simply supported scissor system (Ex. 3) with the same loading and geometry was also tested for further mitigation in bending moment using the proposed method. Ex. 6 was the left part of the morphing system in Ex. 4, with the same geometry and material properties, and all top-free nodes of Ex. 6 were loaded with downward forces of 3 N.

Table 3 shows the maximum bending moment, nodal displacement, and cable axial forces for the three examples (Ex. 5, 3, and 6) for cables before actuation, and for the system with cables after actuation. The results show that, for Ex. 5, the reduction in bending moment after applying actuation to cables 1 and 2 is 90%, with displacements of 0.13 mm and 0.15 mm, respectively. For Ex. 3, a 75% reduction in bending moment was recorded. For Ex. 6, the reduction in bending moment is 3.9% after applying only 0.078 mm of actuation to cable 2. It

can be observed that a small change in the length of members causes a significant change in the bending moment of elements. Using the proposed method to determine the amount of length change and to choose which members' length should be changed is essential. After applying actuation, displacement reductions of 89% and 66.8% were recorded for Ex. 5 and 3, respectively. At the same time, there were increases by 10%, 7.8%, and 2.4% in cable axial forces after actuation.

Table 3 Maximum bending moment, cable axial force and nodal displacements of scissor structures before and after attaching cables and after changing bar lengths (actuation)

Example	Optimal cable layout	Moment (N · mm)			Displacement (mm)			Cable axial force (N)		
		After adding cable	After actuation	Change %	After adding cable	After actuation	Change %	After adding cable	After actuation	Change %
Ex. 5	1&2	-893	-80	-91	-0.29	-0.03	-89.6	90	99	10
Ex. 3	1,2&3	325	80	-75	-0.16	-0.05	-68.7	44.4	48.2	7.8
Ex. 6	1&2	-15831	-15200	-3.9	-18.21	-18.38	1.1	88	90.1	2.4

4.4. Optimal Shape and stress control of geometrically nonlinear trusses (Paper VI)

The applications and validations of the proposed robust gradient-based optimization method for shape and stress control were analyzed in **Paper VI** through several benchmark problems. The results for a saddle-shaped cable net structure (Ex. 1), a space cable structure (Ex. 2), a 2D cable net (Ex. 3), a deployable space truss (Ex. 4), and a large plate-like space truss (Ex. 5) were presented. The efficiency, accuracy, and computational effectiveness of the proposed method were evaluated and compared with those of methods described in the literature, and the benefits of the analytical gradient over numerical differentiation were also examined.

4.4.1 Comparison with methods in the literature

For Ex. 2, which was studied by Yuan *et al.* [61], the reduction in total actuation using the proposed method was 1.7% larger, as compared to the results presented in [61]. For Ex. 3, Xu and Luo [88] attempted to impart a displacement of $[-20, 20]$ mm at joint no. 6 while maintaining stresses near their initial levels, and the displacement target was precisely achieved. The proposed method required 28.7% less actuation than using the approach discussed in [88]. The average error in matching the initial stress denoted in [88] was 9.3%, while it was only 3% when the proposed method was used. The maximum error denoted in [88] was 21.8%, whereas the results of **Paper VI** showed the maximum error of 11.5%. In the case of Ex. 4, Kwan and Pellegrino [19] and Abdulkarim and Saeed [89] attempted to control

tension in cables 3, 5, 7, 8, and 11 at 2 N, and the remaining cables at 1 N, while controlling the stress in compression members at -1 N. The proposed method achieved the goal with 2.23%, and 1.07% less actuation than in [19], and also in [89].

4.4.2 Comparison of convergence for numerical and analytical gradients

A comparison of the computational efficiency between implementing the numerical and analytical gradients was conducted, and the results are shown in Table 4. The table shows significant differences in convergence speed between the numerical and analytical gradients. For Ex. 1, 2, and 3, the analytical gradient converged after performing up to 68.9%, 82.9%, and 85% fewer analyses than the numerical gradient.

Table 4 Number of structural analyses performed for various structures till convergence using numerical gradient (numerical) and the analytical gradient (analytical)

Example	Shape		Stress		Shape & stress	
	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical
Ex. 1	29	9	440	265	522	251
Ex. 2	41	7	47	21	-	-
Ex. 3 Case 1	-	-	-	-	3017	60
Ex. 3 Case 2	-	-	-	-	221	11
Ex. 4	-	-	40	6	-	-
Ex. 5	-	-	-	-	149	9

4.5. Structural weight optimization of geometrically nonlinear trusses (Paper VII)

Several benchmark examples, such as the two-leg truss (Ex. 1), four-bar 3D truss (Ex. 2), and thirty-bar dome (Ex. 3), were tested in **Paper VII** for stability constraints. For displacement constraint, 10-bar truss (Ex. 4), 25-bar truss (Ex. 5), and 52-bar dome (Ex. 6) were examined, and the results were compared to the previous methods known from the literature, including recently developed machine learning techniques. Table 5 shows a comparison of the results (considering stability and displacement constraints) using the proposed method and other methods from the literature. It clearly demonstrates the superiority of the proposed method over other methods in the structural optimization of geometrically nonlinear trusses. One can see that, in almost all examples, the proposed method achieved a lower structural weight at a more cost-effective rate than methods described in the literature. Comparing the proposed method's efficiency with a machine learning technique, it was observed that the machine learning technique achieved the optimal solution for Ex. 5 with 505.49 kg, evaluating 18120 structural analyses in 2748 seconds, while the proposed method

achieved the optimal solution of 501.35 kg, evaluating only 24 structural analyses in 1.7 seconds. It can be concluded that the proposed method needs 99% less time than the method in [69].

Table 5 Comparison between the results obtained using the method proposed and other methods from the literature.

Example	Ref	Constraint	Weight (kg)		Iterations		No. of structural analyses		Time (sec)	
			Ref	This study	Ref	This study	Ref	This study	Ref	This study
Ex.1	[73]	Stability	0.2484	0.2458	5	2	-	5		
Ex. 2	[90]		52.19	52.19			-	20		
	[91]		347.54				-			
Ex. 3	[73]		347.78	331.20	-	-	-	13	-	-
	[90]		347.00				-			
	[69]		332.77				4120			
Ex. 4	[73]	Displacement	4318.00	4288.40				81		
	[92]		507.00	501.35			63			
Ex. 5	[69]		505.49				18120	24	-	1.7
	[92]		5161	2143			-	13	-	1.6
Ex. 6	[69]		2142.4				14100		3948	

4.5.1 Comparison of the number of structural analyses of the proposed method with numerical and analytical gradients.

A comparison was also conducted in **Paper VII** between the implementation of numerical and analytical gradients for eight examples. Ex. 1-3 considered structural stability, while Ex. 4-6 took displacement into account as in the previous subsection. Again, Ex. 3 was tested with both displacement and stability considered as constraints simultaneously. Ex. 7 considered displacement and stress; it was examined to demonstrate the efficiency of the proposed method for large-scale structures. Table 6 tabulates the number of structural analyses and the time required to achieve the optimal solution for each example using the numerical and analytical gradients. It can be seen from the table that, in some cases, for example, Ex. 4, using analytical gradients requires 78% fewer structural analyses than numerical gradients. For Ex. 7 implementing analytical gradients converged after evaluating 264 structural analyses in 513 seconds, while implementing numerical gradients converged after evaluating 701 structural analyses in 1621 seconds. The results indicate that implementing analytical gradients converges much faster than implementing numerical gradients.

Table 6 Comparison of the proposed method efficiency considering numerical and analytical gradients.

Example	Constraint	No. of structural analyses		Time (sec)	
		Numerical	Analytical	Numerical	Analytical
Ex. 1	Stability	8	5	0.07	0.04
Ex. 2		18	10	0.43	0.22
Ex. 3		41	13	10.95	4.05
Ex. 4	Displacement	480	102	22.48	4.71
Ex. 5		120	24	3.85	0.71
Ex. 6		79	13	3.54	0.69
Ex. 3	Displacement and stability	108	44	77.82	32.26
Ex. 7	Displacement and stress	701	264	1621	513

4.6. Optimizing structural weight in space frames (Paper VIII)

4.6.1 Optimal structural weight

The optimal solution for the eleven cases of extra-bar layout on the egg-shaped space frame was analyzed in **Paper VIII**. Table 7 shows the weight reduction for optimal extra-bar locations, as compared to the structure without extra bars (Case 0) across five loading scenarios. The table shows that, for horizontal loading, the optimal case is Case 9, with a 28% weight reduction and a 20% reduction for loadings at joints 18-21 and 26-29, respectively. On the other hand, Case 6 is optimal for vertical loading, with weight reductions of 4% and 6% when joints 1 and 10-17, and 1 and 18-25, were loaded. Finally, Case 7 is optimal for simultaneous horizontal and vertical loading, achieving an 8.5% reduction in total weight as compared to Case 0.

Table 7 Weight reduction for optimal extra-bar locations across five different loading scenarios.

Loading scenario	Optimal case	Weight reduction
Horizontal (joints 18–21)	Case 9	28%
Horizontal (joints 26–29)	Case 9	20%
Vertical (joints 1, 10–17)	Case 6	6%
Vertical (joints 1, 18–25)	Case 6	4%
Combined loads	Case 7	8.5%

4.6.2 Nodal displacement

Aside from weight reduction, the maximum nodal displacement for all optimal cases was smaller than that of the corresponding Case 0. Table 8 displays the nodal displacements at the top joint (joint no. 1) for three loading scenarios. It is clear from the table that the reduction in maximum nodal displacements was 9%, 22.8%, and 19% for horizontal, vertical, and

combined loadings, respectively.

Table 8 Maximum nodal displacement of joint no.1 for different load scenarios

Loding direction	Horizontal		Vertical		Horizontal & vertical	
Cases	Case 0	Case 9	Case 0	Case 6	Case 0	Case 7
Displacement direction	X		Y		Y	
Nodal displacement (mm)	319.13	290.32	-125.22	-106.93	98.32	79.56

5. Conclusions

This PhD thesis studied structural control and optimization applied to flexible structures, such as trusses, frames, and scissor structures. The following main conclusions were derived from the thesis:

- Based on the literature review, most available methods concerning the design of flexible structures assume their linear behavior, which limits their applicability to geometrically nonlinear structures.
- Shape and twisting moment can be controlled simultaneously in space frames with or without extra members by changing the length of certain active members. The proposed method, based on SVD-FM, can effectively control twisting moments and displacements. Linear programming and interior-point methods have been found to be efficient to minimize total actuation.
- The approach proposed in this thesis for controlling nodal displacements in scissor structures yields exceptional results, achieving a 60% reduction in actuation requirements and reducing the turnbuckle by up to 40%, as compared to other techniques described in the literature. This advancement has the potential to significantly improve the efficiency and cost-effectiveness of reshaping scissor structures.
- Bending moments and deformations in structural members can be mitigated by up to 90% through attaching cables in optimal locations and adjusting the length of certain active members. Positioning the cable at the end opposite the joint's displacement direction along the element's axis allows it to engage in tension as the joint moves, helping to resist the resulting bending moment. Slight alterations in the lengths of active members substantially change the bending moment values.
- The shape and stress of geometrically nonlinear trusses, including large systems, can be efficiently controlled through minimal actuation. The total actuation can be reduced by up to 28.7% using the proposed method compared with methods described in the literature. By using the analytical gradient of the constraint functions instead of numerical differentiation, the number of structural analyses can be reduced by up to 85%.
- Using analytical gradients in the structural weight optimization of geometrically nonlinear truss structures significantly outperforms traditional numerical differentiation in terms of both accuracy and computational cost. The proposed method, based on DM, addresses displacement, stress, and stability constraints simultaneously and encompasses non-monotonic

displacement paths. This sets it apart from many existing methods that treat these constraints separately or heuristically. The proposed method achieves an optimal weight resulting in a lighter structure and it's 99% faster than the machine learning technique known from the literature.

- The structural weight of space frames can be substantially reduced by adding cables in optimal locations to mitigate the bending moments. The reduction in structural weight depends on the direction of loading and the location of placement of the extra members, and it can reach up to 28%.

6. Future work recommendations

Different aspects related to the optimization of flexible structures can be further investigated in future studies, as highlighted below:

- It is recommended to extend the studies to consider dynamic loading as well as material nonlinearity in addition to the geometric nonlinearity.
- Another direction for further investigations could be to take into account the effects of material imperfections as well as the imperfections of actuators.
- Incorporating local buckling and flexural behavior by applying the framework to beam and shell elements would enable the treatment of more complex structures.
- It is also suggested to consider employing smart actuators for highly sensitive structures.
- Finally, consideration of the failure modes of actuators can be recommended so as to verify the safety of structures equipped with the devices.

7. References

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Paper I

Paper I: Manguri, A., N. Saeed, and R. Jankowski, *A review: structural shape and stress control techniques and their applications*. Archives of Computational Methods in Engineering, 2025. **32**(2): p. 885-898. <https://doi.org/10.1007/s11831-024-10149-9>



A Review: Structural Shape and Stress Control Techniques and their Applications

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Abstract

This review article presents prior studies on controlling shape and stress in flexible structures. The study offers a comprehensive survey of literature concerning the adjustment and regulation of shape, stress, or both in structures and emphasizes such control's importance. The control of systems is classified into three primary classes: nodal movement control, axial force control, and controlling the two classes concurrently. Each class is thoroughly assessed, showcasing diverse methods anticipated by various scholars. Furthermore, the paper discusses methods to reduce the number of devices (actuators) to adjust and optimize actuators' placement to achieve optimal structural control, considering the cost implications of numerous actuators. Additionally, various actuators are presented in detail, their advantages and disadvantages are also discussed. Moreover, the applications of the presented techniques are reviewed in detail, the essential recommendations for future work are also suggested.

1 Introduction

Shape and/or control of structures is crucial in modern engineering, allowing for the creation of innovative and efficient designs across various fields. The capacity to actively manipulate and adjust structures' shape and/or stress provides unparalleled performance, durability, and safety advantages. Recent progress in materials science and intelligent systems has transformed shape control, leading to exciting applications in aerospace, civil engineering, robotics, and biomedicine. An adjustment involves minor modifications or movements to enhance performance or achieve a desired result. Shape control minimizes or eliminates structural deformation from external disruptions [1]. Consequently, specific structures are intentionally designed with the ability to alter their shapes by adjusting the lengths of components or applying forces to them [2]. Varying bar lengths cause stress changes in indeterminate structures. Thus, internal forces should also be considered.

Structural engineering achieves control or adjustment of structures through altering the length of some bars. These length changings are made possible using embedded devices known as actuators, which produce the necessary length changes [3]. To achieve precise shape control, the devices must be able to lengthen and shorten. "Adjustment" refers to minor joint displacements or axial force changes to achieve the targets. The terms control and adjustment can be used interchangeably in this context. Although the concept of length changing causing changes in shape is straightforward, research on analytical and numerical techniques is limited, especially regarding direct approaches. Former studies in this field can be categorized into three main groups: nodal movement, axial force, and both cases control together.

Thus far, only a few review articles are available in the field of structural shape control; for example, two reviews were conducted in 1999 by Sunar and Rao [4], and in 2002 by Irschik [5], the reviews were dedicated for the studies used piezoelectric actuators. This paper covers the research conducted on shape and/or stress control of structures to date. It also covers the studies about several types of actuators and the research about optimization in the number of actuators and their optimal locations. It's crucial to underscore that for this study, the literature review was conducted using the highly reputable Web of Science database. This database was chosen for its comprehensive coverage and

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rigorous selection process, ensuring the inclusion of only high-quality research. The review focused on the latest advancements in shape and stress control, as well as actuator technologies, specifically for structural applications. To ensure a thorough and comprehensive collection of relevant research, a search strategy combining titles, topics, and keywords was employed. The main key terms used in the search were “shape control”, “displacement control”, “stress control”, “actuators”, and “structures”.

2 Structural Control

2.1 Shape Control

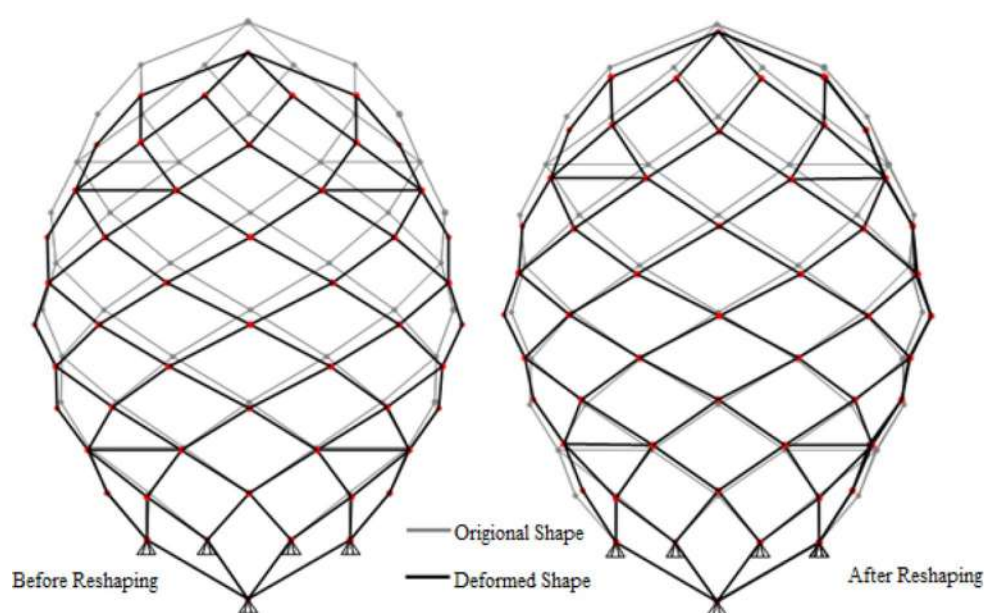
In structural engineering, the configuration of flexible structures holds great importance. Typically, their structural geometry is defined by nodal positions; these structures often support sensitive equipment used in scientific or communication applications, necessitating high geometric accuracy for optimal performance, especially in exacting situations like space [6–9]. The structures in space are intended to be lightweight, leading to high flexibility and shape imperfections from various factors such as manufacturing errors, thermal distortion, and transportation loading. Thus, adjusting these structures with appropriate corrections is crucial to maintain accuracy [10, 11]. The concept of reshaping was initially presented by Weeks [12, 13], who provided explanations for methods concerning the determination and management of static shapes in extensive space systems. Mitigation of static distortion by applying temperature was then developed analytically [14] and computationally [1].

The primary objective of reshaping is to diminish the influence of exterior disturbances on the distortion of the structure through suitable elongation [5, 14]. In cases where shape distortion is undesirable, adjustments to nodal positions are necessary for reshaping, which is done by altering the length of some bars [15, 16]. Shape control is closely related to control engineering and is a branch of structural engineering; an example of shape restoration is illustrated in Fig. 1. One approach to effectively control external displacement is to adjust the length of active members; identifying active bars and determining the required size of actuation for those bars is essential. Researchers like Korkmaz [17] have addressed this issue in several studies on structural control. Additionally, static shape adjustment approaches were studied by Burdisso and Haftka [18], and Ziegler [1].

Over time, more researchers have explored various approaches to shape control. Indirect methods, like Subramanian and Mohan’s successive peak error correction algorithm [20], have proven economical and practical for correcting shape deformation; further improvements were made to the algorithm by Skelton and DeLorenzo [21]. Yoon [22] used finite element analysis (FEA) to control the shape of adaptive one-dimensional structures for performance optimization, and Salama, Umland [23] employed simulated annealing with FEA for controlling a truss system. In another study, piezoelectric sensor distribution was implemented for shape control for intelligent structures based on FEA [24]. In addition, shape-controlling methods have been derived, considering curvatures and slopes as fine-tuning criteria [25, 26] and eigenstrain actuation [27].

Concerning direct methods, Mitsugi, Yasaka [28] studied the direct method of using the generalized inverse of a Jacobian matrix to determine inputs for shape control pertaining

Fig. 1 Reshaping a structure after deformation due to external loading adapted from Saeed, Manguri [19]



to cable length variations. In addition, a direct method based on Maysel's formula was proposed by Nyashin, Lokhov [29] for nodal displacement control of trusses. The surface accuracy method for cable mesh reflectors was presented by Tanaka and Natori [30]. An effective method to control the shape of flexible structures was proposed based on the linear force method [31] and then coupled with optimization algorithms [32]. Tanaka [33] developed the "surface adjustment mechanisms" method, an innovative approach for resembling and correcting the surface deformation of an antenna. This approach creates connections between surface errors in antennas and the resulting alterations in antenna gains caused by deliberate deformations, allowing for correction following the assessment of the initial deformation of the antenna surface based on signal gain impacts resulting from implemented modifications. Tanaka [33] also determined that this method reduced calculation costs and improved applicability. Furthermore, the deformed cantilevers' shape was controlled by employing Timoshenko beam coupling with strain actuation theory using piezoelectric [34] and laminated piezoelectric [35]. Du, Zong [16] suggested an optimization-driven method for modifying the shape of structures.

Regarding shape-controlling applications, it has been utilized in various structures to address shape distortion issues. For instance, Senatore, Duffour [36] conducted intensive studies on adaptive trusses, Yang and Ngoi [37] focused on controlling a beam's shape using piezoelectric. They derived a solution for beam deflection that considered the actuation of piezoelectric and external forces. Although they presented case studies demonstrating the potential of analytical solutions for shape control, they also acknowledged that achieving the desired shape was challenging due to limited actuation [37]. Burdisso and Haftka [18] stated the efficient analysis of shape distortion statistics in antenna structures. For cable mesh antenna reflector surfaces, precise adjustments are required to achieve enhanced amplification or focused signal directionality, enabling the minimizing of ground terminals or capture of faint signals through the alteration of cable length [16]. In addition, the distorted shapes of cable mesh antenna [28, 30, 33, 38–44] for prestressed cable structures [31, 45–47] and plate wings [48] were restored. Trak and Melosh [49] also explored shape control of deployable cable net reflectors based on self-equilibrated stresses, achieving shape control without iteration. They studied the manipulation of a truss's shape by adjusting nodal coordinates to counteract deformation, demonstrating that minor changes in the truss's geometry did not significantly impact the determinate truss bar forces. Further numerical applications of shape control theories were allied to a single-layer egg structure [19] double-layer domes [50, 51].

The number of studies that have been conducted for shape controlling in the disciplines of civil, mechanical and aerospace engineering from 2010 to 2023 are shown in Fig. 2. A formulation was developed by You [52] for nodal movement control of a prestressable pin-jointed assembly presented in Fig. 3. He was interested in prestressing effects and he didn't put the effect of external loading into account.

$$d_s = Ee_{os} \quad (1)$$

Where d_s is the displacements at nodes resulting from the adjustment of displacements, E is the matrix that relates external nodal movement and actuation, and e_{os} is the actuation applied to modify the shape.

Then, it was further developed by insetting external loading effects [11]

$$Ee_{os} + d_p = d_t \quad (2)$$

Where d_p and d_t are nodal displacements due to loading and targeted ones, respectively.

After that, the method was progressed by giving the desired displacement in a domain and involving optimization algorithms [10].

$$-d_t \leq -Ee_{os}d_p \leq d_t \quad (3)$$

Equation (3) is subjected to the optimization function below to minimize the sum of the total actuation and eliminate the actuators out of the domain of $[L_b \text{ and } U_b]$, which are lower bound and upper bounds, respectively.

$$\min f(x) = \sum_{i=1}^n e_{oi} \quad (4)$$

2.2 Stress Control

Specific structures may require more attention to controlling the internal forces within their members rather than focusing solely on displacements or deformations. An example is seen in systems containing cable members that may become slack under specific loads, necessitating tightening to maintain their structural integrity [53]. Conversely, slender struts may face instability due to buckling failure requiring reduced compressive force [54–56].

It is essential to note that bar length actuation impacts axial force only in structures with a high degree of indeterminacy. This discussion centers around controlling axial forces in specific structural members to prevent structural failure caused by over- or under-stressing due to loading.

Fig. 2 Number of studies were conducted from 2010–2023

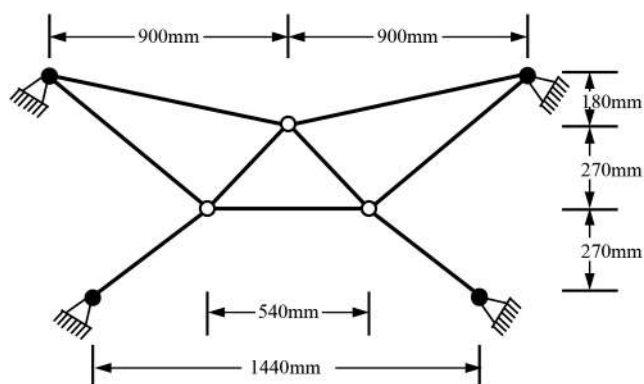
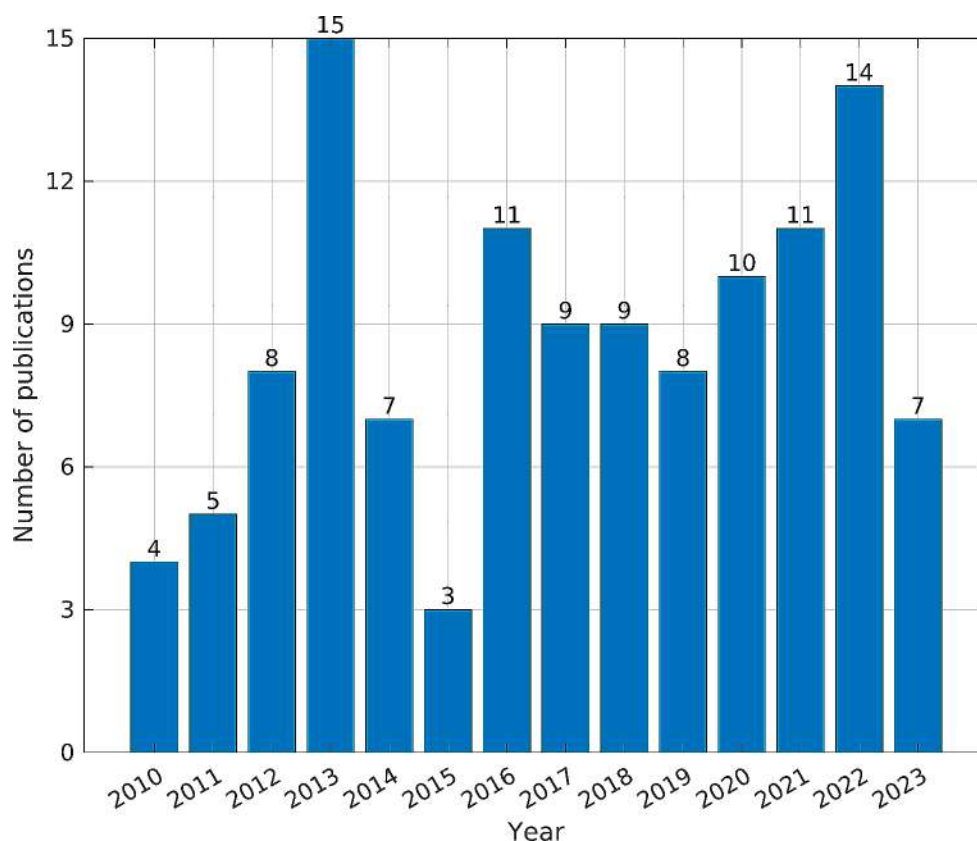


Fig. 3 A 2D pretensioned cable structure You [52]

While the study of internal force control in structures is limited, some related works have been reviewed and summarized in this section, mainly focusing on the broader concept of prestressing in structures. Prestressing, strictly speaking, is typically applied to indeterminate structures. However, all structures experience some degree of “pre-stressing” owing to their deadweight. Enormous space systems, often indeterminate, can be specifically prestressed to reduce overall weight. This is achieved by adjusting the internal forces to be more tensile-dominant, thereby decreasing the reliance on bulky compression struts and enabling the structure to span more considerable distances [57].

The prestressing technique directly involves the length actuation of structural members. Kwan and Pellegrino [58] discussed methods for calculating length actuation before applying the load; the aim is to attain a particular prestress arrangement using the force method (FM). The researchers also investigated finding the best locations for actuators that apply prestress to pin-jointed trusses and the most effective actuator adjustments for improving an existing, incorrect prestress state. Roth and McCarthy [59] proposed a method coupled with a genetic algorithm; the technique suggests pretension on the tensegrity structures so that the compression members can resist six times more than before prestressing. Another approach to controlling forces in prestressed systems is pre-tensioning the bars to an anticipated amount [60]. The process to analyzing the pre-tension in a prestressable space structure relied on linear superposition. The computational outcomes demonstrated the approach’s correctness, reliability, and effectiveness, making it appropriate for analyzing prestressable networks.

In different studies, stress has been controlled and become a constraint in topology optimization studies using bi-directional evolutionary structural optimization (BESO) [61–65]. In addition, the influence of self-stress in tensegrity structures was studied theoretically and experimentally [66, 67]. Furthermore, Dai and Yu [68] developed a new method for creating shell openings that takes advantage of

the powerful modeling capabilities of Grasshopper. This visual programming language is integrated with the Rhinoceros 3D software to control stress in designing shell structures. In addition, the Stress-Aware Optimal Actuator Placement (SOAP) framework is a novel approach to placing actuators on composite structures. SOAP considers the residual stresses generated in the structure when the actuators are activated, and it minimizes the residual stresses by optimizing the actuator placement [69]. Piezoelectric actuators nullify the axial force in members in a one-dimensional structure [70].

The applications of force control can be found in several structures like stress control on a Levy cable domes [71], adaptive cable domes [72]. In addition, prestressing has been done for several structures like cable-strut structures [73–78], cable domes [79, 80] tensegrity systems [81–84]. Studies conducted in force control or prestressing of cable

or tensegrity structures between 2010 and 2023 are presented in the line graph in Fig. 4.

2.3 Shape and Stress Control Simultaneously

In practical applications, it can be challenging to manipulate a sole inconstant without affecting other variables simultaneously. For instance, when restoring an antenna's shape to a predefined target by adjusting specific member lengths, one must be careful not to cause dangerous levels of strut forces or slacken cables due to reduced internal force. This requires controlling nodal displacements and bar forces, which is typically challenging.

Limited research has been conducted on controlling both shape and internal forces. Kawaguchi, Hangai [85] explored an analytical approach using the linear force method to control the stress and shape of prestressable trusses. They validated their proposed method by comparing computational

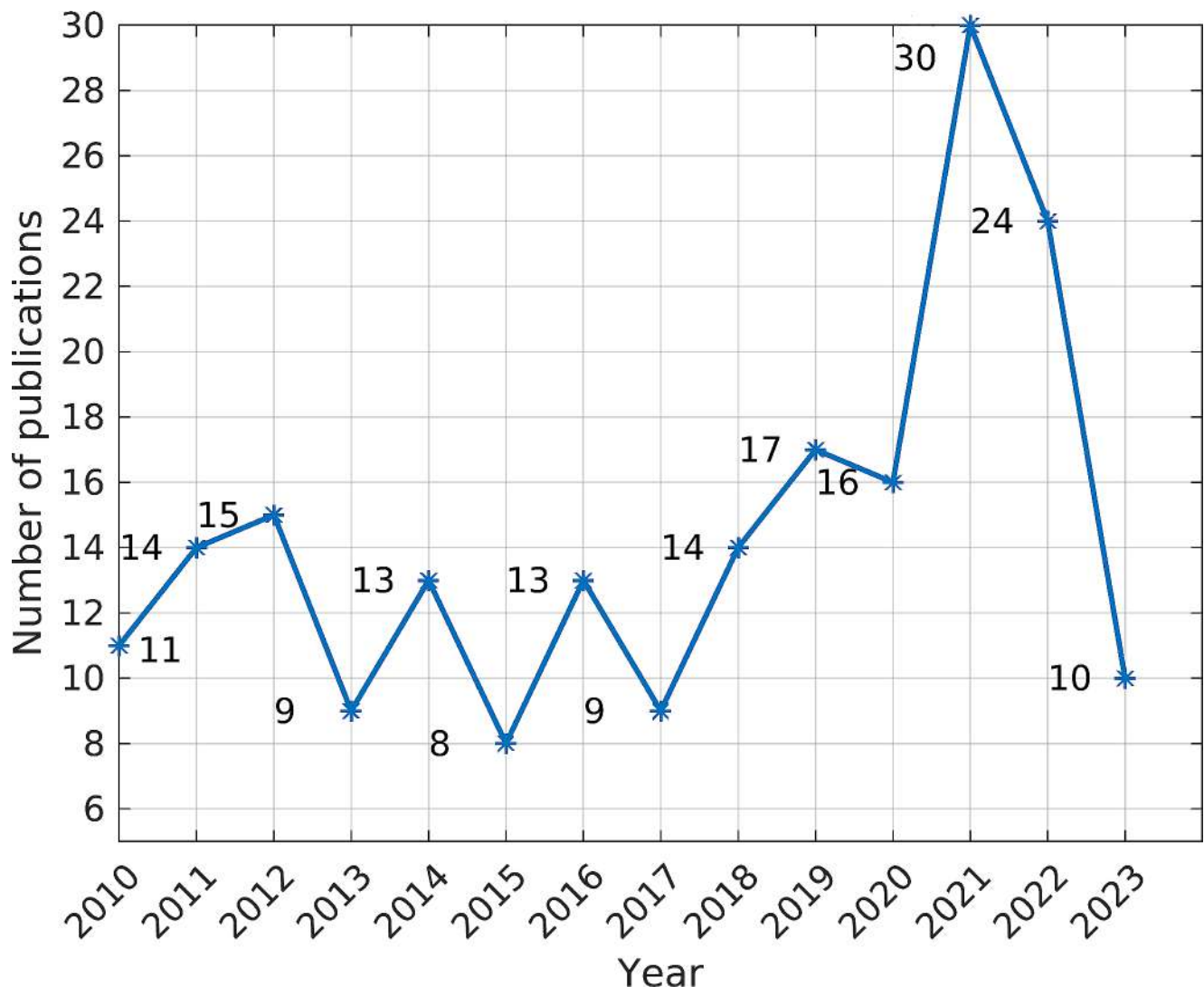


Fig. 4 Research about force control or prestressing cable or tensegrity structures from 2010 to 2023

results with experimental findings on a pin-jointed tensile structure. The experiments focused on identifying difficulties related to controlling displacement components. Yuan, Liang [86] presented a study using a nonlinear force method to simultaneously control cable structures' nodal displacements and axial forces. Another study by You [52] proposed a displacement control method for prestressed cable structures derived from FM and elastic deformations. The technique directly linked length adjustments and displacements in prestressed systems. The validation was done by comparing computational and experimental results on a preattentive 2D cable structure shown in Fig. 3. However, this method is limited to structures made of materials that exhibit linear elasticity and applies only to cases with small deformations. Similarly, Kawaguchi, Hangai [85] investigated stress and shape adjustment of prestressable trusses; they employed a straightforward analytical approach grounded in FM. However, they faced difficulties in confirming computational results by practical testing. Senatore and Reksowardojo [87] developed a study to control the force and displacement of adaptive structures with optimal actuators.

Innovative structures have the ability to adapt their configuration, damping, and stiffness, to environmental deviations [17]. The study found applications in space structures with a high level of precision, for example, reflectors. However, minimal damping, changing temperature status, and slackness in the joint can disturb these structures. Actuator control can help mitigate these effects. Sener, Utku [3] discussed shape adjustment in prestressable adaptive space pin-jointed assemblies to meet precision requirements for supported instruments. They emphasized the need for exact control methods as approximate methods were inadequate for achieving the required micrometer-level geometry control for precise measuring instruments upheld by flexible space trusses. Furthermore, a process of optimizing multiple objectives was presented using a genetic algorithm for shape control of a prestressed cable structure [88]. Wang, Li [46] argued that multi-objective optimization methods were

unsuitable for cable structures that demand high surface precision. This is due to the low accuracy of the solutions obtained and the extensive computational burden associated with large and/or delicate pin-jointed systems. Such methods might lead to substantial misshape or extensive stress in members. To address this, an algorithm was suggested for member and joint exchange that could be used to find alternate arrangements of members and joints in truss structures that result in lower surface errors and axial forces [89].

Furthermore, an iterative procedure that combines the linear displacement control technique and the non-linear force method to control the shape of prestressed cable structures was presented by Xu and Luo [90]. The computational results indicated that the non-linear method agreed with the expected results, while the equivalent outcomes of the linear method showed significant discrepancies. The researchers concluded that the non-linear approach presents a marked improvement over the linear one, particularly when considering maintaining prestressing. Manguri, Saeed [55] proposed a technique based on FM and interior point algorithm through altering the length of some bars, nodal displacements, and axial force controlled; they also considered buckling in their work, the technique applied on several numerical examples see Fig. 5. In a different study, axial force and external nodal displacements of cables and trusses were simultaneously adjusted while the weight of the structures was optimized [91]; their technique was derived from FM and coupled with the interior point optimization algorithm. A linear method for shape and stress control was used to see the validity of the technique on a linear and geometrically non-linear structure [92].

3 Types of Actuators

An actuator is a device that uses a power source to create mechanical motion. They are used in various applications, including engineering, machinery, aerospace, industrial, and medical [93]. This paper discusses the most used actuators to control shape and stress in structures.

3.1 Mechanical Actuators (Turnbuckles)

The turnbuckle consists of two threaded eye bolts, one screwed into each end of a small metal frame, one with a left-hand thread and the other with a right-hand thread, as presented in Fig. 6. When the eye bolts are turned, they screw into each other, shortening or lengthening the turnbuckle. This changes the tension in the rope or cable threaded through the eye bolts [94, 95]. Researchers like Lee and Shin [96], and Olsson [97] developed novel types of turnbuckle with a regular cross-section. Shin and Lee

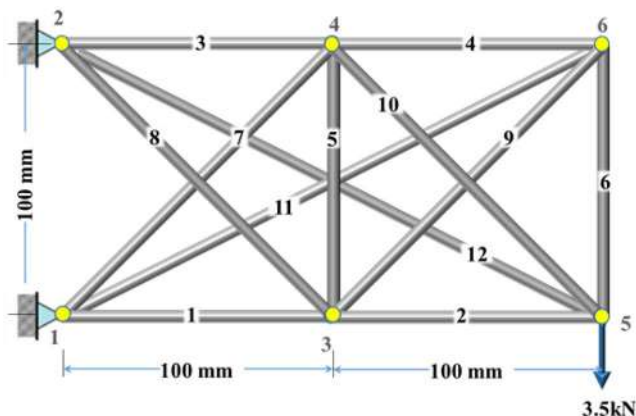


Fig. 5 A 2D prestressable truss [55]

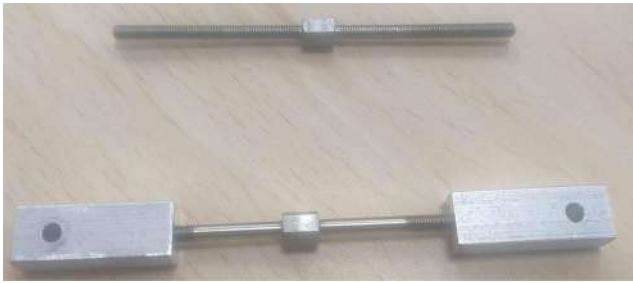


Fig. 6 A mechanical Actuator (turnbuckle)

[95] considered the flexural elements of a turnbuckle as beam-column components under the influence of both tensile force and moment; it becomes feasible to derive the relationship between load and displacement theoretically. Researchers used the mechanical actuator to change the force in members and/or relocation of joints [98, 99]. In addition, Turnbuckles were implemented to develop external post-tension in reinforced concrete beams [100–102].

3.2 Thermal Effect

Employing heating effects as an actuation technique can present a practical solution for mitigating static distortion in a big space system caused by its deformed shape. This method involves altering the temperature of the control elements. To ensure its effectiveness, choosing an active component with a high coefficient of thermal expansion is crucial.

Haftka and Adelman [14] conducted shape adjustment experiments on space structures utilizing heat as the actuation mechanism. Additionally, they conducted a study to anticipate and evaluate the impact of actuator inaccuracy on the shape adjustment process for delicate space systems, with the applied temperature serving as the actuation method [103]. Furthermore, Giusti, Mróz [104] described a method for designing thermomechanical actuators. The method uses topology optimization to maximize the output displacement of the actuator. Topology optimization is finding an object's optimal shape by minimizing a cost function while satisfying constraints. In this case, the cost function is the output displacement, and the constraints are the mechanical and thermal equilibrium equations [105].

3.3 Piezoelectric

The utilization of piezoelectric actuators for shape control in flexible structures is an exciting application of piezoelectric materials [106]. These materials are the most commonly used intelligent materials for shaping systems. Irschik [5], and Sunar and Rao [4] have extensively discussed and reviewed the concept of shape control through piezoelectric

actuation. Numerous researchers have applied piezoelectric actuators to control the shape of flexible structures. For example, Yang and Ngoi [37] and Hadjigeorgiou, Stavroulakis [34] used them to control the shape and identify damage in a beam and a cantilever composite beam, respectively. Additionally, Koconis, Kollar [107], Koconis, Kollar [108], Wang, Chen [24], and Chee, Tong [25], Chee, Tong [26] employed piezoelectric actuators for static shape control of intelligent structures.

A novel form of actuator called the laminated piezoelectric (LP) was applied by Yu, Zhang [35] to shape a cantilever beam. The LP consists of many piezoelectric patches with identical geometric and material parameters. Although Piezoelectric actuators are a valuable tool for shape adjustment, they have some drawbacks. They are highly sensitive to temperature variations, which can affect their performance. They produce relatively small displacements that may not meet the required range for certain applications. Additionally, their inherent nonlinear behavior necessitates complex computational techniques for accurate control. These drawbacks can be mitigated, but in some cases, it may be necessary to use a different type of actuator [109].

Wang, Qin [110] introduced a mathematical model focused on shape control of piezoelectric intelligent structures. The study utilizes a genetic algorithm to analyze optimal shape control outcomes for piezoelectric materials in a cantilever plate configuration, where both ends are subjected to specific loads. The analysis is conducted from various perspectives, such as precision reference or cost reference.

3.4 Shape Memory Alloy (SMA)

SMA is a comparatively recent and valuable material that has gained popularity owing to its exceptional characteristics. For example, considerable strength and an extended range of movement, compact size, and lightweight. Consequently, it has found increasing applications in various fields. One area of recent research involves using SMA for structural control purposes. Song, Ma [111] comprehensively reviewed SMA material applications in structural control. However, it is essential to acknowledge that Peng, Jiang [112] expressed concerns regarding SMA's limited stability and controllability, posing challenges to achieving precise actuation. While SMA holds great promise for structural control, addressing the issues related to stability and controllability is crucial for ensuring accurate actuation. Further research in this domain is necessary to harness SMA's potential in control applications fully.

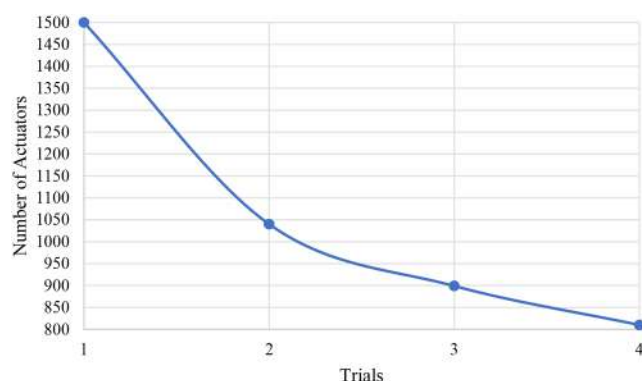


Fig. 7 Minimizing actuator numbers in four iterations

4 Number of Actuators

Controlling a specific structure effectively requires an optimal number of actuators to address shape distortions and bar forces. Actuator numbers are needed contingent on the structure's characteristics and the desired control over nodal displacements and axial forces. Research by Salama, Umland [23] suggests that adding more actuators leads to more accurate shape corrections. However, due to the high cost, there is a limitation of the actuators that can be used for adjusting and resolution. Thus, finding the ideal quantity of actuators for a specific problem is crucial. Haftka and Adelman [113] tackled the task of choosing a certain number of actuator positions (n) from a larger pool of available sites (m) to control the shape of large space structures statically.

Yuan and Yang [114] proposed a technique based on the singular value decomposition method to minimize actuators that adjust a sizeable deployable mesh reflector shape. In another study conducted by AlBahar, Kim [69] developed an SOAP framework to be more effective than conventional approaches in reducing deformation and residual stresses. It has also been shown to find the optimal solution with fewer actuators. Ali, Ghotbi [115] used Genetic Algorithm (GA) and sequential quadratic programming (SQP) to minimize the number of actuators to optimize the structural weight of a truss structure. Chanekar, Chopra [116] suggested a new approach for optimal actuator placement with several advantages over previous approaches. First, it can find the global optimal solution, which is the best possible solution to the problem. Second, it can be applied to both stable and unstable systems. Third, it does not require an initial controllable actuator combination, which makes it more flexible and easier to use. Other researchers like Chen, Jiang [117], Dhingra and Lee [118], and Du, Yue [119] investigated the optimal placement of actuators. Other studies have been conducted to minimize imbedded actuators for deformation control in several iterations in cable structures [120], and domes see Fig. 7 [121].

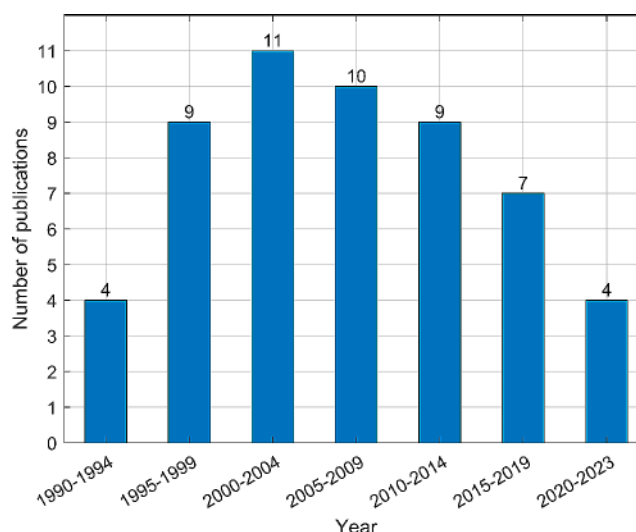


Fig. 8 Studies on actuator placement in 1990–2023

Haftka [122] introduced the notion of “ideal actuators” to approximate the required quantity of actuators, reducing shape errors and evaluating their effectiveness in minimizing the truss bars members needing actuators. In another approach, Kincaid [123] utilized simulated annealing to achieve a near-optimal solution for the arrangement of actuators in assemblies with numerous members. Simulated annealing proved a practical computational technique, providing an efficient approach for actuator placement calculations without exhaustively searching all possible configurations. In another study, optimal actuators were utilized to control force and shape simultaneously [87].

5 Optimal Location of Actuators

The positioning of actuators to achieve the best control of a structure has a great importance [124–128]. Subsequently, various endeavors have been undertaken to optimize and decrease the structural deformation by identifying the most effective locations for actuators, the number of studies has been conducted since 1990 up to 2023 are shown in Fig. 8. This involves determining the least actuators required to accomplish the tasks. Furuya and Haftka [129] discovered that, even for small structures, the potential combinations for placing controllers could be vast. Due to the expensive cost of installing numerous actuators for control purposes, the quantity of actuators is usually restricted. As a result, finding the best locations for actuators becomes critical. Various researchers have tackled this issue by seeking global or nearly optimal solutions. Kwan and Pellegrino [58] stressed the significance of actuator locations in adjusting an inaccurate prestress distribution and obtaining optimal actuator adjustments for required actuation.

Haftka [130] explored analytical methods for optimally placing thermal and force actuators to control static distortions in massive space systems. The derived formulation drew on designing against the worst disturbances and minimized the analytical effort through numerical means. The study emphasized that actuator placement was more crucial for force actuators, though it was also important for thermal actuators. Furthermore, Reksowardojo, Senatore [131] conducted experimental testing of an adaptive truss that can change its shape to respond to the effects of loading. The placed actuators are optimized to ensure the structure can adapt to the optimal form for each load case. The experimental testing showed that the structure can adapt to a wide range of loading conditions and significantly reduce the structural loads.

Skelton and DeLorenzo [21] devised an algorithm known as the SD algorithm aimed at identifying the best locations for positioning actuators and sensors in optimal control. The method assesses the impact of adding up or removing actuators based on quadratic performance criteria at all available areas. It progressively eliminates the sites with minimal impact till the optimum actuators are nominated within the allowed quantity of control sites. Subramanian and Mohan [20] proposed a novel, straightforward algorithm known as the successive peak error correction, claiming its superior speed over the SD algorithm for statically controlling flexible structures while maintaining similar levels of precision.

A significant investigation explored actuator placement in large structures using heuristic integer programming to address shape control [113]. The study devised two iterative algorithms, the Worst-Out-Best-In (WOBI) and the Exhaustive-Single Point Substitution (ESPS). Both algorithms could attain shape control by repositioning actuators, although the outcomes relied on the preliminary assumption. Moreover, the researchers compared the outcomes of WOBI and ESPS with those gained from the SD algorithm for the shape control of a reflector. The study revealed that WOBI and ESPS produced better outcomes when selecting relatively few available sites. Though, the SD algorithm proved numerically more inexpensive than the two other algorithms when a substantial portion of the available sites needed to be nominated. It was also noted that achieving precise surface alignment with a limited number of actuators, even when strategically positioned, was challenging.

Following Haftka and Adelman [113], many researchers presented novel actuator placement methods, including genetic algorithms (GA). Rao, Pan [132] proposed a GA approach to address the precise optimal positioning of actuators in actively controlled systems, treating it as an optimization problem with binary encoding. They stated that their method may yield global-optimal solutions with adequate generations. The derivation was implemented to

determine optimum actuator positions in the truss structure [132]. Similarly, Furuya and Haftka [129, 133] employed genetic algorithms and effectiveness indices to optimize the locations of actuators in significant space structures.

Simulated Annealing (SA) is a different algorithm for optimum actuator positioning. The SA method was applied to the problem of minimizing static distortion by optimizing actuator location in pin-jointed assemblies, as demonstrated on a large pin-jointed assembly for the least shape distortion [40]. Chen, Bruno [134] also employed SA to study the optimum positioning of active and passive members in intricate trusses. In a similar vein, Onoda and Hanawa [135] used a (GA), a modified GA (GA2), and an improved SA (ISA) method to tackle the actuator positioning optimization problem in the context of truss shape adjustment. The researchers determined that the three mentioned algorithms outperformed ESPS, WOBI, and SA when applied to a three-ring tetrahedral pin-jointed assembly.

Maghami and Joshi [136] presented an optimum actuator positioning method for actively controlling large, delicate space structures. They optimized the positioning of actuators to move the transmission zeros from the right-half-plane to the left-half-plane of the imaginary axis, a critical aspect for achieving rapid and optimal regulation and tracking capabilities. Moreover, they successfully applied this technique to a huge delicate system [136]. Additionally, Kwan and Pellegrino [58] investigated the identification of ideal locations for prestress actuators and the most efficient adjustments for these actuators to enhance the pre-existing, inaccurate prestress condition in pin-jointed trusses.

Irschik and Nader [137] also tackled the issue of actuator positioning to adjust the shape of weighted beam. They utilized the extension of the Mohr Analogy to eliminate movement and rotation at designated positions on the beam through piezoelectric. The researchers compared their proposed beam-type method and a two-dimensional FEA, yielding favorable results. Matunaga and Onoda [138] and Wang and Wang [139] recognized the optimum placement for actuators, they also mentioned that the actuators used in smart structures are crucial to their performance. The actuator's category, their positions, and actuation amount must all be carefully considered by designers. Hadjigeorgiou, Stavroulakis [34] developed an FE model and a GA optimization technique for identifying and controlling damage in a piezoelectric-equipped composite beam with a cantilever configuration. The derivation was based on the shear formulation beam theory and the linear theory of piezoelectricity. The researchers demonstrated that although many actuators were typically required, careful positioning and optimum voltage values could also effectively attain adjust of the beam's shape with fewer actuators.

A continuum approximation technique was introduced to enhance the cost-efficiency of actuator location optimization Burdisso and Haftka [140]. This approach allows the users to calculate the statistical characteristics of the adjusted form and demonstrated its performance as being favorable compared to exact statistical analysis. Furthermore, they established that continuous optimization techniques were considerably more economical than the integer programming approaches proposed by Haftka and Adelman [113]. Additionally, they determined that the continuum analysis led to optimized actuator positions, minimizing the weighted root mean square (rms) distortion. Moreover, their study revealed that the optimized design outperformed a design with evenly spaced actuators.

One of the primary drivers of shape control is the advancement of adaptive materials with inherent shape-changing properties. Smart materials, like shape memory alloys (SMAs), piezoelectric composites, and electroactive polymers, can undergo reversible deformations when exposed to external stimuli. For example, SMAs can regain their original shape after being subjected to heat or stress, offering self-repair and reconfiguration capabilities for aerospace structures operating under extreme conditions. In addition, optimal location based on the actuator power was studied [141, 142]. Hu, Li [143], Presented a study that seeks to explore the distributed activation response caused by piezoelectric patches on an adaptively supported paraboloidal cylindrical shell panel that is simply held in place. Moreover, Bruant and Proslir [144] introduced a revised optimization standard for addressing the pair of optimization challenges. It guarantees effective observability or controllability of the structure, while also factoring in residual modes to constrain the potential spill-over consequences. Another study was conducted to find optimal location of additional bars to minimize the total weight of space frames [145]. Reksowardojo, Senatore [146] Used a formulation based on stochastic search and the nonlinear force method to optimize the actuator placement and commands. This step ensures that the actuators are placed in the optimal locations and that the commands are optimally sent to the actuators so that the structure can adapt to the target shapes.

6 Conclusion and Future Work

This paper presents an intensive review about structural controlling in the sponce of shape and stress, as well as types of actuators that can be used for length changing of structural members. In addition, the methods and the research dealt with minimizing number of actuators and finding optimal location of them were presented in detail. This study

suggests further research in the field of structural controlling as follow.

- Most of the methods established for shape and stress control of structures are liner techniques, while some structures are geometrically nonlinear especially cables structures. Linear methods are not as accurate as non-linear ones, thus establishing structural control methods based on non-linear methods for example dynamic relaxation method is suggested.
- Using other powerful optimization algorithms to find optimum number of actuators.
- Implementing different algorithms to find optimal location of actuators.
- For smart systems, intelligent actuators can be designed that can reshape and redistribute stresses upon the desired levels.

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1007/s11831-024-10149-9>.

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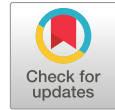
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Paper II

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REVIEW

Topology, Size, and Shape Optimization in Civil Engineering Structures: A Review

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ABSTRACT: The optimization of civil engineering structures is critical for enhancing structural performance and material efficiency in engineering applications. Structural optimization approaches seek to determine the optimal design, by considering material performance, cost, and structural safety. The design approaches aim to reduce the built environment's energy use and carbon emissions. This comprehensive review examines optimization techniques, including size, shape, topology, and multi-objective approaches, by integrating these methodologies. The trends and advancements that contribute to developing more efficient, cost-effective, and reliable structural designs were identified. The review also discusses emerging technologies, such as machine learning applications with different optimization techniques. Optimization of truss, frame, tensegrity, reinforced concrete, origami, pantographic, and adaptive structures are covered and discussed. Optimization techniques are explained, including metaheuristics, genetic algorithm, particle swarm, ant-colony, harmony search algorithm, and their applications with mentioned structure types. Linear and non-linear structures, including geometric and material nonlinearity, are distinguished. The role of optimization in active structures, structural design, seismic design, form-finding, and structural control is taken into account, and the most recent techniques and advancements are mentioned.

KEYWORDS: Structural optimization; topology optimization; size optimization; shape optimization; multi-objective optimization

1 Introduction

Optimization focuses on enhancing performance, and structural optimization involves designing a material assembly to withstand loads as efficiently as possible [1]. The theory of optimization is linked to both natural events and the human desire for a positive outcome [2]. Structural optimization approaches aim to discover the optimal design by balancing structural safety, material performance, and cost [3]. This field is a relatively new discipline with rapidly changing methodologies and focus [4,5]. Structural optimization resulted in a new era in structural engineering [6] since identifying the appropriate solution to a problem is critical for cost efficiency. Reducing weight can significantly lower manufacturing, transportation, and operational costs while enhancing material efficiency [7]. In addition, because the construction industry is a significant contributor to material use, energy usage, greenhouse gas emissions, and waste generation [8], the current design techniques aim to reduce the built environment's energy consumption and carbon emissions to the lowest practicable level due to increasingly rigorous requirements [9]. Automated design



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processes that may guide structural design toward lower embodied carbon solutions are becoming more vital [10]. Structural optimization is crucial to lessen the built environment's carbon footprint in light of the upcoming environmental catastrophe [11]. Minimizing whole-life energy results in adaptable solutions with a lower ecological impact than passive structures [12]. Optimized active structures have the potential to be sustainable as they use less material, consume less energy, and emit less Greenhouse gas GHGs over their lifespans [13].

According to Vanderplaats [2], the first analytical study in structural optimization was by Maxwell [14] and Michell [15]. This research establishes theoretical lower constraints on truss weight, contributing significantly to structural optimization and design. In addition, several studies were conducted to enhance analysis and design techniques by looking at recent advancements in computer analysis, mathematical research, and physical applications. These issues prompted the development of intelligent structures employing these assembly techniques. Using optimization to design structures is a crucial step toward the long-term growth of technology and structural designs [16]. Many design constraints, such as those pertaining to stiffness, frequency, and buckling load, may apply to a structure. Various standards for material deletion must be used based on the types of design constraints [17]. According to Xiao et al. [18], an ideal structural system design may include maximizing a system's size, shape, or topology. Numerous studies have been conducted to optimize shapes and sizes. However, because of its complexities, topology optimization has received significantly less attention [19]. Size optimization focuses on adjusting the cross-sectional dimensions of structural components to minimize material usage without compromising structural integrity. In contrast, form optimization involves modifying the shapes of structural elements to enhance their performance. Additionally, topology optimization determines the optimal distribution of materials within a given design space.

Structural optimization approaches can be advanced through applications with other structural design and control fields. Research has been done to regulate uniform stress and shape while optimizing structures. Manguri et al. [20] employed interior point optimization with matrix condensation to regulate nodal displacements in pantographic structures. A computational technique was presented [21] for reducing material consumption in designing current spatial frame structures by prestressing only active members, considering shape control. Moreover, Saeed [22] presented an optimization-based technique to control the displacement of nonlinear pin-jointed assemblies. On the other hand, Wang et al. [13] concluded that using many materials can lead to a lighter design that performs better than one composed of a single material. They focused on applying optimization methodologies to multi-material structures because the methodology's initial focus was on single-material designs. Moreover, a flexible form-finding strategy for tensegrity structures is discussed by Zhang et al. [23], emphasizing the restricted optimization challenge. In addition, Cai et al. [24] provided an optimization-based form-finding technique for regular and irregular tensegrity structures. Meanwhile, Brütting et al. [8] described structural optimization strategies for designing truss structures through reuse. They stated that reusing structural materials reduces waste, saves energy, and lowers the need for new materials. To overcome inefficiency in time-consuming and aesthetic knowledge, Qian et al. [25] introduced a self-sparse generative adversarial network (GAN), guaranteeing over 11% improvement in the Frechet inception distance (FID) score compared to traditional GAN approaches. In another study, Qian et al. [26] proposed That topology description functions enhanced the neural network (TDF-NN) to perform topology optimization 10 times faster than traditional approaches while enabling diverse structural designs for practical applications. In addition, an optimum number of actuators was used [27] to control the shape and buckling of truss structures. Similarly, Reference [28] used optimization in number actuators and actuation to control displacement and stress of truss structures.

Optimization techniques [13] are widely used in structural engineering with various structures and targets, such as truss structures [29–31], tensegrity structures [32–35], frame structures [36–39], reinforced concrete structures [40,41], origami structures [42,43] and pantographic structures [20]. However, there is a lack of review papers that focus on optimization techniques in structural engineering covering different techniques, structures, and applications; for this reason, this paper presents a comprehensive review of structural optimization techniques, including size, shape, topology, and multi-objective approaches and their applications for different types of structures such as truss, frames, and tensegrity. Such an article is essential for evaluating the improvement and widespread adoption of techniques with detailed comparisons in the face of theory and application. It is crucial to highlight how optimization techniques improve efficiency and reduce the environmental impacts of structures. Finally, Recent breakthroughs include using artificial intelligence and machine learning in optimization procedures with future directions covered in this article.

It is essential to emphasize that the literature review for this study was conducted using the highly reputable Web of Science database. This database was selected due to its broad coverage and rigorous selection criteria, ensuring the inclusion of only high-quality research. The review focused on the most recent developments in structural optimization over the past 20 years (2005–2024). A search strategy combining titles, topics, and keywords was applied to achieve a comprehensive and exhaustive collection of relevant studies. The primary search terms included ‘topology,’ ‘size,’ ‘shape,’ ‘optimization,’ ‘truss,’ ‘frame,’ ‘tensegrity,’ and ‘structures.’ Separate searches were performed for each type of structural optimization, using combinations such as ‘topology,’ ‘optimization,’ and ‘structures.’ Fig. 1 shows the distribution of literature according to years and Fig. 2 presents the correlation map of keywords based on literature that corresponds to topology, size, and shape optimization.

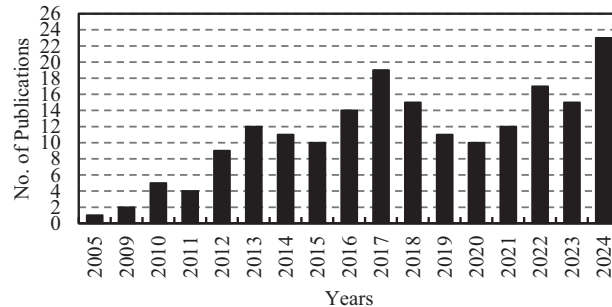


Figure 1: Classification of literature worked on structural optimization according to years

2 Truss Structures

A truss is a structural system composed of small components connected at joints, typically at both ends of each member. Common architectural components used in trusses include metal bars, channels, wooden struts, and angles [16,44]. Given the widespread application of these structures, optimizing trusses can significantly influence flexibility, cost, environmental impact, and overall efficiency. This paper reviews various studies that focus on trusses, employing different optimization methods, as shown in Fig. 3. This section discusses the optimization of truss structures using these techniques.

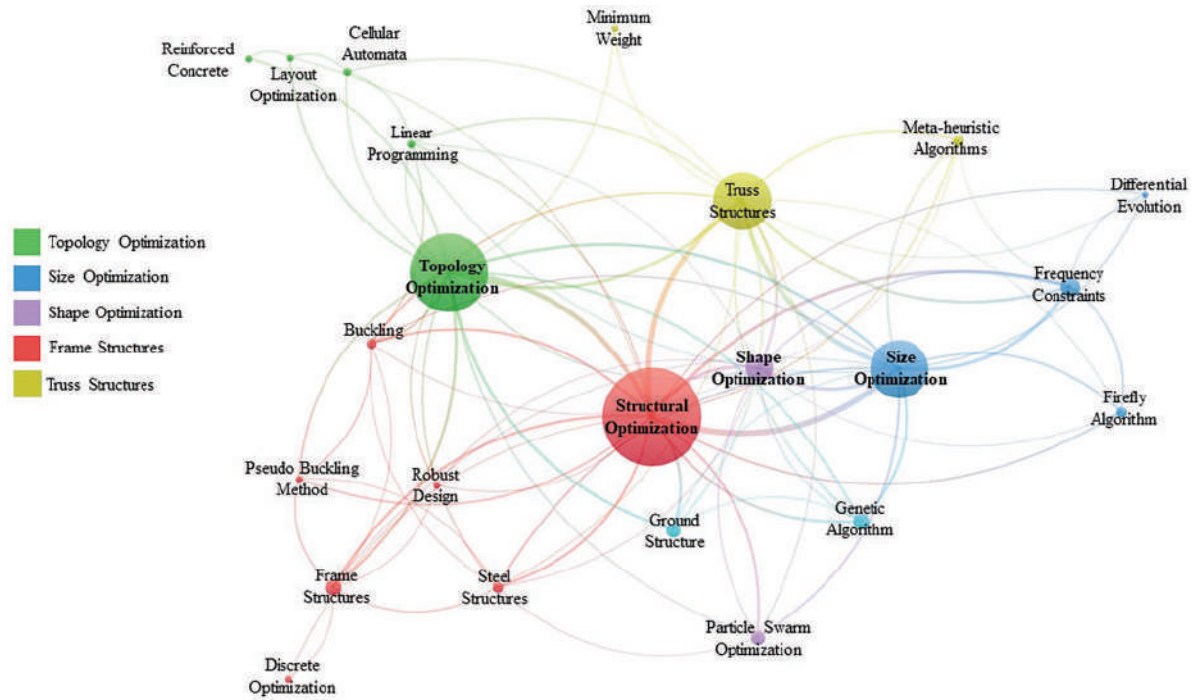


Figure 2: Correlation map of keywords based on literature

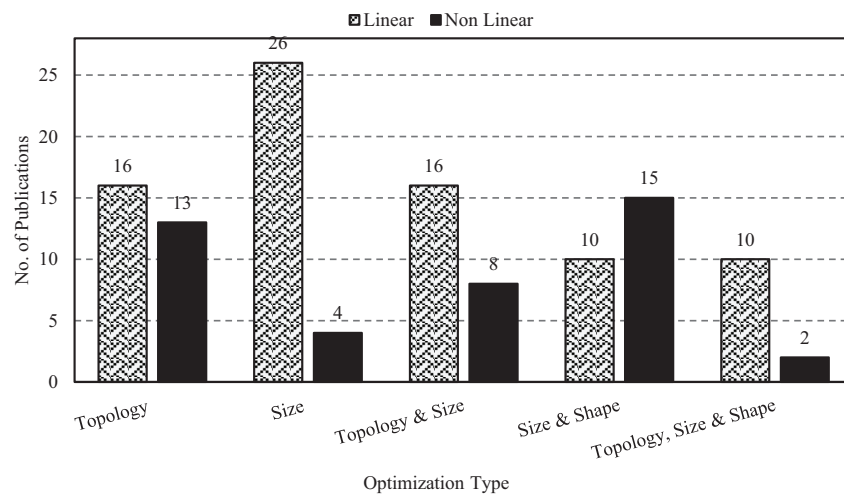


Figure 3: Classification of literature worked on truss structure optimization

2.1 Topology Optimization

2.1.1 Linear Approaches

Topology optimization finds the optimal material distribution inside a design space. Topology optimization will be critical to provide solutions that increase stiffness and reduce weight without compromising

structural functionality. In civil engineering, topology optimization has been used to design structures with the best possible material distribution, making the most use of available space and resources. Two examples of applications include complex support systems and the frames of high-rise buildings. The ground structure technique, developed in the 1960s, significantly enhanced topology optimization [45]. An interesting ground-structure-based representation for truss topology optimization is suggested by Shakya et al. [46]. Before designing the final trusses, the recommended diagram eliminates unnecessary components. Kinematically unstable and nonsensical zero-force components are examples of undesirable components. Also, a new strategy is suggested by Gao et al. [47] to produce ground structures based on primary stress trajectories that can be used in both regular and irregular design domains. The optimization model is a standard linear programming issue based on the plastic design handled by the interior point approach.

Asadpoure et al. [48] offered a method for considering manufacturing costs while optimizing the topology of light, rigid, periodic lattice structures. The regularized Heaviside step function approximates the number of elements in the design domain. Gradient-based optimization is made possible by the smoothness and differentiability of the cost function. The optimization will be more effective and faster when the analytical gradient of the functions is implemented [49]. Moreover, Tyburec et al. [50] offered a creative method for creating truss modules with little compliance through design and fabrication. To handle a variety of optimization challenges, modularity is essential. Besides, genetic algorithms used by Richardson et al. [51] enhance truss-like structures' topology for both single- and multi-objective uses. The process has two phases: chromosomal repairs and producing stable individuals within the original population. In addition, Asadpoure et al. [52] provided a computational method based on known uncertainty and second-order statistics for optimizing structural topology. The technique combines a perturbation approach based on material characteristics and geometry with deterministic topology optimization to quantify uncertainty in structural stiffness. Similarly, a powerful topology optimization technique with discrete design parameters, material, and loading uncertainties was investigated for truss structures using Non-dominated Sorting GA-II (NSGA-II) by Richardson et al. [53]. The problem is defined by truss length, spatially correlated material stiffness, and load uncertainty. Also, Guo et al. [54] suggested a novel approach to truss-like continuous topology optimization under varying loads. The robust optimization formula minimizes the weighted sum of expectation and standard deviation for structural compliance within volume restrictions. According to this hypothesis, applied loads have independent and unexpected sizes and directions.

Furthermore, Zhang et al. [55] proposed a randomized optimization technique that considerably reduces the computational cost of designing truss and continuum topologies under varying loads. This technology lowers compliance under various load conditions and improves the layered minimum-end compliance topology by integrating density-based techniques with ground structure. On the other hand, Mashayekhi et al. [56] developed a Modified Gravitational Harmony Search Algorithm (MGHSA) that effectively enhances the topology of two- and three-layer grid layouts. The recommended solution worked well with respect to size optimization and topological concerns for double and triple-layer grid layouts. When compared to alternative meta-heuristic methods, MGHSA demonstrated strong competitiveness. While Nwe et al. [57] declared that, given a structural mass, the traditional approach to addressing a truss optimization problem is to minimize linear elastic strain energy. Based on this theory, the ideal solutions might produce thin members prone to buckling or giving way. Cai et al. [58] developed a Nominal Disturbing Force (NDF) technique to handle kinematic instability in the best-in-class truss assemblies. This method combines the ground structure approach with unstable nodes. It is suggested that a mixed-integer linear optimization model be used in conjunction with the NDF technique to reduce the weight of trusses with discontinuous cross-sectional areas. Constraints such as bar buckling, allowed stress, nodal displacement, bar crossing, and overlapping should be considered in this model, as shown in Fig. 4. The figure illustrates the domain (a) and

the ground structure (b), considering neither kinematic stability (WNF) nor necessary conditions (WONC) (c). It also shows incorporating the Nominal Disturbing Force (NDF) approach while not considering necessary conditions (WNC) (d) and incorporating the Nominal Lateral Force (NLF) approach while not considering necessary conditions (WNC) (e). The figure also shows employing the Nominal Disturbing Force (NDF) approach without considering necessary conditions (WONC) (f).

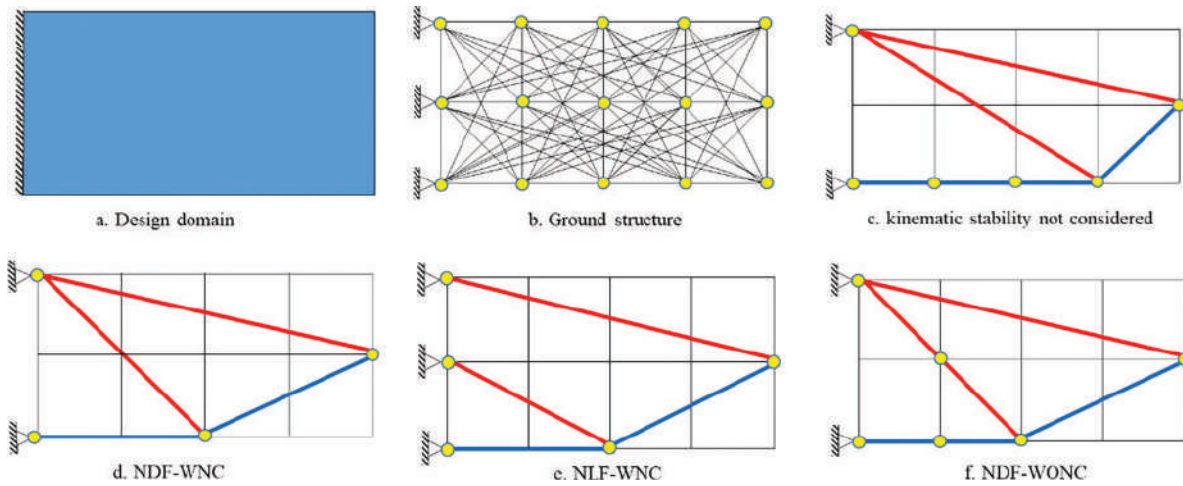


Figure 4: Cantilever beam optimization results (a. Design domain, b. round structure, c. kinematic stability not considered, d. NDF-WNC, e. NLF-WNC, f. NDF-WONC), adapted from [58]

To be optimal for practical applications, truss topologies need to consider both local and nodal buckling stability. Using widely used methods might result in processing that is highly costly or poorly conditioned optimization problems such as concavity; due to that, Cai et al. [59] provided two fundamental techniques: nominal Per Turbine Force (NPF) and acceptable stress iteration (ASI). Numerical examples show that nodal and local buckling stability significantly influence optimum designs. Also, Cai et al. [60] demonstrated that gradient-free Proportional Topology Optimization (PTO) may effectively enhance truss topology, which is considered unpredictable for static and dynamic loads. The objective function is estimated using the Univariate Dimension Reduction method and Gauss-type (UDRG) quadrature and the univariate dimension reduction approach to account for load uncertainty. The topology optimization of truss structures under unknown loads is reduced to a simple deterministic issue by using the UDRG approach. Moreover, He et al. [61] proposed a double-layer optimization (DLO) strategy to address the truss topology problem's local buckling constraints. This method enables efficient calculation by considering local buckling limits as linear functions of optimized variables. Also, Ching et al. [10] suggested a technique for improving the topology of a two-material truss system to lessen global warming potential. New design variables are developed using the ground structure approach to regulate each truss component's cross-sectional area and material composition. Moreover, Senatore et al. [62] provided a method for topology optimization of adaptive structures, which was the first of its kind to enable the development of flexible structures by optimizing both structural topology and actuator placement at the same time. This unique technique generates adaptive solutions that significantly outperform passive topology-optimized solutions. The study's findings extend the domain applicability of flexible structures to design problems when stiffness is not the primary factor. Fig. 5 shows the relationship between actuation and structural topology may enhance solution quality by concurrently changing element size, actuator position, control instructions, and structural topology.

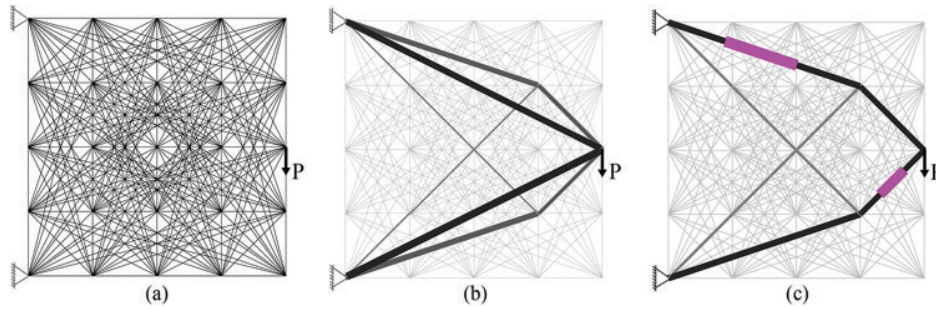


Figure 5: Ground structure (a), traditional (passive) solution (b), and adaptive solution (c) [62]

2.1.2 Non-Linear Approaches

This subsection focuses on applications of topology optimization on non-linear structures, including geometry and material non-linearity. Descamps et al. [63] proposed a fast and reliable method for optimizing the geometry and topology of truss systems for various load scenarios. Recasting the issue with consistent substitutions can help fix topological and geometric differences. Stragiotti et al. [64] investigated maximum stress, local buckling limits, and volume reduction under various load conditions in 3D truss designs. It depends on the starting position and non-convex due to kinematic compatibility limitations and non-linearities from buckling. The proposed two-step optimization procedure minimizes the effect of the starting point on the optimized structure while solving numerical problems using a heuristic approach. Also, Leng et al. [65] described a new method of truss design optimization that prevents element intersection, as shown in Fig. 6. The intersection factor, element cross-sectional area, and local structural arrangement stability were computed using the Heaviside function and with application of Sequential Quadratic Programming (SQP) to solve the nonlinear optimization problems. Tangaramvong et al. [66] described a mathematical nonlinear programming (NLP) technique for improving the lowest-weight topology of truss-like structures. The process considers the properties of the material that become pliable under pressure and lead to bodily instability. Canelas et al. [67] presented a novel concept for optimizing interior points. This methodology uses failure probability as a direct measure of resilience, in contrast to earlier approaches. Adopting this technique is challenging because of its nonlinearity, nonconvexity, and complexity, which are all well-known issues in reliability-based design optimization. While Li et al. [68] offered a topology optimization technique that considers geometric nonlinearity for prestressed cable-truss constructions. They developed a multi-node cable element model using massive deformation kinematics theory that considers the geometric nonlinearity of prestress and external loads.

Applications of topology optimization that considered material nonlinearity were taken into account. Research by Fairclough et al. [69] intended to improve the fail-safe truss topology of plastic design. Damage is a moving circular zone with zero strength under the given load situation. New high-resolution criteria for “fail-safe” structure formation have been devised using stiff polymers and global reduction. The study examines how location, damage radius, and structural simplicity affect multi-span trusses, short cantilevers, and square cantilevers. Furthermore, the ground structure approach improves nonlinear topology for hyperelastic materials by including the total potential energy concept into both the target function and the equilibrium solution [70]. Kozłowski et al. [71] have a unique technique that optimizes the geometry and topology of planar trusses without the necessity of a ground structure by using virtual displacements and strain fields. Yang et al. [72] outlined a brand-new fail-safe topology optimization method for systems with many scales. Partial damage is considered in the structural design process both at the macro and micro levels. For fitting challenges, less complex models of partially damaged truss-like microstructures are used to save

processing costs. They also concluded that heaviside projection also speeds up convergence and preserves a distinctive configuration. Rozvany et al. [73] examined plastically constructed multi-load, least-volume trusses and their significance in optimally designed trusses. Moreover, Zhang et al. [74] provided a helpful reduced-order modeling-based (ROM) filtering method to optimize two- and three-dimensional truss networks while considering nonlinear constitutive behavior under different load conditions. By optimizing for suitable load levels, the suggested method uses just truss members to create a displacement field devoid of artificial stiffness. Similarly, Zhang et al. [75] proposed an efficient multi-material topology optimization formula that solves structures with material nonlinearity. The proposed formulation, Zhang-Paulino-Ramos (ZPR) accommodates various materials with varying properties, allows for freely specified layers, and offers a generic volume limitation option, as shown in Fig. 7.

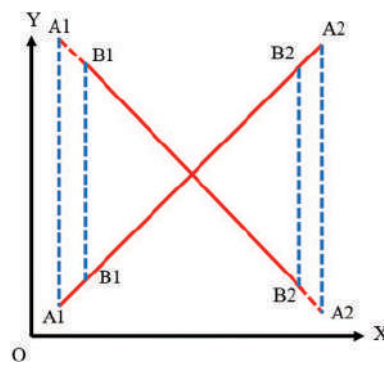


Figure 6: An explanation of element intersection in analytical adapted from geometry adapted from [65]

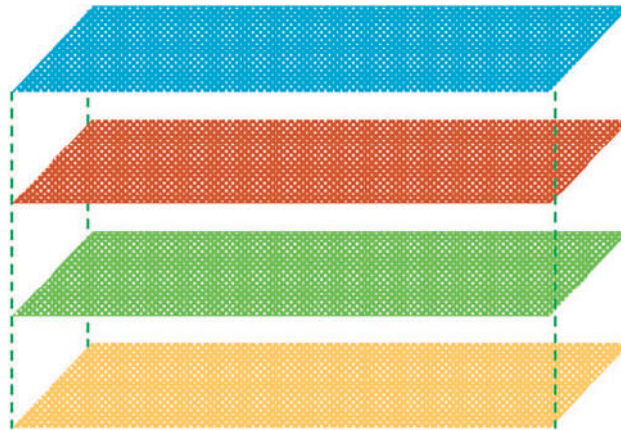


Figure 7: (Continued)

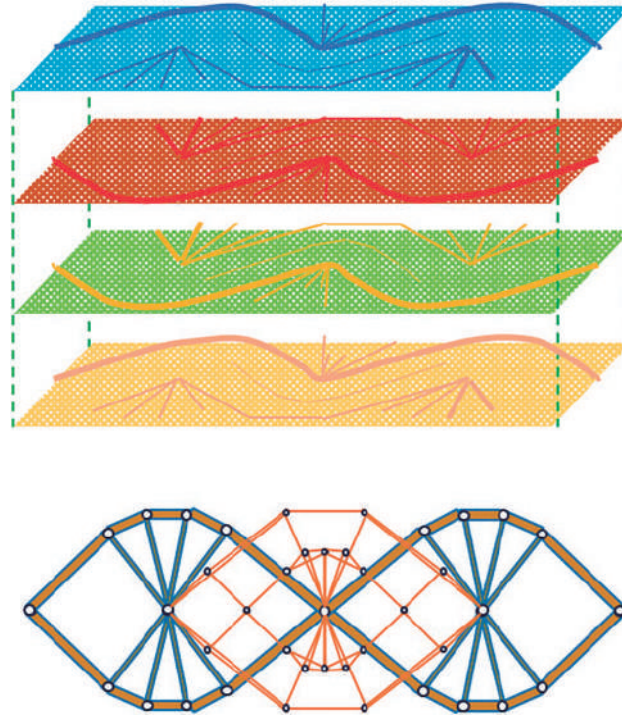


Figure 7: Initial material dispersal and the optimal structure for materials that share the whole domain adapted from [75]

In general, gradient-based optimizations are implemented for easy convergence and cost-effectiveness for linear techniques. In addition, GAs are implemented in multi-objective optimization, where strategies such as chromosomal repair and stability enhancement are employed to generate robust solutions through iterative population-based searches. Robust optimization frameworks have been included to handle load-impression uncertainties, using stochastic elements and Monte Carlo simulations to improve structural resilience. The application of advanced meta-heuristic algorithms like the (MGHSA) for non-linear techniques integrates principles from harmony search and gravitational optimization to solve complex, non-convex design problems. Moreover, (SQP) is discussed as an effective solver for handling nonlinear optimization with constraining local buckling regarding neural network applications, examination of architectures that utilize convolutional layers, and transfer learning techniques to optimize structural design processes by leveraging large datasets, enabling improved accuracy and reduced computational overhead.

2.2 Size Optimization

Studies have demonstrated notable reductions in material usage while accomplishing objectives related to structural performance. Heuristic approaches like genetic algorithms and gradient-based approaches are examples of common methodologies. These approaches ensure both material economy and cost-effectiveness. Constructing lightweight truss bridges, which lower material costs without losing safety, is one well-known use of size optimization. Optimization research was conducted to minimize the weight of space structures by adding extra members in crucial locations [76]. Also, there are applications for using genetic algorithms to design truss structures for minimum weight [77–80]. Moreover, research was conducted to perform size optimization of truss structures using a metaheuristic algorithm [81,82], and by using the method of centers and force formulation [83]. Moreover, Kaveh et al. [30,84], presented optimization

techniques for size optimization for truss structures using a swarm-based algorithm. While Kaveh et al. [85] presented a Hybrid Big Bang–Big Crunch (HBB–BC) optimization algorithm based on the Big Bang and Big Crunch hypothesis (BB–BC) to solve the truss optimization issue. One of the primary reasons why the size optimization problem was investigated more frequently than shape optimization, also known as geometry optimization in this context, was that it might occasionally be described as a linear programming problem [86].

2.2.1 Linear Approaches

Cheng et al. [87] described a novel use of the Harmony Search (HS) method. The hybrid harmonic search (HHS) technique replaces the randomization function with the global-best particle swarm optimization (PSO) and neighborhood search, maintaining the harmonic memory and pitch adjustment features of the HS algorithm. Furthermore, a new linear method is proposed to find the optimal weight arrangement for trussed structures subjected to self-weight and external loads [88]. The cross-sectional areas of the members are design variables. The force method determines inequality limits since isostatic structures require maximum and lowest axial stress. Moreover, Quyen et al. [89] provided a unique technique for optimizing truss size using the finite element approach. The purpose of truss weight reduction was achieved. Constraint criteria were developed to ensure the building conformed with equilibrium equations and standards for strength, stiffness, and compatibility. On the other hand, A novel and efficient method by combining border-search operator (BSO) and jump operator (JO) for optimizing spatial trusses has been created by Dizangian et al. [90]. First, an efficient random technique is used to find the global optimal point. Next, the initial local optimal point is discovered using a sensitivity-based technique to complete a series of evaluations.

An enhanced variant of the colliding bodies optimization (CBO) method, ECBO, is proposed by Kaveh et al. [91] to address problems such as delayed convergence and entrapment in local minima. Introducing chaos into metaheuristic algorithms is an effective way to improve their performance. Moreover, Flager et al. [92] examined the Fully Constrained Design (FCD) method for discrete component optimization in steel truss and frame systems. In contrast to other deterministic approaches, FCD does not need the first derivative of the goal and constraint functions in terms of design variables. A separate study uses an altered Heat Transfer Search Algorithm (HTS) version to improve truss designs [93]. Sizing optimization aims to reduce a structure's weight while considering stress and displacement constraints. Another paper by Degertekin [94] presented two harmony search optimization techniques (EHS and SAHS) that perform better in robustness and convergence than traditional HS. It put the new methods for optimizing truss construction size to the test. Additionally, the newly created Marine Predators Algorithm (MPA) is used for the first time to improve the design of truss structures by Bodalal et al. [95]. Finally, because of its excellent performance and small number of performance factors, MPA is a reliable optimization metaheuristic for truss sizing issues.

Modified Simulated Annealing Algorithm (MSAA) is used by Millán-Páramo [96] to optimize the weight of truss structures with discrete size variables. MSSA is a more sophisticated version of the simulated annealing (SA) method. Moreover, Manguri et al. [97] described a new computing method that, when combined with the conventional force method, finds the optimal number of actuators to minimize the cost of producing the cross-sectional area of assemblies made up of pin-jointed structures. According to their findings, the most active members were chosen for prestressing to control the internal force of high-stress bars and distribute stress evenly throughout the structure. Because of the reduced stress in critical sections, the designers were able to employ smaller cross-sectional areas than before. The optimized results of their work are presented in Fig. 8. Furthermore, The objective of Sawada et al. [98] is to improve the accuracy ratio

and convergence for the minimum-weight optimization of a discrete truss design by combining the unique randomized line search approach with GA. Another study [99] described a novel, ranked-based method for optimizing structure size. This approach uses violation-based sensitivity analysis and the borderline adaptive sliding technique (ViS-BLAST) on the feasible non-feasible (FNF) design space boundary.

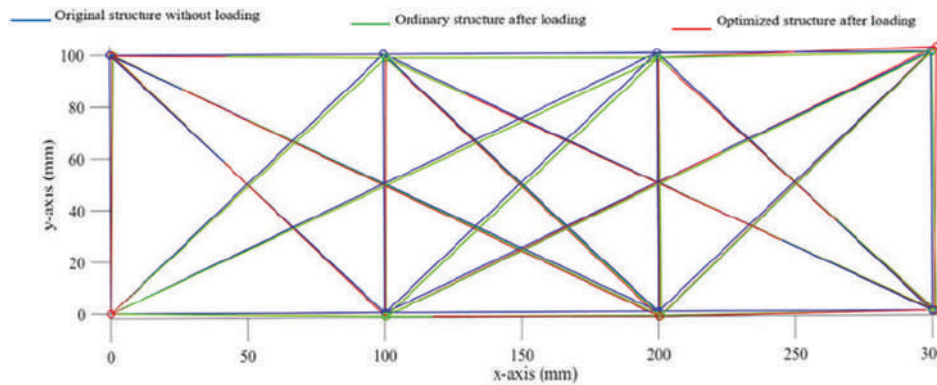


Figure 8: The distorted shape of the standard and optimized 20-bar truss and the original structure's shape [97]

Alkhraisat et al. [100] introduced the GWOM, a modified version of the grey wolf optimizer (GWO). The application of both discrete and continuous design factors optimizes the sizes of truss structures. The results show that mutation operators are successfully included in the GWO architecture to enhance exploration and exploitation capabilities while preventing local optima from becoming imprisoned. Moreover, Baghlani et al. [101] offered a teaching-learning-based optimization (TLBO) approach used to optimize the size of truss structures while addressing constraints efficiently. This method greatly enhanced the search for the perfect structure. The proposed technique can provide totally viable optimal designs. The resilience and quality of the solution are determined by the optimized structure's geometry shown in Fig. 9, TLBO MS outperforms the other two variants, with TLBO-PF (penalty function) coming in second, then fly-back mechanism (TLBO-FB), as the results are shown graphically in Fig. 10. The study's conclusions also show the success of the recommended TLBO-MS algorithm compared to other metaheuristic optimization methods.

Since it is essential to optimize and manage natural frequencies in structures and prevent resonance, the computation of truss structure natural frequencies in various moods based on weight reduction has been highlighted in research by Joubari et al. [102] and stated that It's not always the best course of action to optimize solely based on weight in several buildings that have sustained substantial damage from wind, earthquakes, or storms. Weight loss and natural frequency improvement have garnered much attention lately. The newly developed flower pollination algorithm lightens the truss framework's load by Bekdaş et al. [103], considers the varied size of the design as well. The novel algorithm, which draws inspiration from the process of plants self-pollinating and cross-pollinating, has the potential to combine local and global searches efficiently. A novel approach to weight optimization of truss structures using continuous variables that is straightforward was presented by Kooshkbaghi et al. [104]. The artificial coronary circulation system algorithm (ACCS), a relatively recent metaheuristic method, improves the size of truss structures. The optical center point of populations is used in all iterations of the ACCS optimization technique. Degertekin et al. [105] optimized truss structures using a cutting-edge meta-heuristic search approach known as "teaching-learning-based optimization" (TLBO). The method makes connections between how optimization issues are solved and how students learn. Eleven additional well-known and strong solvers, as well as a discrete optimizer, called the Soccer League Competition (SLC) approach., have also been used in

the search for a reliable and effective technique for optimizing truss designs are examined by Moosavian et al. [106] and statistical analysis is performed for them. Furthermore, optimization of the size of large-scale dome structures under dynamic restrictions is presented in [107]; the smallest size of design variables is discovered in the optimum design of these structures utilizing the Jaya approach. The design variables are the cross-sectional areas of the steel truss bar components.

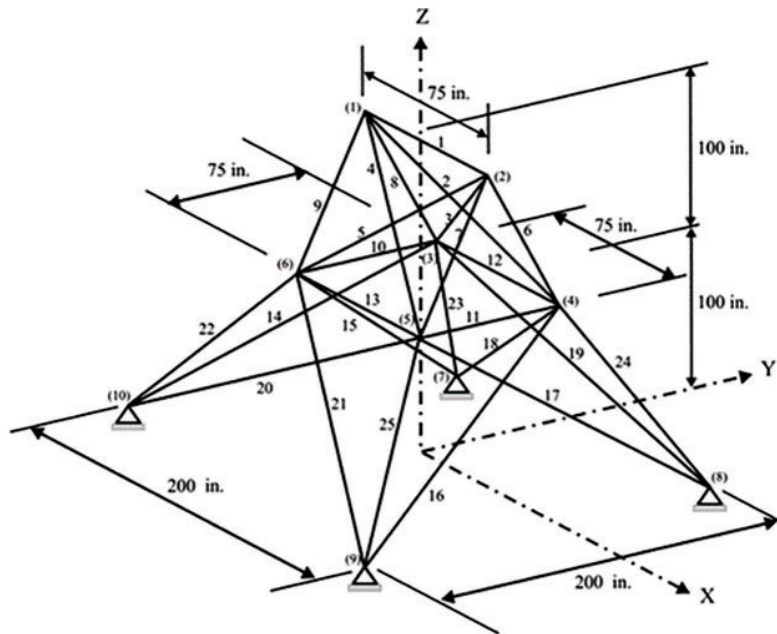


Figure 9: 25-bar spatial truss structure [101]

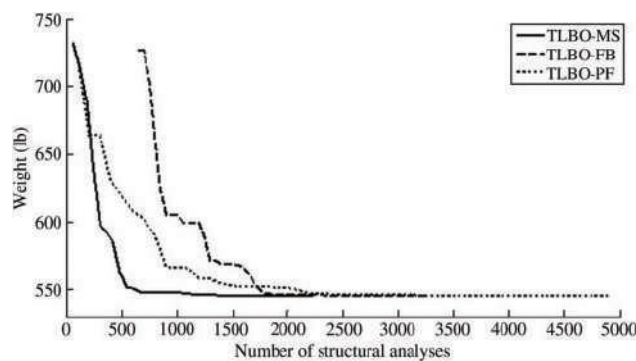


Figure 10: A comparison of the 25-bar spatial truss structure's TLBO variants' rates of convergence [101]

2.2.2 Non-Linear Approaches

Over the past 20 years, structural optimization problems have grown more difficult due to their severely non-linear design restrictions and complicated solution domains. This has prompted the creation and use of several cutting-edge metaheuristic algorithms [108]. Within the context of structural optimization,

Awad [108] has used the recently announced political optimizer (PO) technique. To find the best balance between exploration and exploitation, the algorithm should split the search agents into political parties vying for control of seats (electoral districts). This strategy is based on the multiphase political framework prevalent in parliamentary democracies. Khatibinia et al. [109] resulted in the acceleration of the pace of convergence of the gravitational search algorithm (GSA) and the creation of the accelerated multi-gravitational search algorithm (AMGSA). Combining the breeder genetic algorithm (BGA) operator mutation and simplex crossover (SPX) the multi-gravitational search algorithm (MGSA) produces AMGSA, an algorithm with a high exploration-exploitation balance. Moreover, by Pham et al. [110] metaheuristic techniques were applied to provide a computationally efficient solution to the discrete variable truss problem. This new technique is called “k-nearest neighbor comparison,” or simply “k-NNC”. The k-NNC determines the value of a new design by comparing it to the k-closest designs (k-nearest neighbors) of an existing design in the population. In addition, Farshi et al. [83] presented a novel method for creating truss constructions with the least weight possible. The force notion is included in a center-point optimization algorithm. The design variables are the member cross-sectional areas and redundant forces measured for each loading scenario the structure encounters. The optimization method employs the greatest hyperspheres within the available space.

To sum-up, several hybrid optimization algorithms were introduced for size optimization and linear approaches, such as the (HHS), which combines elements from (PSO) to promote global search efficiency while maintaining the benefits of harmonic memory. Additionally, enhanced variants of metaheuristic methods like the (ECBO) utilize chaos theory to prevent entrapment in local optima. These modifications address common challenges in optimization, like slow convergence and local minima issues. For nonlinear approaches, cutting-edge algorithms like the Political Optimizer were included, which simulates the dynamics of political parties to balance exploration and exploitation phases, and the (AMGSA), which integrates genetic mutation and crossover techniques to enhance convergence speed. Furthermore, techniques such as (k-NNC) optimize discrete variable truss designs by leveraging proximity-based evaluations to guide the search process.

2.3 Topology and Size Optimization

2.3.1 Linear Approaches

Truss structure topology and size may be simultaneously optimized with the help of a probability-based regulating method provided by Mortazavi [111]. To minimize the number of ineffective repetitions, the suggested method focuses on topology or size action. It uses the Boolean characteristics of the topological variables to forecast the search algorithm's behavior. In their paper, Faramarzi et al. [112] proposed a two-step procedure for size and topology optimization of two-dimensional truss designs. The topology problem was replaced with an appropriate size optimization problem to find the optimal truss topology. To preserve the topology found in the first phase, the optimal truss size has to be ascertained in the second. This presented a fresh difficulty for size optimization. Furthermore, Kaveh et al. [113] suggested a two-step process for optimizing two-dimensional truss designs regarding size and topology. A suitable size optimization problem was substituted for the topology issue to determine the ideal truss topology. The ideal truss size must be determined in the second phase to maintain the topology discovered in the first. This introduced a new level of difficulty to size optimization. Ali et al. [114] examined a combined strategy to increase the size and topology of structures under active management. A domain for the structure, load application sites, and support locations must be determined to attain the optimal structural topology. The optimal topology is reached by lowering the strain energy. Control system scaling and optimization are done iteratively once the optimal design has been determined. According to Brütting et al. [115], a formulation is provided

for optimizing the size and topology of truss structures under stress and displacement constraints using mixed-integer linear programming (MILP) problems.

Assimi et al. [116] present a multi-objective genetic programming (GP) technique to optimize the topology and size of the truss. The aim is to find the optimal cross-sectional areas and connectivity between the nodes, subject to constraints like nodal deflections, maximum allowable stress in members, and kinematic stability, to arrive at a set of trade-off solutions that satisfy all optimization objective functions. Similarly, Su et al. [117] optimized the size and topology of the truss structure with a genetic algorithm. Individual identification and sparse node matrix encoding can save processing time by reducing pointless structural analysis. Furthermore, an adaptive multi-island search strategy for multi-objective optimization problem (AMISS-MOP) is developed to enhance the dissemination and convergence of non-dominated optimal solutions for multi-objective optimization problems. On the other hand, Kumar et al. [118] employed a discrete meta-heuristic (MH) technique to optimize the size and topology, thereby enhancing truss design performance. Heat transfer search (HTS), ant lion optimization (ALO), whale optimization (WOA), dragonfly algorithm (DA), and teaching-learning-based optimization (TLBO) are the five basic machine learning algorithms that are being studied. Also, the size and topological optimization of the truss structure was done by Lee et al. [119], QbHS (Quantum-based Harmony Search) is a recently suggested methodology combining classical HS (Harmony Search) approaches with quantum computing. In addition, by figuring out the best joint connectivity and cross-sectional areas, Assimi et al. [79] aimed to reduce the weight of the search space. For the structural optimization problem, the maximum allowable stress, deflection, and kinematic stability are utilized. This approach determines the optimal response by letting computer programs emerge from a variable-length representation of potential responses. Zhong et al. [120] provided the cooperative coevolutionary genetic algorithm with independent ground structures (CCGA-IGS) for simultaneously optimizing the topology and size of discrete structures.

In addition, Mortazavi [121] confirmed that the interactive fuzzy search algorithm (IFSA) can search for structural optimization problems and demonstrated the limitations of this class of optimization problems, including various natural frequencies. Furthermore, the structural dimensions and layout considerations are taken into consideration separately. Moreover, The Shuffled Shepherd Optimization algorithm (SSOA), a recently created meta-heuristic algorithm, was explored by Kaveh et al. [122] for its potential in structural layout optimization. Also, Jawad et al. [123] described a technique to optimize the combined size-layout of planar and spatial truss systems using a simulated bee colony. The Artificial Bee Colony algorithm (ABC) seeks to compete with honey bees' intricate foraging techniques. The design variables of form optimization for the truss structure system are the joints' nodal coordinates and the members' cross-sectional areas.

2.3.2 Non-Linear Approaches

To optimize the weight of truss constructions, an inventive, non-uniform cellular automated technique was introduced by El Bouzouiki et al. [124]. The method was evaluated on several benchmarking issues. The suggested approach (see Fig. 11) was quicker than previous techniques, but the results indicated it was still relatively competitive regarding structural weight. Moreover, the effectiveness of the proposed technique may be further enhanced by using the parallelism property of the Cellular Automata (CA) methods, which makes them ideal for resolving large-scale structure optimization problems. On the other hand, Moghaddam et al. [125] developed a practical plan that minimizes seismic damage to nonlinear truss-like structures while optimizing their size and topology. The approach explains a large percentage of the variability in ground motion. This is accomplished by closely evaluating each member's Mean Annual Frequency of Exceedance (MAFE) in light of the relationships displayed and the results of the nonlinear analysis. The outcomes showed that the topology of the final structures and the cross-sectional values of the components are independent

of the initial design and that the ultimate optimal design is constant throughout a range of beginning cross-sections.

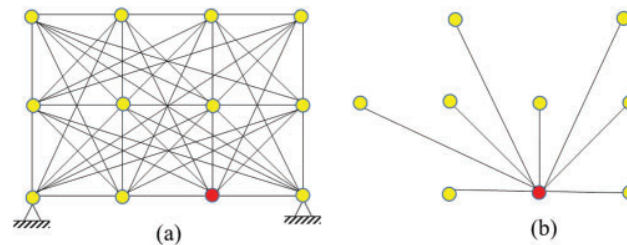


Figure 11: The representations of (a) Two-dimensional truss construction with non-uniform CA representation, and (b) the non-uniform unit cell on the border adapted from [124]

Moreover, Avci et al. [126] showed the effectiveness of the Improved Stochastic Ranking Evolution Strategy (ISRES) method for optimizing the size and design of benchmark truss structures. The results show how effectively the algorithm converges to optimal solutions, lowering the weight of the truss and satisfying complex design specifications. In addition, the electromagnetism-like mechanism algorithm and migration strategy (EM-MS) hybrid optimization approach is presented by Hosseinzadeh et al. [127]. The EM approach optimizes size in truss structure designs, and the migration strategy incorporates frequency constraints. Using the modified EM algorithm as a global optimizer, the hybrid approach under discussion guides the particles toward the region of the global optimum in the optimization problem's solution space, and depending on this migration method, the particles can more quickly and efficiently explore their search zone. Kaveh et al. [128] provided an updated version of the spiral optimization algorithm (SPO) that maximizes the shapes and sizes of truss structures while accounting for different frequency constraints. One such technique is the hypotrochoid spiral optimization algorithm (HSPO), which has been proposed. To improve the accuracy of the results and the method's capacity to break out of local optima, it reorganizes the search point movement operation and uses various tactics. Furthermore, The optimization of truss structure dimensions and arrangement is assessed using the newly developed Improved Vibrating Particles System (IVPS) method, which is an improved version of the VPS metaheuristic algorithm by Kaveh et al. [129]. The new technique almost always yielded results that were comparable, if not superior to, the best findings from its earlier complex algorithms. While Mortazavi et al. [130] examined how well the new particle swarm optimizer, called the integrated particle swarm optimization technique (iPSO), blends the improved fly-back mechanism with the weighted particle concept works.

Researchers used the probability space with boolean variables for linear approaches to improve the search and limit unnecessary iterations in complete optimization processes. Hybrid forms such as the (QbHS) incorporate the principles of optimization and quantum computing, enhancing the convergence and accuracy of the solution. Mixed techniques were also incorporated, and MILP and multi-objective genetic algorithms were used to improve topology and size while satisfying some constraints like nodal deflections, stress limits, and structural stability. For non-linear approaches, the most recent enhancements of meta-heuristic algorithms were underscored, which attempt to resolve the challenges related to the non-linear design space. For instance, the HSPO and IVPS algorithms employed near search dynamics to assist the HS takeover during local optima entrapment. Also, the iPSOM method employs weighted particles with a fly-back mechanism to solve optimization problems for structures' dimensions and topology. These advanced algorithms allow for a balanced exploitation-exploration process, which is of utmost importance, especially for natural frequencies or dynamic loading conditions constrained designs. Incorporation of

Cellular Automata, for instance, is a parallel method whereby optimization processes of large truss structures are made faster.

2.4 Shape and Size Optimization

2.4.1 Linear Approaches

The innovative bi-level hierarchical technique for optimizing member size and shape in both determinate and indeterminate truss systems is described by Flager et al. [131]. The method employs a novel algorithmic hierarchy: to optimize discrete sizing. The Fully Constrained Design (FCD) approach is paired with SEQOPT, a gradient-based optimization tool for continuous form variables. Lopez et al. [132] provided a unique way to enhance truss structure design while considering shape and size based on worldwide dependability. Response surface approximation, global optimization with the firefly approach, and global constraint formulation are used for reliability analysis. Also, using a job search-inspired (JSI) methodology and genetic operators of mutation, selection, and crossover, Serpik [133] proposed a meta-heuristic technique for the discrete optimization of truss size and form. Furthermore, an iterative approach to concurrently solve a size and form optimization sub-problem, subject to local stress constraints and Euler buckling, was proposed by Schwarz et al. [134] using Sequential Linear Programming (SLP).

To adhere to the SLP problem statement, a first-order Taylor expansion for the nodal movement and the buckling constraint is created. Cao et al. [135] stated that particle swarm optimization (EPSO) provided a better way to increase the size and form of truss constructions. The proposed EPSO combines partial swarm optimization (PSO) with a particle classification mechanism to reduce the need for extra structural studies throughout the optimization process while also increasing the computing efficiency of PSO-based structural optimization. To tackle the problem of early convergence in the traditional methods, Li et al. [136] proposed a novel PSO and integrated a structural design form optimization technique. Hosseini et al. [137] presented an approach to multi-objective optimization and asserted that, according to the statistics, the Multi-Objective Particle Swarm Optimization (MOPSO) technique generated more accurate answers. Moreover, the optimization of the size and form of truss structures is achieved by applying a recently developed method called ray optimization (RO) by Kaveh et al. [138]. Their optimization strategy aims to reduce the truss weight within the required bounds. SeyedOskouei et al. [139] used an advanced metaheuristic approach to deliver the optimum truss structural design. Using the Artificial Rabbit Optimization (ARO) approach, the algorithm's principal search loop considers rabbits' natural survival techniques, such as detour foraging and random hiding. The optimization of size and shape for several large-scale truss structures is investigated by Azizi et al. [140]. It uses the recently developed metaheuristic optimization method known as Chaos Game Optimization (CGO).

2.4.2 Non-Linear Approaches

Optimizing the size and shape of truss structures is a complex task due to its NP-hard nature, highly nonlinear design constraints, and conflicting design variables [141]. The nonlinear relationship between nodal coordinates and cross-sectional areas further complicates optimization, particularly for large-scale truss structures. A costly optimization engineering challenge arises when the shape and bar size variables are connected, resulting in a multi-modal search space with dynamic limits [142]. Lieu et al. [143] gave rise to the Adaptive Hybrid Evolutionary Firefly Algorithm (AHEFA), which considers a range of frequency restrictions and optimizes the size and form of truss structures. The Differential Evolution (DE) and Firefly Algorithms (FA) are combined in this strategy. Moreover, Roulette Wheels Selection-elitist-Differential Evolution (RWeDE), an improved version of DE, was proposed by Ho-Huu et al. [144]. The proposed approach is a specific form of the DE algorithm with two modifications. Initially, rather than employing

random selection, a Modified Roulette Wheel Selection (MRWS) with stochastic acceptance is utilized to choose members for the mutation phase. Also, Venkatesh Varma et al. [145] offered a comprehensive method for maximizing truss system size and shape while considering buckling limitations. The Change of Measure-Based Evolutionary Optimization (COMBEO) method, which manages complicated inequality constraints without violating them, is enhanced by this study.

The suggested approach uses a forward model to prevent buckling and capture the structure's geometric nonlinear reactions. Jalili et al. [146] developed the Chaotic Biogeography-Based Optimization (CBBO) algorithm, which effectively optimizes by utilizing a combination of chaos theory and biogeography-based optimization (BBO). Huynh Van et al. [147] recommended the use of Gaussian Local Search (GLS) in combination with Enhanced Comprehensive Learning Particle Swarm Optimization (ECLPSO) to maximize the size and form of truss structures while respecting design constraints. On the other hand, Ma et al. [148] offered a technique for employing subset simulation to optimize size and shape design for discrete and continuous variables. A special adaptive conditional sampling technique is used to generate fresh conditional samples in addition to the component-wise Metropolis-Hasting approach. Furthermore, Bodalal [141] introduced an innovative human-based metaheuristic method for truss structural configuration optimization called the Heap-Based Optimizer (HBO). Search agents are logically structured into hierarchies based on their fitness ratings, similar to the Corporate Rank Hierarchy (CRH) concept. Fig. 12 illustrates the application of Bodalal [141]. Moreover, Seyed Taheri et al. [149] used the Enhanced Biogeography-Based Optimization (EBBO) method to optimize the form and dimensions of the truss system while respecting its intrinsic frequency constraints. Additionally, a completely new strategy was described by Ahrari et al. [150] that optimizes truss structures' size and form simultaneously. A software known as the Fully Stressed Design (FSD) Evolution Strategy (FSD-ES) combines evolutionary algorithms' wide global search capabilities with the well-known deterministic approach to entirely stressed design. In addition, a newly proposed metaheuristic method is suggested by Farshchin et al. [151] for optimization issues, including size, shape, and frequency restrictions.

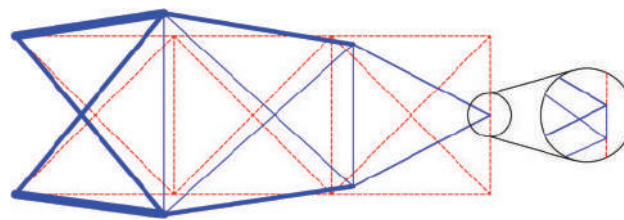


Figure 12: A. The best configuration for the 15-bar truss, B. detailed zoomed in for nodes 4 and 8 adapted from [141]

These optimization issues are thought to be very non-convex and non-linear. The suggested solution expands on using a single optimization process by utilizing many cooperative optimization processes. Moreover, for the first time, Miguel et al. [152] employed two of the most sophisticated metaheuristic algorithms developed in the last ten years, Harmony Search (HS) and Firefly Algorithm (FA), to optimize truss size and design under a variety of natural frequency limitations. However, the hybrid sine cosine firefly algorithm (HSCFA) is recommended to get better results with less finite element analysis. Meanwhile, to get better results with less finite element analysis, the Hybrid Sine Cosine Firefly Algorithm (HSCFA) is recommended by Tao et al. [153]. The Firefly Algorithm (FA) is analyzed, and the factors affecting its precision and computational effectiveness are revealed. Additionally, Teaching-Learning-Based Optimization (TLBO), a recently created methodology, was employed by Baghlani et al. [154].

Within the class of linear techniques, one such method is bi-level hierarchical optimization presented, wherein the fully constrained design is combined with SEQOPT to optimize both discrete and continuous variables. Other advanced meta-heuristics like (EPSO), (ARO), and (CGO) have further advanced the field by effectively searching much larger complex design spaces because of their classification mechanisms, natural survival strategies, and the principles of chaos, respectively. Within the non-linear optimization approaches, AHEFA and RWeDE combine evolutionary and swarm intelligence approaches to mitigate nonlinearity and achieve faster convergence with constraints. Likewise, (HBO) also adopts a hierarchical ranking model, while (TLBO) mimics learning processes to increase effectiveness in truss model exploration. Altogether, these techniques utilize a range of optimization functions and critical dimensions relevant to the design of trusses shapes and sizes.

2.5 Topology, Size, and Shape Optimization

Size, form, and topology are the three main categories structural optimization often falls into. In the past few decades, much work has gone into establishing methods for optimizing structures considering one or more of these aspects [155]. The most efficient truss optimization strategy takes topology, shape, and size (TSS) into account [156]. Dehghani et al. [157] introduced the future hybrid meta-heuristic algorithm CA-ICEA. The Imperialist Competitive Algorithm (ICA) was improved by the use of the Dolphin Echolocation algorithm (DE) and the Cellular Automata (CA) approach. The truss system's size, topology, and layout were optimized using the suggested manner. The numerical results of the solved instance demonstrated the computational advantages of using the CA-ICEA approach to determine the best topology of large-scale skeletal systems. Degertekin et al. [158] provided an improved iteration of the Jaya Approach (JA) for optimizing discrete truss structures, taking into account topology, size, and placement. Discrete Advanced JA (DAJA) is a unique approach that generates a sequence of descending pathways around each candidate design to boost the population. In Miguel et al. [159], the Firefly Algorithm (FA) was used to optimize the truss structure's topology, size, and shape simultaneously. The findings indicate that mixed variable optimization problems, a prevalent situation for these problems, are particularly well-suited to the technique. Potentially produce outcomes better than or equivalent to those reported in the literature of their paper, all at a reduced computing cost. Moreover, the integrated Particle Swarm Optimizer (iPSO) was suggested by Mortazavi et al. [160] as a way to optimize truss structures.

To solve simultaneous structural optimization issues, the iPSO effectively integrates the best aspects of the fly-back approach, weighted particle definition, and classic PSO into a single effective solution. Additionally, Carvalho et al. [161] examined fresh structural optimization problems using Generalized Differential Evolution 3 (GDE3) and presented some surprising but exciting results. Trade-off curves depict the number of unique cross-sectional areas employed in the solutions in relation to the ideal weights. The Jaya Algorithm (JA), a metaheuristic technique, is used by Degertekin et al. [162] to maximize the size and placement of truss structures. The outcomes demonstrate that JA can produce superior designs than other cutting-edge gradient-based and metaheuristic optimization techniques. In comparison, Tejani et al. [163] modified optimization using a framework for teaching and learning. To evaluate the efficacy of TLBO (MTLBO), it is utilized to simultaneously optimize the topology, form, and size of space and planar trusses. Stress, displacement, kinematic stability constraints, and discrete and continuous design factors are present in every benchmark challenge.

There are also nonlinear analysis applications in optimizing truss structures in this form. Souza et al. [164] provided a generic technique for optimizing transmission line tower topology, size, and shape. Stripped bracing may be readily incorporated using the recommended method. In every case studied, staggered bracing was shown to be the optimal design. The results show that the recommended strategy

has much potential, but further research is still required. El Bouzouiki et al. [165] offered a bi-level non-uniform Cellular Automata (CA) method that may be employed to resolve the minimal weight optimization problem for truss designs in the case of stress and displacement constraints. Regarding the algorithms in this subsection, CA-ICEA outperformed the Imperialist Competitive Algorithm and Dolphin Echolocation and Cellular Automata for truss structure optimization. In another study, the Discrete Advanced Jaya Approach appears to improve the iteration process of the design candidate's development to improve truss topology, design optimization, and size. As for trusses and related structures, researchers employed the Firefly Algorithm to optimize engineering structures with mixed variables. At the same time, another group of researchers integrated Particle Swarm Optimiser used PSO with combinations of weighted parameters to perform optimal topology, shape, and size optimization. For the nonlinear case, a bi-level Cellular Automata approach was used to optimize designs with a minimal weight that conforms to stresses and displacements. In addition, the details of most literature references about trusses and solvers are tabulated in Table 1.

Table 1: Implemented optimization and solvers on types of pin-jointed structures

Ref.	Type of optimization	Structural types	Solvers
[10]	Topology	Truss, Timber, and Steel	MILP
[21]	Size	Single-layer dome	Interior-point
[48]	Topology	Truss structures–Periodic lattices	–
[51]	Mult objective	Trusses	GAs
[53]	Topology	Trusses	NSGA-II
[55]	Topology	Truss and Beams	–
[56]	Topology	Large-scale spatial skeletal structures	MGHSA
[61]	Topology	Trusses, Truss-like plates	DLO
[65]	Topology	Trusses	SQP
[66]	Topology	Truss and Beams	NLP
[68]	Topology	Cable-truss	–
[72]	Topology	Truss-like microstructures	MMA
[74]	Topology	Truss lattices,	ROM
[75]	Topology	Truss and Lattice structures	ZPR
[83]	Weight	Trusses	–
[85]	Size	Space trusses	HBB-BC
[90]	Size	Spatial truss	BSO, JO
[94]	Size	Truss	HS
[95]	Weight	Trusses	MPA
[96]	Weight	Trusses	MSAA
[97]	Size, Actuation	Cable & Trusses	SQP
[98]	Size	Trusses	GA
[99]	Size	Trusses	ViS-BLAST
[100]	Size, Shape and Topology	Trusses	GWOM
[101]	Size	Trusses	TLBO
[102]	Size	Trusses	ABC

(Continued)

Table 1 (continued)

Ref.	Type of optimization	Structural types	Solvers
[108]	Size	Trusses	PO
[113]	Size and Topology	Trusses	CBO
[116]	Size and Topology	Trusses	GA
[116]	Multi-objective	Trusses	GP
[117]	Multi-objective	Trusses	AMISS-MOP
[119]	Size and Topology	Trusses	QbHS
[120]	Size and Topology	Bridges and Towers	CCGA-IGS
[121]	Sizing and Layout	Trusses	IFSA
[122]	Sizing and Layout	Trusses	SSOA
[123]	Sizing and Layout	Trusses	ABC
[125]	Size and Topology	Truss-like structures	–
[126]	Sizing and Layout	Trusses	ISRES
[130]	Sizing and Layout	Trusses	iPSO
[133]	Size and Shape	Trusses	JSI
[135]	Size and Shape	Trusses	EPSO
[136]	Size and Shape	Trusses	PSO
[137]	Size and Shape	Trusses	MOPSO
[138]	Size and Shape	Trusses	RO
[156]	Size, Topology, and Shape	Trusses	FSD
[158]	Size, Topology, and Layout	Trusses	DAJA
[159]	Size, Topology, and Shape	Trusses	FA
[160]	Size, Layout, and Shape	Trusses	iPSO
[161]	Size, Layout, and Shape	Domes	GDE3
[162]	Size, Topology, and Layout	Trusses	JA
[163]	Size, Topology, and Shape	Space and Planar trusses	MTLBO
[165]	Size, Topology, and Layout	Trusses	CA

3 Frames

A basic frame structure consists of joints and beams that are built to sustain specific loads [166]. Like truss structures, optimization is vital in designing frames, especially in lightweight frames and steel sections. In this part of the review, the different optimizations are focused on as classified in Fig. 13.

3.1 Topology Optimization

Fu et al. [167] suggested a method to optimize the topology, materials, and beam types of complex structures made up of shells, bars, beams, or combinations of these; the extended ground structure is the basis for the optimization problem and is made up of different components or cross-sectional beams with other materials at each location. Kaveh et al. [36] demonstrated the process of utilizing Improved Ant Colony Optimization (IACO) to create planar steel structures. The process is divided into two stages: local search and global search. The findings demonstrate a considerable reduction in the computation time required for the improvement.

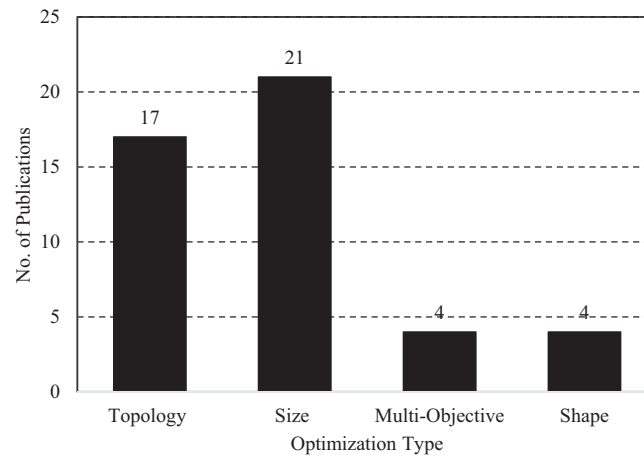


Figure 13: Classification of literature worked on frame structure optimization

A periodic spaceframe structure is intended by Lim et al. [37] to maintain the highest torsional and flexural rigidities while maintaining the maximum mass. Once the spaceframe has been parametrized using its component unit cells, its shape, topology, and size as a generic structure should be optimized using multi-objective Genetic Algorithm (GA) (MOGA) optimization. Moreover, a performance-based optimum seismic design for frame structures is demonstrated by Kaveh et al. [168] using the Ant Colony Optimization (ACO) technique. A nonlinear analysis utilizing a basic computer-based push-over approach that considers first-order elastic and second-order geometric stiffness characteristics determines the structural response at different seismic performance levels. Yamada et al. [169] gave frequency limitations, and suggested a method that offers a locally optimum solution for frame topology optimization. The approach uses a nonlinear programming methodology to gradually reduce the relaxation value and solve a relaxation problem. For frame systems with unexpected geometric or material properties, a resilient topology optimization strategy is proposed in [170]. Based on a comparison of the resilient designs, the final designs exhibit greater load variety under both sources of uncertainty than deterministic designs. Also, Ahmadi et al. [171] offered a strategy for optimizing the topology of skeletal constructions with deficient structural components, even when the members are not perfectly straight. In addition, in [172], seismic analysis was performed on steel frame structures with brace configurations using topology optimization and a truss-like material model. The truss-like components in the first topology optimization design domain were chosen to match the original steel frame construction, establishing the first truss-like continuity.

In the face of geometric uncertainty, Changizi et al. [173] developed a practical stress-based topology optimization approach for frame systems. The suggested approach makes minimum assumptions about the lengths of the frame members and propagates uncertainty up to the response level defined as the highest anticipated value of von Mises stresses throughout the domain using the stochastic perturbation technique. Furthermore, the stress-based topology optimization method for steel frame structures with cross-sectional properties mapped from I-beam sections of a design manual is described in Changizi et al. [38] in a computationally efficient manner. Two methods were suggested to enhance frame gradient-based topology optimization. The structural design was initially based on actual data for components that were manufactured in factories. The significant variance in the data set was explained using quantile regression, which was also used to quantify the moment of inertia and other cross-sectional characteristics as a function of member cross-sectional area. A workable optimization strategy for the ideal seismic design of nonlinear truss structures is presented. For the optimal seismic design of non-linear truss systems, a feasible optimization

approach is provided by Hajirasouliha et al. [174]. Uniform deformation may enhance the topology of rigid constructions subjected to seismic and gravitational forces. While each seismic excitation has its own ideal topology, one may reach practical results by averaging the best distribution patterns for a series of earthquakes.

Furthermore, Gao et al. [175] researched buckling constraint optimization of continuum structure topology. New methods, such as buckling load factors and volume, are being developed to decrease structural compliance. In [176], the goal was to construct the stiffest frame structure possible using hollow beam members using method of moving asymptotes (MMA) while accounting for the required load and stability. The proposed method may be used to build frame systems with beam members having distinct cross sections. Additionally, An approach for the topology optimization of multi-material active buildings is suggested by Wang et al. [13] to lessen its negative environmental effects, as shown in Fig. 14. The approach is based on the density-based topology optimization concept. Passive and active materials must be arranged as effectively as possible throughout the topology optimization approach while considering constraints and optimization goals. Changizi et al. [177] proposed a computationally efficient topology optimization technique for steel frame structures that incorporates user-defined safety factors against global and individual member instabilities. While Miyazu et al. [178] suggested an optimization method for creating flexible arch-supporting structures that lessen the acceleration response in the direction of the roof structure.

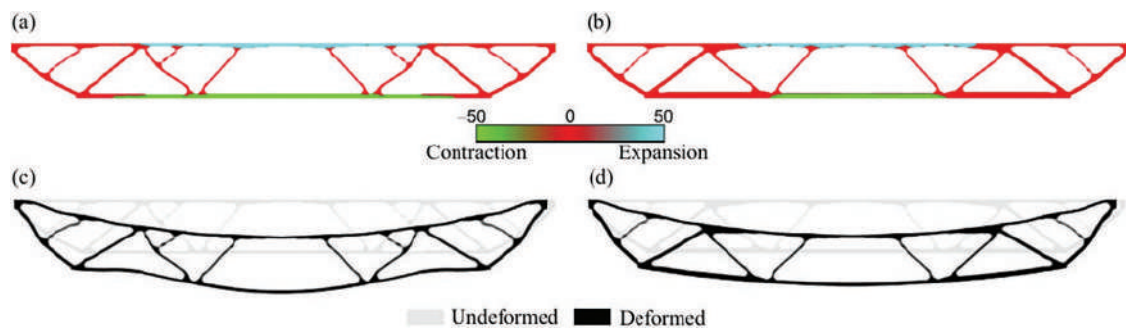


Figure 14: a and b exhibit the ideal designs for embodied and total energy savings, respectively, whereas c and d reflect the two structures' final deformations. Actuation effects of active material in optimal active structures and the associated deformations [13]

In this subsection, Modern optimization techniques incorporate multi-objective strategies, such as genetic algorithms to balance rigidity and total mass in spaceframes or improved ant colony optimization to minimize computation time in 2D steel structure design and optimize local and global search. The seismic performance is evaluated with a push-over method considering dynamic loads, which optimizes the performance-based level to achieve an optimal frame for durability against seismic damage. Moreover, stress-based topology optimization, including stochastic perturbations that control uncertainties in structural parameters, has been conducted to enhance design reliability for steel frames. Adaptive methods such as (AIO) optimize structural topology with actuators' location and control strategies to fulfil different load/performance actions and simultaneously meet different load and performance criteria. Further, they incorporate buckling limits and environmental effects through the optimization of material density or compliance, which results in structural designs that are lighter, more anti-fragile, stable, and resilient.

3.2 Shape Optimization

Research by Cui et al. [179] used nodal or elemental sensitivity data to develop a morphogenesis approach for spatially limited framed structures. As numerical examples demonstrate, this method effectively minimizes structural bending moments while producing structures that largely transfer external force via axial stress. While Shimoda et al. [180] described a non-parametric optimization approach for constructing solid, shell, and frame structures with strong shapes in the face of uncertainty. The suggested non-parametric form optimization method for robust design consists of four fundamental phases derived from structural eigenvalue analysis and gradient functions. Moreover, a two-phase genetic algorithm (GA) is proposed by Kociecki et al. [181] to conserve weight and build free-form steel space-frame roof structures using commercially available rectangular hollow structural sections (HSS), modifying the roof form, member and column sizes, and topology will increase structural efficiency.

3.3 Size Optimization

Scale optimization of high-rise structures is complicated due to their discrete design elements, implicit optimization function, and expensive individual assessment [182]. A hybrid meta-heuristic approach that combines Particle Swarm Optimization (PSO) and Harris Hawk Optimization (HHO) is used by Mashayekhi et al. [183] to design Castellated beams. The desired purpose determines the whole cost, including construction and material expenses. Constrained functions have both geometric and coding limitations. A working set technique with interesting theoretical elements has been suggested by Verbart et al. [184] for size optimization problems in structures subjected to dynamic loads, with few variables but many restrictions. This method may improve the structural performance of frame structures subjected to various loads and stress limits. Moreover, An improved fireworks algorithm (IFWA) is suggested by Gholizadeh et al. [185] to lessen the shortcomings of the initial method. Also, Su et al. [186] recommended using standard member sizes in applying the Binary Comprehensive Learning Particle Swarm Optimization (BCLPSO) method to get the optimal design for nonlinear steel structures. Furthermore, Aydogdu et al. [187] offered four updated iterations of the well-known grasshopper optimization algorithm (GOA), which may be applied to tackle complex, large-scale, real-size steel truss and frame-building problems. GOA imitates the swarms of grasshoppers during their feeding. The objective is to improve the algorithmic performance of the GOA.

Moreover, Kaveh et al. [84] offered a revised iteration of the whale optimization technique to maintain a propensity toward balance in intensity and variation. This endeavor aims to improve and accelerate the convergence of the solution by stabilizing the original WOA formulation. The unique Extended Whale Optimization Algorithm (EWOA) is evaluated using truss and frame structure size optimization problems. Gholizadeh et al. [188] investigated and compared two distinct algorithms: the improved multiverse (IMV) algorithm for the discrete size and optimization of structures based on cosmology inspired by nature and the improved black hole (IBH). The numerical results show that the suggested improved algorithms outperform the metaheuristics mentioned in their literature. In addition, Kaveh et al. [189] introduced the Improved Shuffled-based Jaya (IS-Jaya) technique, a new iteration of the Jaya algorithm. The suggested technique simultaneously adds the concept of the shuffling process and a path out of local minima to the conventional Jaya algorithm. Furthermore, Ma et al. [190] present an interval size optimization for geometric analysis concerning beam structures based on beam theory. This optimization makes the beam construction lightweight and guarantees its lifetime. Moreover, the restricted size optimization based on the LRFD-AISC is carried out by Dede [191] utilizing the W-shape's cross-sectional area and integer design variables.

The challenge's goal is to find the minimum weight of the grillage assembly. Furthermore, the advantages of the two-phase GA optimization method and the technology provided by Kociecki et al. [192] include automation of the design process for complex and unique structures, reduced iterative design time, and

more cost-effective design. Also, the optimization issue is represented by Van Mellaert et al. [193] as a Mixed-Integer Linear Programming (MILP) problem with structural analysis equations as constraints. Also, to enhance Teaching-Learning-Based Optimization (TLBO)'s performance, Quantum Teaching-Learning-Based Optimization, or QTLBO, is presented by Kaveh et al. [194]. Kaveh et al. [195] introduced Multi-Adaptive Harmony Search (MAHS), an enhanced Harmony Search technique. MAHS's novel pitch-adjusting mechanism produces better results and faster convergence since the intensification and diversification stages of the optimization search process closely complement each other. While a non-parametric shape size optimization method was developed by Shimoda et al. [196] to alter the large-scale frame designs' static deformation. The Lagrange multiplier approach, the material derivative method, and the adjoint variable method are theoretically used to determine this issue's form gradient function, size gradient function, and optimality criterion.

In summary, high-rise structures are complex due to the nature of their design variables and, therefore, computationally expensive to optimize. Advanced hybrid algorithms (for example, combining Particle Swarm Optimization with Harris Hawk Optimization) address geometric and functional constraints for minimizing material cost and maximizing structural efficiency. Newer algorithms such as the (EWOA) and the (IS-Jaya) focus on systematic structural performance enhancement by finding stable solutions, thereby preventing reaching local minima. (MILP) effectively embeds structural constraints, and the Quantum and Multi-Adaptive, Harmony Search methods, improve convergence speed by balancing exploration and refinement while enhancing solution quality. Similar non-parametric shape optimization methods also help reduce weight, improve durability, and produce efficient large-scale structural designs. The types of optimizations and algorithms used to optimize different types of frame structures are tabulated in Table 2.

Table 2: Implemented optimization and solvers on types of moment-resisting structures

Ref.	Type of optimization	Structural types	Solvers
[3]	Size and Layout	Trusses and Frames	SA
[37]	Weight	Space Frames	MOGA
[84]	Size	Trusses and Frames	EWOA
[167]	Topology	Beams and Shells	GA
[169]	Topology	Frames	–
[170]	Topology	Frames	–
[171]	Topology	Frames	Interior point
[176]	Topology	Hollow cylindrical beam	MMA
[177]	Topology	Steel Frames	Interior-point
[180]	Shape	Solid, Shell, and Frame	–
[181]	Shape	HSS Frames	GA
[182]	Size	Frames	Dividing RECTangles
[184]	Size	Frames	–
[185]	Size	Trusses and Frames	IFWA
[186]	Weight	Steel Frames	BCLPSO
[187]	Weight	Frames	GOA
[188]	Size	Steel and RC Frames	IBH and IMV
[189]	Size	Trusses and Frames	IS-JAYA

(Continued)

Table 2 (continued)

Ref.	Type of optimization	Structural types	Solvers
[190]	Size	Beams	MMA
[191]	Size	Steel Grillage Structures	JAYA
[192]	Size	Space Frame Roofs	GA
[193]	Size	Frames	MILP
[194]	Size	Truss & Frames	TLBO
[195]	Size	Truss & Frames	MAHS
[197]	Size, Layout and Topology	Truss & Frames	PGO

3.4 Multi-Objective Optimization

A Plasma Generation Optimization (PGO), a novel physics-based metaheuristic algorithm, was used by Kaveh et al. [197] to maximize the growth, organization, and structure of the skeletal system. PGO is utilized to mimic the creation of plasma, and the processes that go into making plasma are repeated. Six design examples illustrate how to assess PGO resilience and performance under different structural modifications. The optimization findings suggest that PGO could find the best solutions with fewer structural analyses. Zhao et al. [198] investigated a frame structure's combined topology and size optimization problem, considering displacement, stress, and stability constraints. The study proposed a combined topology and size optimization approach for frame structures that prioritizes structural weight by balancing displacement, stress, and stability restrictions. In addition, Moghaddam et al. [125] offered a practical technique for optimizing the size and topology of nonlinear truss-like structures to survive seismic activity while accounting for a significant degree of uncertainty in ground motion. The best design solutions are produced by using an adaptive optimization technique. Also, Daneshfar et al. [199] investigated how to employ beam components in structural analysis models to optimize the size and topology of the frame structure according to the ground structure technique.

4 Tensegrity Structures

Tensegrity structures are light but stable systems composed of compression members called struts called in and tension components known as cables arranged to integrate their component parts in self-stressing equilibrium. The optimization in such structures is done to minimize total weight and enhance structural resistance. In a study about tensegrity robots, axial forces in cables attached to a tensegrity structure were calculated using a developed quadratic optimization (QO) approach [200]. In addition, a novel tensegrity sphere structure was introduced using spherical logarithmic spirals and Double Helix Tensegrity topology to optimize minimal mass under torsional loads [201]. Wang et al. [202] developed Total Energy Optimization (TEO) to enable the creation of minimal energy configurations for tensegrity implementing branch-and-bound (BNB), hybrid Quesada-Grossman (HQG), and mixed-integer sequential quadratic programming (MISQP). In addition, Masic et al. [32] are also concerned with designing tensegrity structures with the ideal mass-to-stiffness ratio. Also, particle swarm optimization is used by Yildiz et al. [203] to solve the prestress optimization and size of tensegrity booms.

Nonlinear finite element models of tensegrity structures and solution techniques provide an initial starting point. Additionally, Mills et al. [204] examined the appropriateness of intricate airplane wing designs and contrasted the structural performance of such designs with a typical conventional wing structure through

simulation. In addition, a generalized objective function for the topology finding of tensegrity structures is proposed by Xu et al. [205], in order to quickly identify numerous tensegrity structures through a s and give selection priority to various members. Moreover, Wang et al. [206] investigated topology optimization based on mixed integer nonlinear programming (MINLP) to identify the optimal active tensegrity structure design; their typical benchmark examples demonstrated that topology-optimized active designs obtained using the proposed approach could consume less material than sizing-optimized active tensegrity designs, resulting in lighter structures.

5 Concrete Structures

Structural optimization for concrete structures minimizes total weight or improves the structural capacity against possible loads considering concrete codes. A nonlinear programming approach is implementing SQP to minimize the total cost of the reinforced concrete structure considering American Concrete Institute (ACI) 2005 code requirements [207]. Sirca Gene et al. [208] proposed a mixed integer-discrete nonlinear programming (MIDNLP) approach to minimize the total cost of precast prestressed reinforced concrete beams in bridges. Amir [209] proposed a topology optimization approach to reducing the total structural weight by minimizing the steel and concrete distribution, considering load-bearing capacity. A computational methodology proposed by [210] achieved 30% material savings while enhancing sustainability. In addition, a unique method based on topology optimization is given by Bruggi et al. [211] for the thoughtful placement of fiber reinforcement in masonry constructions. Rizzian et al. [212] offered a feasible numerical optimization method for comparing the seismic spectra and reactions to similar solicitations of a base-isolated and a traditional fixed-base reinforced concrete ordinary building, depending on what is appropriate for each scenario. Regarding optimization for improving structural stability, Khan et al. [210] used a diagonal approach to increase the systemic resistance of brick masonry structures with RC stiffeners. Also, Bondarabadi et al. [213] offered a unique cascade optimization strategy that minimizes the torsion of each RC structure narrative as a fitness function while satisfying the necessary constraints. In addition, Lee et al. [41] examined the stress interpolation function and the stress discrepancy problem to determine the best reinforcing layer design under in-plane load.

6 Other Structures

In this section we deal with the structural optimization conducted on engineering structures other than structural types mentioned above and we refer to the most significant studies conducted in structural optimization. A computational method based on a project algorithm (PA) was proposed to minimize self-deploying energy by considering the force density vector model [214]. In research by Gillman et al. [43], origami was used to generate auxetic metamaterials, and topology optimization provided an independent and reliable means of determining these structures. The potential value of formal structural optimization for foldable and deployable structure design was investigated in [215].

Similarly, the topology and size of geometrically nonlinear dome structures are modified in [216] to reduce total weight and joint displacements while maximizing load-carrying capacity. Moreover, Cucuzza et al. [217] optimized the guyed radio mast's dimensions and design using a genetic algorithm, considering the effects of wind, ice, and seismic forces. They suggested a novel method for considering self-weight loading while improving structural topology in three spatial dimensions. Furthermore, Marotte Luz Filho et al. [218] proposed a new technique for optimizing structure topology in three spatial dimensions while considering self-weight loading. Also, Guo et al. [219] used movable bars to solve topology optimization in geometrically nonlinear three-dimensional structures. Moving morphable bars have fewer design variables due to their different geometrical forms, which speeds up convergence and improves processing efficiency. NURBS

effectively optimized the design of elastic arches using an iso-geometric analysis based on non-uniform B-splines.

To sum-up the last three sections, the methods and optimization algorithms are briefly summarized. Advanced optimization algorithms for adaptive structures can enhance performance selections and efficiency. Total Energy Optimization (TEO) or particle swarm optimization (PSO) allows the determination of minimal energy configurations and prestress for tensegrity structures. Nonlinear solvers (Newton-Raphson method and augmented Lagrangian approaches) are used to tackle problems related to force density and deformation. Moving morphable bars (MMB) is a design variable in topology optimization that simplifies the topological problem and speeds up convergence. Focus on neural networks' iterative-based management, especially convolutional neural networks (CNN), and the ability to predict optimal designs from large datasets using new approaches that result in designs without needing engineering experience. SQP reduces the use of materials, decreases costs, and applies nonlinear programming techniques to concrete. Seismic loading conditions are primarily considered when optimizing reinforcement layouts when designing structures, often coupled with gradient-based optimization integrated with a finite element model. [Table 3](#) presents the objectives and optimization algorithms of various structural types presented in this section.

Table 3: Objectives, solvers of other structural types

Ref.	Objectives	Structural types	Solvers
[200]	Weight	Tensegrity robots	QO
[201]	Weight	Tensegrity	–
[202]	Energy	Tensegrity	BNB, HQG
[203]	Size	Tensegrity	PSO
[204]	Performance	Tensegrity aircraft wings	MILP
[205]	Topology	Tensegrity	MILP
[206]	Topology	Tensegrity	MINLP
[207]	Cost	prestressed concrete	MIDNLP
[209]	Topology	prestressed concrete	–
[210]	Volume	RC stiffeners in masonry	–
[211]	Volume	RC stiffeners in masonry	–
[212]	Size	RC	MOGA-II
[213]	Torsion	RC	Cascade
[215]	Size and Shape	Foldable	SQP
[217]	Size and Shape	Mast	GA

7 Recommendations for Future Studies

- Applying numerical gradient of the objective and constraint functions in case of gradient-based optimizations.
- Further enhancements to teaching-learning-based optimization methods.
- Exploration of adaptive teaching factors in various applications.
- Exploring more complex loading scenarios optimization and load combinations in topology.
- Further studies on structural optimization considering material nonlinearity and/or dynamic loadings.

- Investigating alternative methods for failure probability handling.
- Further investigations on structural optimization considering geometry and material uncertainties.
- Structural optimization techniques can be further developed through applications with other structural design and control disciplines. Form-finding, structural control, seismic design, active structures, and structural design all depend on optimization approaches.
- Since Artificial Intelligence (AI) is already contributing to growth in different sectors, we suggest establishing and expanding connections between AI and optimization in various structures to find new methods and significant applications in this area.
- The applications of machine learning techniques such as Artificial neural Networks, linear regression, Non-linear Regression, and Multi-linear Regression should be improved to achieve better results in structural optimization.

8 Conclusion

Applying optimization techniques is crucial for improving the effectiveness and performance of civil engineering constructions. Topology, size, and shape may all be enhanced to make an ideal structural system. Several studies were undertaken to optimize the forms and proportions. The literature leads to the following conclusions that can be made:

- By utilizing multi-objective optimization, size, shape, and topology, engineers may create more dependable, affordable, and high-performing designs. Given the impending environmental catastrophe, structural optimization is essential for reducing the built environment's carbon footprint.
- In addition to boosting material efficiency, reducing weight may result in considerable cost savings on production, shipping, and operating expenditures. Furthermore, modern design strategies strive to minimize the energy consumption and carbon emissions of the built environment to the lowest feasible level, given the construction sector's significant contributions to material usage, energy consumption, greenhouse gas emissions, and waste creation.
- The number of studies conducted on pin-jointed structures is higher than that on other structures like frames. This may be due to the fact that, the former are the simplest form of structural systems with less variables compared to frames.
- The most used optimization algorithms in the studies presented in this review implemented gradient-based optimization algorithms like SQP, MMA, and free gradient optimizers like GA, PSO, SA, ACO, DE, Artificial Bee Swarm (ABS), and GWO.
- Nowadays, machine learning techniques, such as supervised, unsupervised, and deep learning techniques, use a combination of gradient-based and non-gradient-based methods.
- Numerous studies exist in this field, so we had to select only a limited number of articles. Moreover, this review only considers papers published between 2005 and 2024; there are other significant articles published before the period

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Paper III

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Deformation mitigation and twisting moment control in space frames

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ABSTRACT

Over the last five decades, space frames have centered on the modernization of touristic zones in view of architectural attractions. Although attempts to control joint movement and minimize axial force and bending moment in such structures were made sufficiently, twisting moments in space frames have been underestimated so far. In space frames, external load or restoring the misshapen shape may cause twisting in members. We herein developed a robust computational algorithm to reduce the induced torsional moment through shape restoration within a desired limit by changing the length of active bars that are placed in space frames. Applying optimization algorithms like interior-point and Sequential quadratic programming (SQP), a direct correlation was pursued between bar length alteration and twisting in structural members. A numerical model of a single-layer space frame resembling an egg captures the twisting moment in all members, achieving a specified limit. The overall length change of the active members using an iterative process based on a heuristic that considers a threshold on the minimum length change of the active members.

1. Introduction

In the last five decades, constructing large space frames has become a feature of modern urban landscapes, seamlessly blending architectural finesse with artistic expression [1]. These structures, revered in architecture and structural engineering circles, revolutionize the approach to spanning vast areas. Space frames, characterized by their lightweight yet robust design, efficiently distribute loads across their framework, obviating the need for numerous internal supports [2]. Widely deployed in airports, exhibition halls, and sports arenas [3], space frames epitomize adaptability and versatility, enabling the realization of architectural designs. However, ensuring the safety of individual elements from the forces they endure presents a challenge in space frame engineering. Often, structures experience shape deformation under the action of external loadings. Researchers have been developing methodologies to overcome safety and serviceability problems by reducing the displacement response through the length change of active members.

Structural shape adjustment has been studied by a few researchers before the 90th, among them Weeks [4] and Haftka and Adelman [5]. Kwan and Pellegrino [6] reported the relationship between bar length change and axial force in members of space trusses; their study was based on the force method (FM) based on the Singular Value

Decomposition of the equilibrium matrix (SVD-FM). In a study, Rekso-wardojo and Senatore [7] demonstrated the equivalence between the intuitive Integrated Force Method (IFM) and the computationally efficient Singular Value Decomposition Force Method (SVD-FM). Recently, Wang and Senatore [8] introduced a new method named the Extended Integrated Force Method (EIFM) to model the response of prestress-stable structures extending the IFM to kinematically indeterminate systems. Saeed and Kwan [9] proposed a study to control nodal displacements and axial forces simultaneously in pin-jointed assemblies. The study was developed to assign a domain for displacements of nodes in cables [10]. Shape control of a loaded cable structure was experimentally applied on a 3D cable net [11]. Regarding studies on shape control in space frames, an attempt has been made to reshape a single-layer egg-shaped dome space frame [12]. While research on adaptive structures has focused on truss systems, few studies have been conducted on adaptive tensioning systems for concrete slabs [13] and high-speed railway bridges [14] for efficient load management and significant material reduction. However, formulating the relationship of actuator length change to shape and internal forces previously studied in relation to adaptive structures [7,9], these studies were not interested in the effect of structural member length change on the twisting moment in space frames.

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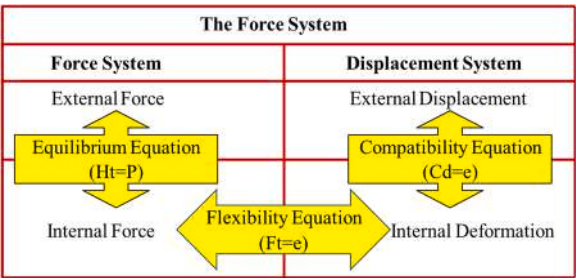


Fig. 1. The diagram of the force method system.

When it comes to structural optimization in adaptive structures, bar size optimization can be applied to cable and truss structures using actuators [15]. Recently, Senatore and Wang [16], formulated a new integrated structure-control All-In-One (AIO) method for topology optimization of adaptive structures. This method obtains global optima of the total system mass (structure + actuators) through simultaneous optimization of the structural topology, actuator placement, and control commands.

Different types of space frames and their analysis methodology were studied by Lan [17]. In addition, the analysis and design of space frames were presented by Ramaswamy et al. [18]. Furthermore, Galishnikova et al. [19] conducted a comprehensive study for finite element analysis applied to space frames, encompassing configurations with and without torsional restraints. The methodology entails delineating the orientation and approach for conducting first-order analyses on space frame elements. Generally, the twisting moment in structures involves the rotation of an element due to external loads, impacting stability and stress distribution within the structure [20,21]. Manguri et al. [22] reported mitigating bending moment in space frames by adding extra members in optimal locations.

When it comes to mitigation of twisting moments in frame structures, attention has been paid to the induced torsion caused by dynamic loadings. For instance, a study reported that a significant twisting moment developed during earthquakes in columns [23]. Likewise,

another study reported the twisting moment in symmetric buildings derived from the soil-structure interaction [24]. Anagnostopoulos et al. [25] presented a detailed review of the effects of earthquake torsion on buildings. Several devices, such as triple-friction pendulum systems [26], multiple magnetorheological dampers [27], and multiple-tuned liquid dampers, have been investigated [28,29] to mitigate torsion-induced damages during earthquakes. However, dynamic loading is not the only source of torsion. Other sources exist, such as eccentric static loadings [30] and structural irregularity [31].

External loads and shape control can cause significant changes in twisting moments in space frames. So far, less attention has been paid to the influence of bar length change on the twisting moments. This study addresses the mitigation of deformation and twisting moments caused by external loads in space frames through shape restoration. The method developed and examined herein seeks to simultaneously control twisting moment and shape deformation in a single-layer egg-shaped space frame while minimizing the actuator length changes and computation time using the force method. The algorithm iteratively approaches minimum actuator length changes considering the constraints. It also attempts to iteratively eliminate ineffective actuators so as to reduce the number of implemented actuators.

2. Methodology

2.1. Method formulation

The proposed technique is based on the force method. Although the EIFM method formulated by Wang and Senatore [8] is equivalent and is more intuitive than SVD-FM, we have chosen to continue using the force method due to its familiarity and the convenience of extending our previous codes to this application. A structure comprising b beams, j joints, and c nodal constraints. The equilibrium (H) is expressed as follows.

$$Ht = P \tag{1}$$

Where t and P are the internal and external forces, including axial, bending, and twisting moments for a frame system, respectively, the

Table 1
Nodal coordinates of the egg-shaped numerical model (in mm).

Joints	X	Y	Z
1	0	1000	0
2-9	200.677, 141.9, 0, -141.9, 200.677, -141.9, 0, 141.9	925	0, 141.9, 200.677, 141.9, 0, -141.9, 200.677, -141.9
10-17	241.1262, 99.8778, -99.8778, -241.1262, -241.1262, -99.8778, 99.8778, 241.1262	840.6	99.8778, 241.1262, 241.1262, 99.8778, -99.8778, -241.1262, -241.1262, -99.8778
18-25	321.3093, 227.2, 0, -227.2, -321.3093, -227.2, 0, 227.2	756.2	0, 227.2, 321.3093, 227.2, 0, -227.2, -321.3093, -227.2
26-33	315.2083, 130.5636, -130.5636, -315.2083, -315.2083, -130.5636, 130.5636, 315.2083	669.3	130.5636, 315.2083, 315.2083, 130.5636, -130.5636, -315.2083, -315.2083, -130.5636
34-41	361.0487, 255.30, -255.3, -361.0487, -255.3, 0, 255.3	582.4	0, 255.3, 361.0487, 255.3, 0, -255.3, -361.0487, -255.3
42-49	328.6659, 136.1379, -136.1379, -328.6659, -328.6659, -136.1379, 136.1379, 328.6659	501.2	136.1379, 328.6659, 328.6659, 136.1379, -136.1379, -328.6659, -328.6659, -136.1379
50-57	350.4421, 247.8, 0, -247.8, -350.4421, -247.8, 0, 247.8	402	0, 247.8, 350.4421, 247.8, 0, -247.8, -350.4421, -247.8
58-65	299.9869, 124.2586, -124.2586, -299.9869, -299.9869, -124.2586, 124.2586, 299.9869	340.8	124.2586, 299.9869, 299.9869, 124.2586, -124.2586, -299.9869, -299.9869, -124.2586
66-73	298.9647, 211.4, 0, -211.4, -298.9647, -211.4, 0, 211.4	261.6	0, 211.4298.9647, 211.4, 0, -211.4, -298.9647, -211.4
74-81	226.36293.7622 -93.7622 -226.362 -226.362 -93.762293.7622226.362	182.4	93.7622, 226.362, 226.362, 93.7622, -93.7622, -226.362, -226.362, -93.7622
82-90	191.0603, 135.1, 0, -135.1, -191.0603, -135.1, 0, 135.1	103.2	0, 135.1, 191.0603, 135.1, 0, -135.1, -191.0603, -135.1
91	0	0	0

Table 2
Joint connectivity of the egg-shaped numerical model.

Members	Joint connectivity
1–8	1–2, 1–3, 1–4, 1–5, 1–6, 1–7, 1–8, 1–9
9–18	2–10, 10–19, 19–27, 27–36, 36–44, 44–53, 53–61, 61–70, 70–78, 78–87
19–28	3–11, 11–20, 20–28, 28–37, 37–45, 45–54, 54–62, 62–71, 71–79, 79–88
29–38	4–12, 12–21, 21–29, 29–38, 38–46, 46–55, 55–63, 63–72, 72–80, 80–89
39–48	5–13, 13–22, 22–30, 30–39, 39–47, 47–56, 56–64, 64–73, 73–81, 81–82
49–58	6–14, 14–23, 23–31, 31–40, 40–48, 48–57, 57–65, 65–66, 66–74, 74–83
59–68	7–15, 15–24, 24–32, 32–41, 41–49, 49–50, 50–58, 58–67, 67–75, 75–84
69–78	8–16, 16–25, 25–33, 33–34, 34–42, 42–51, 51–59, 59–68, 68–76, 76–85
79–88	9–17, 17–18, 18–26, 26–35, 35–43, 43–52, 52–60, 60–69, 69–77, 77–86
89–98	2–17, 17–25, 25–32, 32–40, 40–47, 47–55, 55–62, 62–70, 70–77, 77–85
99–108	3–10, 10–18, 18–33, 33–41, 41–48, 48–56, 56–63, 63–71, 71–78, 78–86
109–118	4–11, 11–19, 19–26, 26–34, 34–49, 49–57, 57–64, 64–72, 72–79, 79–87
119–128	5–12, 12–20, 20–27, 27–35, 35–42, 42–50, 50–65, 65–73, 73–80, 80–88
129–138	6–13, 13–21, 21–28, 28–36, 36–43, 43–51, 51–58, 58–66, 66–81, 81–89
139–148	7–14, 14–22, 22–29, 29–37, 37–44, 44–52, 52–59, 59–67, 67–74, 74–82
149–158	8–15, 15–23, 23–30, 30–38, 38–45, 45–53, 53–60, 60–68, 68–75, 75–83
159–168	9–16, 16–24, 24–31, 31–39, 39–46, 46–54, 54–61, 61–69, 69–76, 76–84
169–176	82–90, 83–90, 84–90, 85–90, 86–90, 87–90, 88–90, 89–90
Axial members	18–19, 20–21, 22–23, 24, 25, 66–67, 68–69, 70–71, 72–73

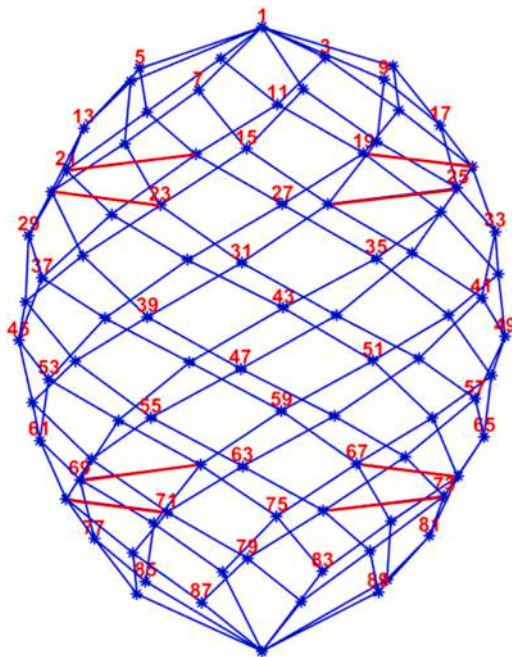


Fig. 2. The single-layer egg-shaped structure with eight additional bars and odd joint numbers.

compatibility matrix (C), which is the transpose of H , can be written as follows.

$$Cd = e \quad (2)$$

Where d and e are external and internal deformations, respectively, the flexibility matrix (F) relating internal forces and deformation is written as.

$$Ft = e \quad (3)$$

The relationships of the three matrices are illustrated in Fig. 1.

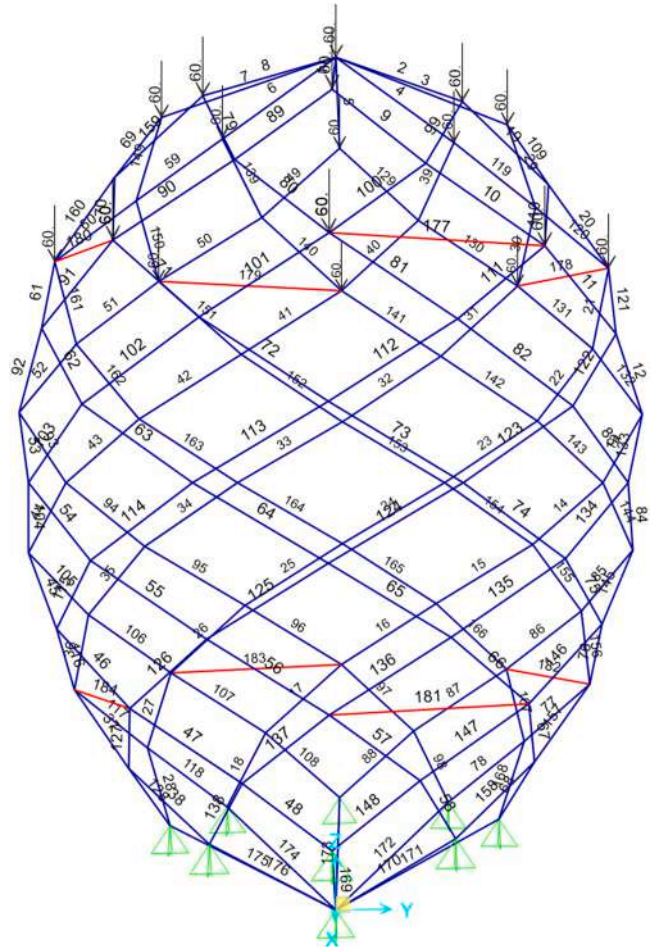


Fig. 3. Numerical model with eight additional bars, member numbering, support reactions, and loadings.

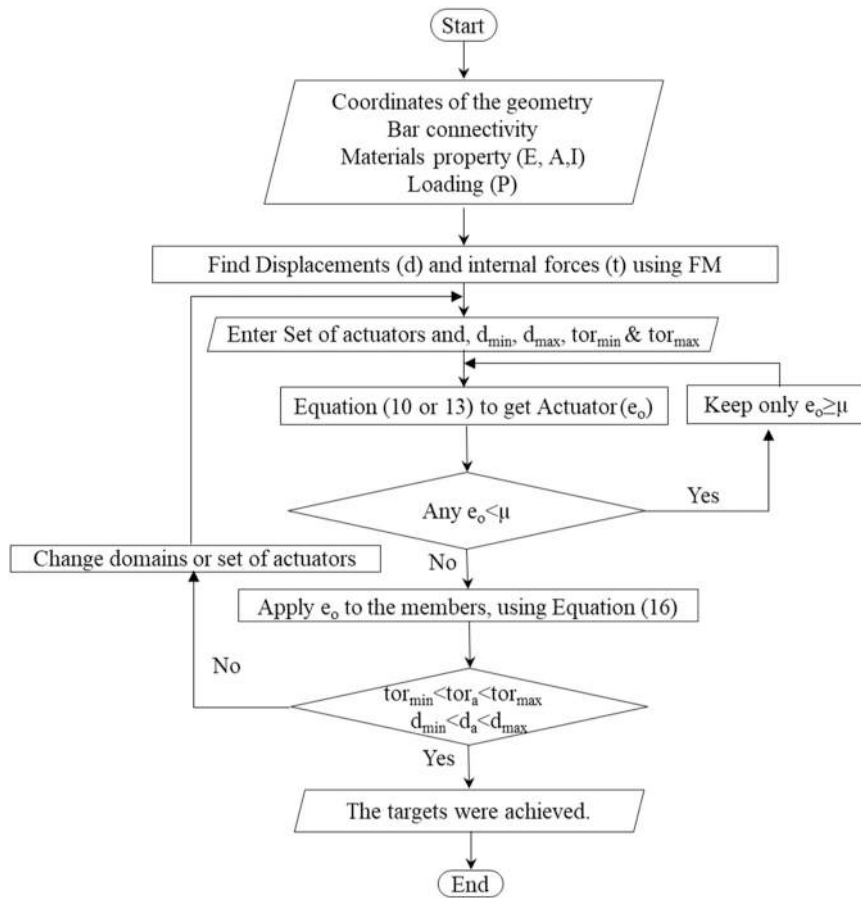


Fig. 4. Flow diagram of the algorithm applied in this work.

In some cases, such as structural imperfection or, in our case, altering bar length (e_o) to adjust shape and stress, the flexibility matrix is written as:

$$e = e_o + Ft \quad (4)$$

The internal force (t) can be expressed as follows:

$$t = t_H + S\alpha \quad (5)$$

Where t is characterized by two components: t_H , which is in equilibrium with P but may not satisfy the compatibility, and the second part is independent states of self-stress (S). While α is the vector of (b-r), b is the number of bars of a structure, and r is the rank of the equilibrium matrix (H). Substituting (5) into (3) gives

$$e = e_o + F(t_H + S\alpha) \quad (6)$$

The compatibility of a set of e can be determined by checking if it belongs to the set of compatible column spaces of C . e must be orthogonal to the left-null space of C , which is also the null space of H since H and C are transposed of each other; thus, the condition of compatibility is $S^T e = 0$, or $S^T e_o + S^T F(t_H + S\alpha) = 0$ thus.

$$-\alpha = (S^T FS)^{-1} [S^T e_o + S^T Ft_H] \quad (7)$$

In the force method, the process of obtaining internal forces and external displacements involves substituting α in (5) and the compatibility equation. The relation of bar length change and nodal coordinates is presented in Eqs. (8 and 9) by substituting (7) in (6) and then in the compatibility equation.

$$\begin{aligned} Cd &= e_o + F(t_H - S(S^T FS)^{-1} [S^T e_o + S^T Ft_H]) \\ d &= C^+ e_o + C^+ F(t_H - S(S^T FS)^{-1} [S^T e_o + S^T Ft_H]) \\ d &= [C^+ - C^+ FS(S^T FS)^{-1} S^T] e_o + [C^+ F - C^+ FS(S^T FS)^{-1} S^T F] t_H \end{aligned} \quad (8)$$

$$Ye_o + d_p = d_t \quad (9)$$

Y is $C^+ - C^+ FS(S^T FS)^{-1} S^T$ that relates external deformation and structural members length change, while d_p is $[C^+ F - C^+ FS(S^T FS)^{-1} S^T F] t_H$ which is also equal to $C^+ e$, while C^+ is the pseudo inverse of C and d_t is the targeted external displacements. Eq. (9) is now expressed to let the displacements have a value within a given domain $[d_{\min} d_{\max}]$ as follows [32].

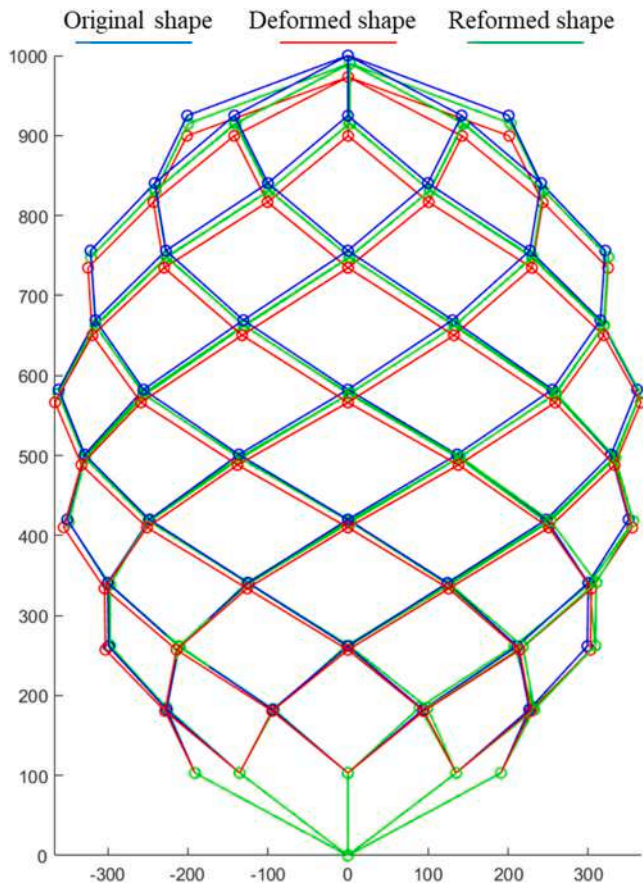


Fig. 5. Space frame configuration before loading, deformed (after loading), and reformed shapes.

$$d_{\min} \leq Y e_o + d_p \leq d_{\max} \quad (10)$$

The original size of the Y matrix is $(6j-c, 6b)$; we are only interested in the joint translations. Thus, the matrix size is reduced to $(3j-c, b)$, and all internal deformation parameters except bar length change (e_o) will be removed. The algorithm iteratively removes inactive beams, retaining only the active beams whose required length changes exceed 0.1 mm, as detailed in Section 2.3.

$$\theta_{y1} \theta_{z1} e_o \beta \theta_{y2} \theta_{z2}$$

$$Y = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{bmatrix} \begin{bmatrix} d_{x1} \\ d_{y1} \\ d_{z1} \\ \psi_{y1} \\ \psi_{z1} \\ \eta \end{bmatrix}$$

where θ and Ψ are external and internal rotations, d is the nodal displacement e_o is the bar length alteration, β and η are the external and

internal angles of the twist. In this study, the relation of torsion and e_o is derived by subbing (10) in (8) as follows.

$$t = t_H - S(S^T F S)^{-1} [S^T e_o + S^T F t_H] \quad (11)$$

$$= [I - S(S^T F S)^{-1} S^T F] t_H - [S(S^T F S)^{-1} S^T] e_o$$

$$tor_t = tor_p - Z e_o \quad (12)$$

Where tor_t is the targeted torsion value, tor_p is the torsion due to loading, and Z is the matrix relates twisting moment and e_o . The size of the original Z matrix is $(6b, 6b)$.

This study deals with the effect of bar length change (3rd column of Z) on the torsion (4th row of Z). After implementing the optimization algorithm, the Z matrix is reduced to (b, b) , and the column size will be reduced, i.e., only active beams will remain.

$$\theta_{y1} \theta_{z1} e_o \beta \theta_{y2} \theta_{z2}$$

$$Z = \begin{bmatrix} c_{11} & c_{13} & & & & \\ & c_{22} & c_{23} & & & \\ & & c_{33} & & & \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ & & c_{53} & & c_{55} & \\ & & c_{63} & & & c_{66} \end{bmatrix} \begin{bmatrix} m_{y1} \\ m_{z1} \\ t_n \\ tor \\ m_{y2} \\ m_{z2} \end{bmatrix}$$

m is the bending moment, t_n is the axial force, and tor is the twisting moment. In reality, requiring control over one (displacement or torsion) will also have some effects on the other. For example, while it is critical to restore the structure's shape to a prescribed profile via member length change, it is also necessary that the resultant set of e_o does not increase torsion to dangerous levels. In many situations, there is thus a need to be able to simultaneously control both the shape geometry and the element torsional forces via the same set of e_o . Thus, the combined equations become

$$d_{\min} \leq Y e_o + d_p \leq d_{\max} \quad (13)$$

$$tor_{\min} \leq Z e_o - tor_p \leq tor_{\max}$$

Where, tor_p is the induced twisting moments in members due to loadings or reducing shape deformation and $[tor_{\min} \ tor_{\max}]$ is the desired domain of the twisting moment after applying the length changing of active bars. Moreover, Z is the matrices that link the twisting moment in members with the structural elements equipped with actuators. e_o is the required change of length of the actuators to keep the torsion in all structure members within the allowable domain. The optimization function, presented in Eq. (14), is subjected to Eqs. (10, 12, and 13) to minimize the actuator length changes. In addition, removing actuators that are required to perform length changes smaller than 0.1 mm (μ) in this study can reduce the total number of actuators.

$$\min f(x) = \sum_{i=1}^n e_{oi} \quad (14)$$

Our numerical model employs the linprog function, which uses the interior point optimization algorithm. We also use fmincon, which uses interior point or SQP. These algorithms iteratively search for a set of e_o

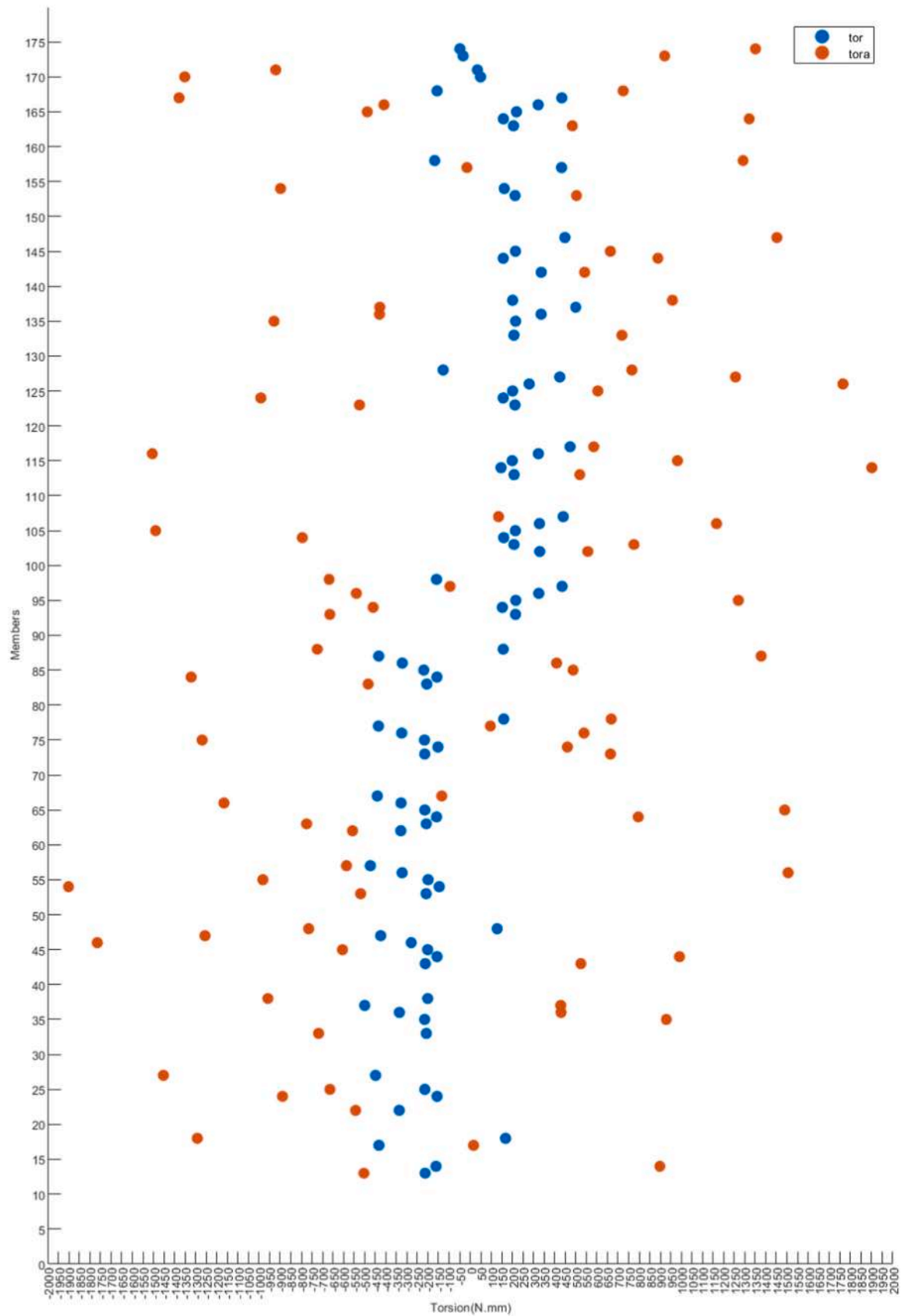


Fig. 6. The twisting moment in members that are greater than 400 N.mm before /after shape adjustment.

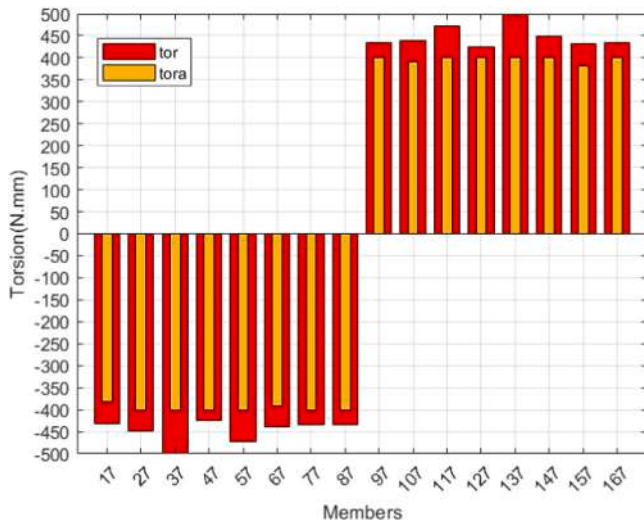


Fig. 7. Twisting moment in critical members before/after actuator length changes using Eq. 12.

that provide a minimum summation of actuator length changes while considering the constraints, and the upper bound (U_b) and lower bound (L_b) for actuator length change.

$$|L_b| \leq |e_o| \leq |U_b| \quad (15)$$

(L_b and U_b) have been thoughtfully determined in this paper, considering the members' length and the actuator's length change capacity to guarantee the practicality and real-world applicability of the model. The limits of -5 mm and 5 mm have been set to avoid any significant changes in the structural geometry. In practice, reducing the number of actuators is more cost-effective than making inefficient or less effective actuator length changes; thus, the actuators required to perform length changes smaller than 0.1 mm (μ) will be removed iteratively. After applying the obtained e_o , the twisting moment in members (tor_a) and nodal displacement (d_a) are obtained as follows.

$$\begin{aligned} tor_a &= tor_p - Z * e_o \\ d_a &= d_p + Y * e_o \end{aligned} \quad (16)$$

2.2. The numerical model

A single-layer egg-shaped space frame is numerically modeled and analyzed in MATLAB. The structure's thin skin is highly susceptible to distortion, making precision crucial and imperfections unacceptable. Similar structures have already been studied for shape control [12], where twisting moments were not considered but were found to be very high after reducing deformation. The numerical model is formed by 90 joints connected by 176 flexural members. Eight additional axial members were horizontally added to mitigate the deformed shape. The members are made of 6 mm diameter steel with Young's modulus of 200 GPa. Tables 1 and 2 give the coordinate system of the nodes and the members' connectivity. Figs. 2 and 3 show the structures' nodal and member numbering and visualize the supported nodes and loadings. The

numerical model is supported at the nine bottom nodes (Joints 82 - 90) in the x, y, and z directions. Nodes 1–9 and 18–25 were subjected to 60 N downward loading (see Fig. 3), which induced a noticeable deformation, as studied by Saeed et al. [12], to reduce the nodal displacement.

2.3. Computational procedure

After inputting the structural geometry and loadings, nodal displacement and torsion in all members are determined using SVD-FM. In this stage, limits for the nodal displacements and torsional members are assumed, and the minimum actuator length change is determined through Eqs. (10, 12, and 13) in several iterations, as presented in Fig. 4. Twisting moments in members and nodal displacements and the amount of length change per actuator are considered. First, it is assumed that all structural members are equipped with actuators; then, step by step, the actuators that are required to perform a length change smaller than 0.1 mm ($e_o < 0.1$ mm) are removed. It should be noted that the proposed approach operates through an iterative process guided by a heuristic that takes into account a minimum threshold for the necessary change in actuator length. However, this method does not ensure any optimal conditions regarding the number of actuators needed or their positions.

3. Numerical example

3.1. Case 1 deformation control by member length change

In this case, the deformed shape of a single-layer space frame has been reshaped after loading. The maximum displacement occurs at the top joint (J1) in the direction of loading. The goal is to keep the nodal displacements within ± 10 mm in all directions using Eq. 10. Fig. 5 shows the structural configuration in three stages. However, the shape is controlled through actuating Member [3 4 5 8 28 38 58 68 78 98 108 118 138 148] by [1.1498 1.9896 1.1498 0.4998 0.3715 6.6073 4.9097 5.6454 0.2077 0.2077 5.6454 4.9097 6.6073 0.3715] mm respectively within ± 10 mm for all nodes. Despite mitigating the deformation by actuating some bars, it caused a significant increase in twisting moments of some members. Fig. 6 shows the momentous change in twisting moment in critical members from 500 N.mm to just under 2000 N.mm.

If only Eq. 12 is implemented to control twisting moment without regard to the shape, the twisting moment can be controlled within $[-400 400]$ N.mm in all members through actuating Members [27 47 57 67 77 87 97 107 117 127 147 167] with $[-0.13373 -0.17404 -0.14832 -0.13572 -0.057838 -0.10709 -0.057838 -0.13572 -0.14832 -0.17404 -0.13373 -0.10709]$ mm respectively. However, the twisting moment is controlled, as presented in Fig. 7. The maximum nodal displacement is -13.5 mm, which is slightly higher than the maximum displacement before mitigating the twisting moment.

Due to the substantial increase in the twisting moment after shape adjustment and noticeable displacement in controlling only the twisting moment, both aspects simultaneously should be considered. Now, Eq. 13 is implemented to reduce the deformed shape and control the twisting moment within ± 400 N.mm in all members simultaneously. It can be seen from Fig. 8 that the twisting moment in all members kept within ± 400 N.mm. Meanwhile, the nodal displacement in all nodes stayed

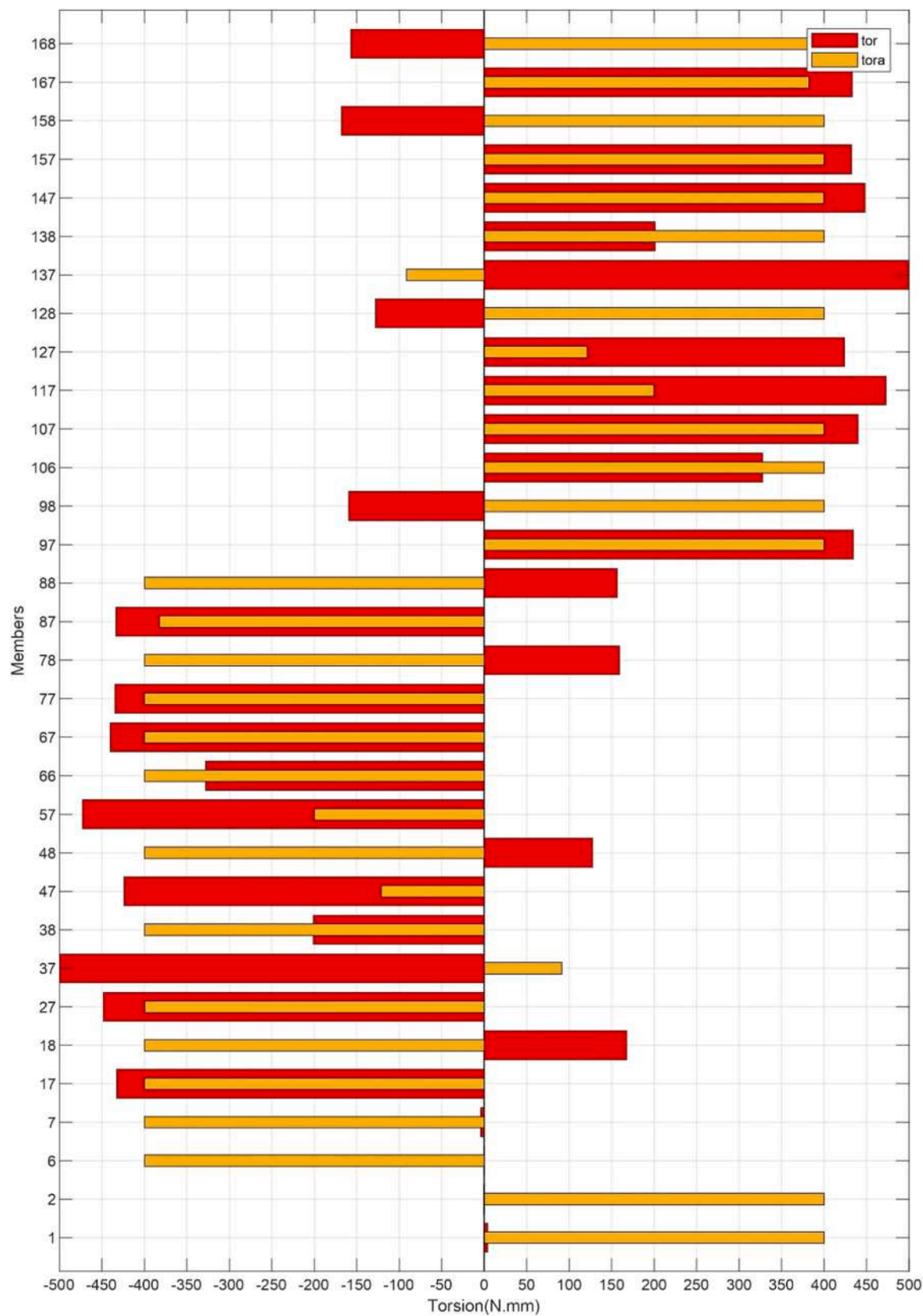


Fig. 8. Twisting moment in critical members before/after reshaping using Eq. 10.

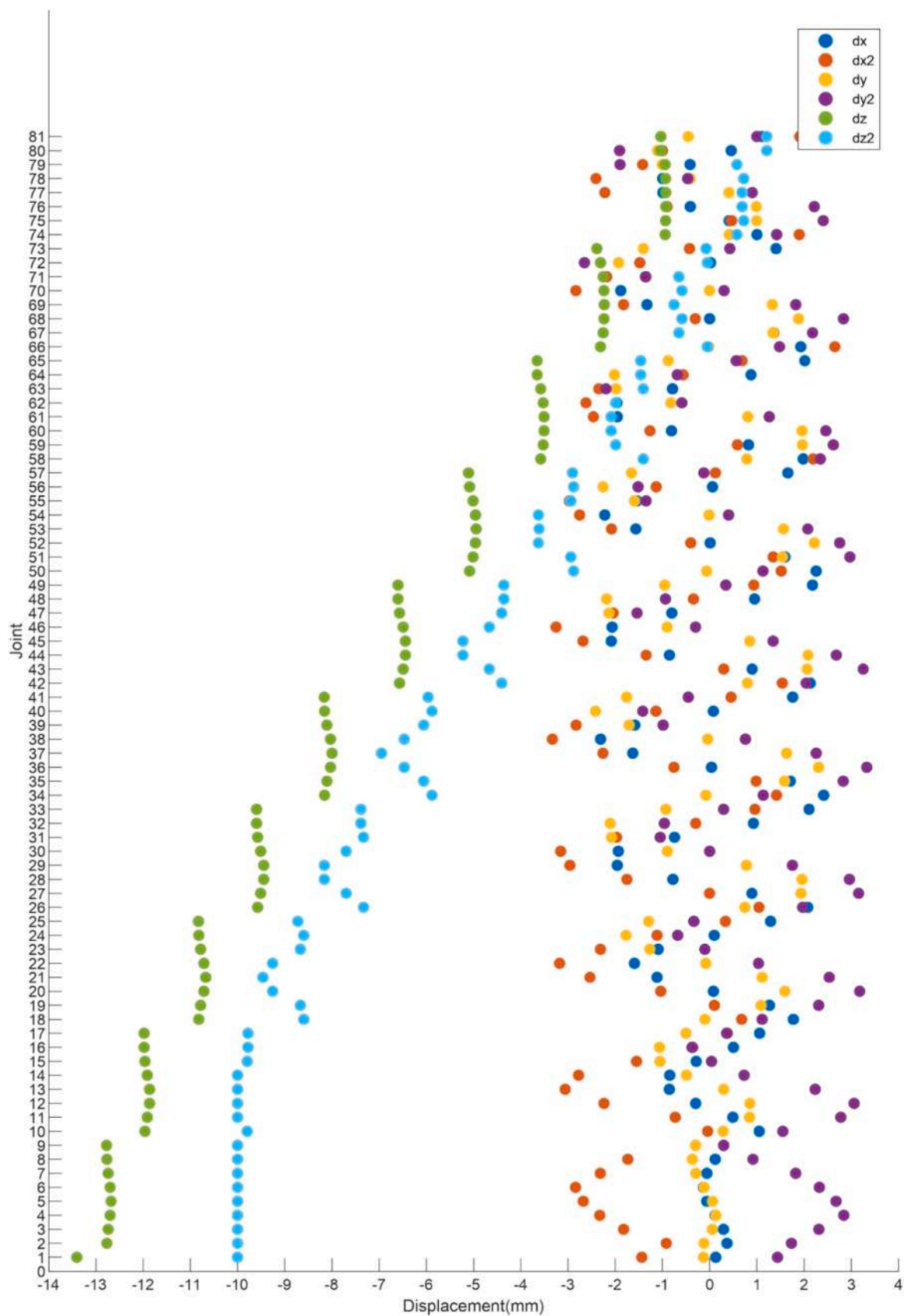


Fig. 9. Nodal displacements before /after actuator length changes of the space frame using Eq. 10.

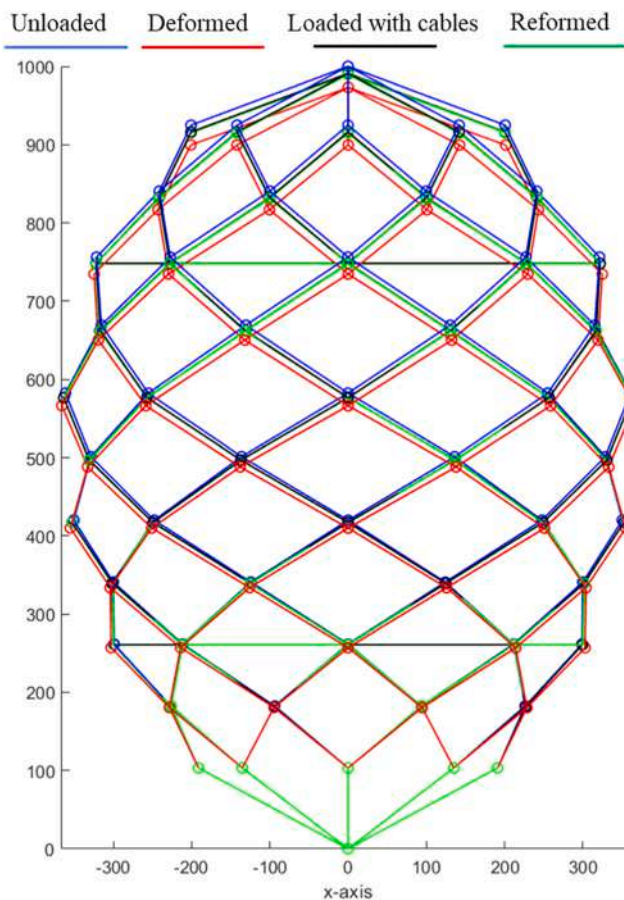


Fig. 10. Structural configuration in four various stages.

within ± 10 , as presented in Fig. 9. Fig. 8 shows a rapid increase in twisting moment in Members 1, 2, 6, and 7 and changes from negative twisting moment to positive in Members 37, 96, 128, 158, and 168 and from positive to negative in Members 18, 48, 78, 88, 137.

3.2. Deformation mitigation by adding extra members

Saeed *et al.* [12] investigated several scenarios for adding extra horizontal members to reduce the deformation of the single-layer egg-shaped space frame. They found that a suitable case is adding eight extra bars in the upper and lower quarters of the structure's height (See Fig. 3) to prevent stress concentration in specific areas and effectively reduce deformation across the structure. This is evidenced by the mitigation of the maximum displacement in the top node from -13.41 mm to -8.7 mm (See Fig. 10). However, this addition did cause an increase in twisting moments in some members, from 499.6 N.mm to up to 576 N.mm. Nonetheless, using Eq. 13 allowed us to maintain

control over the twisting moment within the limit $[-400\ 400]$ N.mm, as shown in Fig. 11 and keep the nodal displacements in all nodes within $[-10\ 10]$ mm.

3.3. Computational optimization

Two distinct functions were utilized in MATLAB: `linprog` for linear optimization problems and `fmincon` for both linear and nonlinear problems. In Case 1, which involves displacement control without considering the twisting moment, the number of iterations and computational time were analyzed to compare the performance of these two functions. Table 3 indicates that both functions successfully determined the optimal actuator length change of 40.27 mm. The `fmincon` function achieved this optimal solution in twenty-eight iterations and 0.15 s using the interior point algorithm and in fifteen iterations and 0.03 s using the SQP algorithm. Conversely, the `linprog` function, employing the interior point algorithm, attained the optimal solution in only ten iterations and 0.05 s. Both the interior point and SQP methods in `fmincon` converged close to the optimal solution by the 23rd and 10th iterations, respectively, as evidenced by the first-order optimality (FOO) conditions. Similarly, the interior point method in `linprog` approached the optimal solution by the 8th iteration, with the complementary slackness conditions (CSC) approaching zero.

4. Conclusion

A simple technique has been presented in this work to simultaneously control the deformed shape and twisting moments in members of space frames through axial length changing of members. The technique used and developed in this work can reduce twisting moments within prescribed limits. The approach is based on the force method coupled with optimization algorithms to minimize the actuator length changes and keep twisting moments within the limit $[-400\ 400]$ N.mm. It also keeps the nodal displacements within $[-10\ 10]$ mm. In addition, the targets are obtained by minimizing actuator length changes by using optimization algorithms through the `linprog` and `Fmincon` functions in MATLAB. The numerical model was modeled using MATLAB and SAP2000 software to verify the accuracy of the technique.

CRediT authorship contribution statement

Najmadeen Saeed: Writing – review & editing, Software, Methodology, Conceptualization. **Ahmed Manguri:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Conceptualization. **Robert Jankowski:** Writing – review & editing, Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

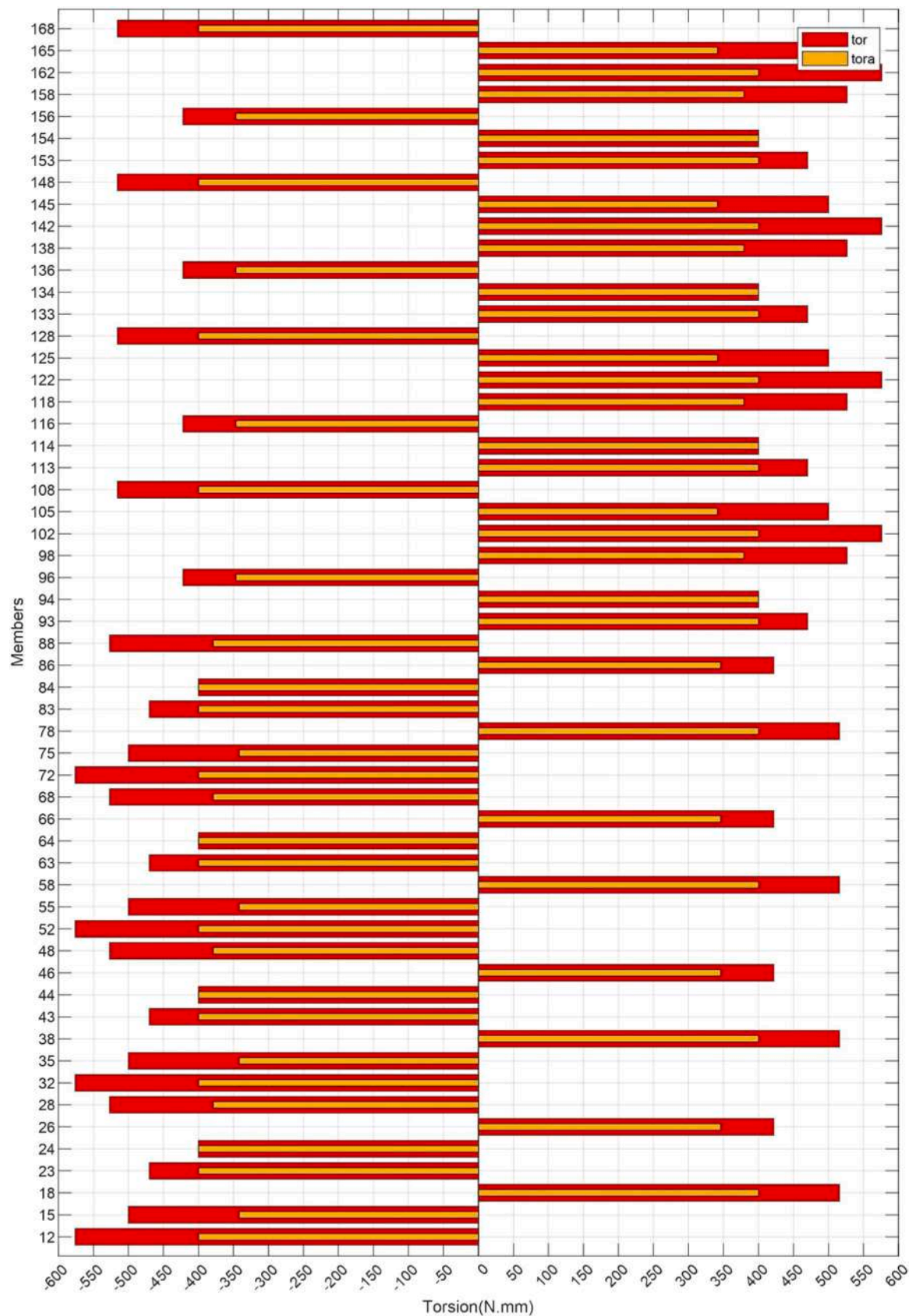


Fig. 11. Twisting moment in critical members before /after actuator length changes using Eq. 13.

Table 3
Total actuator length change using two functions.

Function	fmincon				linprog	
	Interior point		SQP		Interior point	
Algorithm						
Iterations	e _o (mm)	FOO	e _o (mm)	FOO	e _o (mm)	SCS
0	14.00	2	14.00	1.00	231.51	106.1745
1	16.41	1.769	41.41	2.87	202.33	38.2321
2	38.43	0.3843	40.91	0.43	107.65	13.6801
3	41.83	0.3744	40.54	0.12	63.35	4.5501
4	41.36	0.1846	40.52	0.02	58.45	1.5522
5	41.65	0.1001	40.52	0.02	45.30	0.4109
6	40.68	0.0548	40.51	0.02	40.74	0.0719
7	40.70	0.0312	40.47	0.02	40.46	0.0264
8	40.72	0.02	40.29	0.02	40.27	0.0006
9	40.56	0.0185	40.29	0.00	40.27	0
10	40.54	0.0185	40.29	0.00	40.27	0
11	40.43	0.0185	40.29	0.00	-	-
12	40.33	0.017	40.29	0.00	-	-
13	40.36	0.0121	40.27	0.00	-	-
14	40.34	0.0042	40.27	0.00	-	-
15	40.30	0.0023	40.27	0.00	-	-
16	40.31	0.0023	-	-	-	-
17	40.31	0.0023	-	-	-	-
18	40.30	0.0023	-	-	-	-
19	40.29	0.0023	-	-	-	-
20	40.28	0.0015	-	-	-	-
21	40.29	0.0015	-	-	-	-
22	40.28	0.0009	-	-	-	-
23	40.27	0.0002	-	-	-	-
24	40.27	0	-	-	-	-
25	40.27	0	-	-	-	-
26	40.27	0	-	-	-	-
27	40.27	0	-	-	-	-
28	40.27	0	-	-	-	-
Time (sec)	0.15		0.03		0.05	

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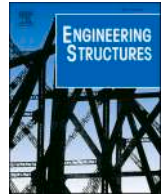
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Paper IV

Paper IV: Manguri, A., N. Saeed, and R. Jankowski, *Controlling nodal displacement of pantographic structures using matrix condensation and interior-point optimization: A numerical and experimental study*. Engineering Structures, 2024. **304:** p. 117603. <https://doi.org/10.1016/j.engstruct.2024.117603>



Controlling nodal displacement of pantographic structures using matrix condensation and interior-point optimization: A numerical and experimental study

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ABSTRACT

This study presents an innovative approach for the precise control of nodal displacements in pantographic structures. The method is founded on the Matrix Condensation of Force Method, seamlessly integrated with an Interior Point Optimization algorithm. This combination offers a unique advantage by allowing users to manipulate displaced nodes within a defined coordination domain. Furthermore, this approach introduces the Interior Point Optimization algorithm as an indispensable tool to eliminate inactive turnbuckles and minimize overall actuation requirements. Traditional control methods typically demand a substantial number of turnbuckles and extensive actuation efforts to attain the desired nodal coordinates. The interconnected nature of node movements, wherein changes in one node affect others, adds complexity to determining the impact of bar length alterations on each node. To address this challenge, precisely control power of the Interior Point Optimization algorithm systematically explores numerous scenarios to identify solutions that minimize both actuation and turnbuckle usage. The current technique's effectiveness is validated through rigorous comparisons with established methods, experimental modeling, and rigorous testing using SAP 2000 software. Notably, the current approach yields remarkable results, requiring a staggering 60% less actuation and reducing the reliance on turnbuckles by up to 40% compared to previous methods. This innovation promises to significantly enhance the efficiency and cost-effectiveness of controlling pantographic structures, marking a substantial advancement in this field.

1. Introduction

Pantographic structures are also known as scissor-like elements [1] or the lazy tong system [2], derived from the concept of pantographs used in mechanical engineering, have emerged as a promising area of research in civil engineering. These structures are characterized by a network of interconnected beams or trusses arranged in a geometric pattern that allows for complex and adaptable mechanical behavior [3]. The unique design of pantographic structures enables them to exhibit remarkable mechanical properties, including high flexibility, structural stability, and load-bearing capacity [4]. In 1961, Emilio P é rez Piñero, a Spanish architect, unearthed and built the three-dimensional scissor

structure. Various fundamental unit types for pantographic structures can be generated through alterations in the placement of the intermediate hinge or the configuration of the bars. These include translational, polar, and angulated units[5]. An extensive review on pantographic structures has been done by Moy, Tan [6].

Moreover, the concept of the foldability constraint for developing kinematic formulations was introduced by Escrig [7]. Subsequently, various kinematic formulations, including the retracting equations have been proposed by Langbecker [8]. It should be highlighted that the majority of the methods cannot analyze kinematically indeterminate structures [9]. Many researchers dealt with analyzing pantographic structures using linear [10,11] and nonlinear methods [12]. The current

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study developed from a linear elastic method proposed by [13] based on matrix condensation of force method to analyze scissor-like-element structures. Pantographic structures have found applications in various fields of engineering, such as bridge construction [14,15], architectural design [16], foldable stairs [17,18], solar roof panels [19,20], reflectors [21,22], tensegrity [23] and foldable robotics [24]. In addition, their ability to undergo large deformations while maintaining structural integrity makes them suitable for dynamic loading or seismic events [25–27]. By distributing loads efficiently and redistributing stresses, pantographic structures can enhance the resilience and safety of civil infrastructure.

In the field of structural engineering, the configuration of flexible structures carries significant importance. Typically, the geometric layout of these structures is defined by the positions of nodal points. Such structures often support sensitive equipment used in communication applications, where high geometric precision is essential for optimal performance. This precision is particularly critical in demanding environments, such as space missions [28,29]. Structures deployed in space are designed to be lightweight, resulting in a high degree of flexibility and susceptibility to shape imperfections caused by various factors like manufacturing discrepancies, thermal effects, and loading during transportation. Consequently, adjusting these structures with appropriate corrections becomes paramount to maintaining their accuracy and functionality [30].

Researchers developed methods to adjust nodal displacements or to shape control of structures using the Force Method [31–33], Subramanian and Mohan's successive peak error correction algorithm [34], and finite element analysis (FEA) [35,36]. In addition, shape adjustment has been made for many structures, such as trusses [37], double-layer domes [38,39], beams [40], and cable structures [41–43]. A detailed review about structural shape control using piezoelectric actuation was done by Irschik [44]. In terms of pantographic structures, only a few studies have been conducted to control the nodal displacement of such systems.

Pellegrino, Kwan [13] developed a method based on the Force Method to reduce the size of matrices by eliminating corresponding rows and columns to the unloaded joints. Saeed and Kwan [45] progressed the study by formulating the direct relationship between bar elongation and nodal coordinate position. In their research, to control the nodes' position, the users must input the exact value of the targeted position; in some cases, it is impossible to obtain. This paper addresses this issue by allowing the input of the targeted displacement as a domain. Moreover, using Saeed and Kwan [45]'s method, many members must be lengthened or shortened (i.e., many turnbuckles have to be attached to the structure, and a large amount of actuation is required). This is because the members are interconnected, and the relationship between the bar actuation and nodes is complicated. In this study, the interior point algorithm in the fmincon function is subjected to the developed formulation to minimize the total actuation. While the actuation is minimized, the members with minor actuation are assumed to be inactive; thus, they are eliminated.

Optimization algorithms are widely used in structural engineering to minimize the total structural weight [46], and the cost of construction and computational costs [47]. Optimization algorithms were employed to shape optimization [48], and shape and size optimization [49] of deployable structures. Regarding optimization in the number of actuators, researchers stated that using many actuators and a large amount of actuation is unpragmatic and uneconomic [50–52]. In addition, the actuators' position substantially influences the reduction of their numbers [53,54]. Researchers dealing with pantographic shape control were trying to find the effect of the length change of each bar on each node; they called it the process of bar sensitivity [45,55]. In the process, the impact of each bar length changing on each node movement has to be determined, which is time-consuming and ineffective. Since the movement of the nodes is interconnected, their movement cannot be calculated separately; they are dependent on each other. On top of that,

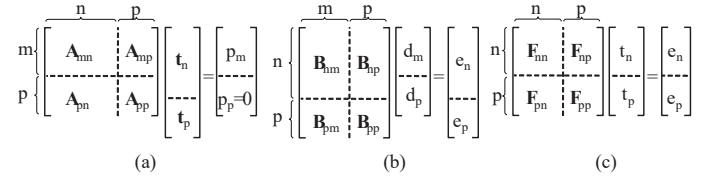


Fig. 1. Different arrangements of (a) equilibrium (b) compatibility, and (c) flexibility matrices [13].

the method is nearly impossible for complex element structures. The current technique involves an interior optimization algorithm to examine many scenarios to reach the goal; the algorithm selects the most cost-effective solution.

2. Methodology

2.1. Formulation for condensing matrices

Pantographs are considered complex element structures since two macro elements are connected with a shear connector while the ends of their two ends are connected with pins. It should be highlighted that the external loads are applied only at pin joints. Applying a direct load to a shear connector in such elements would result in an inaccurate representation of the actual load distribution and behavior. Scissor-like elements are restricted to loading exclusively at the pin joints, as these joints represent the sole connection points between the two bars constituting the element. Since The bars of the element are not designed to withstand loads applied elsewhere, if a load is applied elsewhere on the element, it will cause the element to rotate at the pin joints, and this will eventually cause the element to fail. Pellegrino, Kwan [13] divided and categorized the A (equilibrium matrix), B (compatibility matrix), and F (flexibility matrix) based on the loadable and unloadable joints, as shown in Fig. 1. The three matrices based on the Force Method are presented as follows.

$$At = p \quad (1)$$

Where t and p are the internal force and external loads, respectively.

$$Bd = e \quad (2)$$

Where d and e are the external nodal displacement and internal bar elongation.

$$Ft = e \quad (3)$$

(m,p) and (n,p) are dimensions of the vectors (external nodal displacement (d), applied loads (p)), and (n,p) (axial force) t , (bar elongation) e respectively. The external force vector ' p ' can be divided into sub-vectors: ' p_m ' with ' m ' non-trivial components and ' p_p ' with ' p ' trivial components. Likewise, the displacement vector ' d ' is divided into sub-vectors ' d_m ' and ' d_p ', where ' d_m ' corresponds straight to ' p_m ', and ' d_p ' corresponds to ' p_p ', as depicted in Fig. 1(a) and (b). The condensed matrices are presented as follows [13].

$$A^* t^* = p^* \quad (4)$$

The ' A^* ' matrix is the reduced equilibrium matrix, which is expressed as follows based on Fig. 1.

$$A^* = A_{mn} - A_{mp}A_{pp}^{-1}A_{pn} \quad (5)$$

' p^* ' represents the generalized external (non-trivial) force, equivalent to ' p_m '. It should be noted that ' A^* ' dimension is ' m ' by ' n '. Whereas ' t^* ' represents the generalized axial forces, which is condensed and contains only internal bar forces excluding minor bending (t_p) that induced near the shear connectors, its equivalent to ' t_n ' that can be presented as follows.

$$\mathbf{t}_n = \mathbf{t}_{nH} + \mathbf{S}^* \boldsymbol{\alpha}^* \quad (6)$$

In the condensed equilibrium equations, $\mathbf{t}_n$ is the summation of a particular solution that satisfies Eq. (4) and the complementary homogeneous solution. The particular solution, denoted as $\mathbf{t}_{nH}$, is obtained from $\mathbf{t}_{nH} = (\mathbf{A}^*)^+ \mathbf{p}_m$. The complementary homogeneous solution $\mathbf{S}^* \boldsymbol{\alpha}^*$ is the set of reduced bar forces solutions that satisfy $\mathbf{A}^* \mathbf{t}_n = \mathbf{0}$, derived from the null space $(\mathbf{A}^*) = \mathbf{S}^*$. $\mathbf{S}^*$ represents the condensed states of self-stress, which depend on the static redundancy. The vector $\boldsymbol{\alpha}^*$ is a set of s^* coefficients reflecting a collection of combinatorial constants.

Regarding, the reduced compatibility matrix which is the transpose of ' $\mathbf{A}^*$ ' can be written as follows.

$$\mathbf{B}^* \mathbf{d}^* = \mathbf{e}^* \quad (7)$$

Here, ' $\mathbf{d}^*$ ' represents the generalized joint displacement, which is equivalent to ' $\mathbf{d}_m$ '. Additionally, ' $\mathbf{B}^*$ ', which establishes the relationship between the ' $\mathbf{m}$ ' nodal movements (associated with the ' $\mathbf{m}$ ' non-trivial force components) and the ' $\mathbf{n}$ ' selected condensed bar elongation (associated with the chosen ' $\mathbf{t}^*$ '). The condensed bar elongation is expressed as follows.

$$\mathbf{e}^* = \mathbf{e}_n - \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{e}_p \quad (8)$$

The condensed flexible matrix is expressed as

$$\mathbf{F}^* \mathbf{t}^* = \mathbf{e}^* \quad (9)$$

where $\mathbf{F}^*$ is represented as.

$$\mathbf{F}^* = \mathbf{F}_{nn} - \mathbf{F}_{np} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn} - \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{F}_{pn} + \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{F}_{pp} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn} \quad (10)$$

Saeed and Kwan [45] introduced a relationship between the bar actuation and the position of the nodes as presented in Eq. (10); the detailed formulation is available at [45] and [56].

In general, structural deficiencies may arise from either inherent construction imperfections or, as is pertinent to our investigation, intentional extensions designed to regulate the nodal displacement within the structure, consequently inducing alterations in axial force (a phenomenon particularly pronounced in structures characterized by a high degree of indeterminacy). The elongation of a bar can be attributed to two distinct origins. One component stem from the axial force acting on the bar, denoted as $\mathbf{F}^* \mathbf{t}_n$ (Eq. 9), while the other component, represented as $\mathbf{e}_0^*$, results from either a lack of fit or purposeful elongation aimed at controlling nodal displacement, as exemplified in our study. It is noteworthy that although each bar may possess an $\mathbf{e}_0^*$, only those bars designated as active are equipped with turnbuckles. A comprehensive examination of this scenario will be presented in the subsequent optimization section. It is important to note that, for the purposes of our analysis, every bar is deemed capable of actuation, rendering $\mathbf{e}_0^*$ universally applicable and accounted for in our considerations thus.

$$\mathbf{e}^* = \mathbf{e}_0^* + \mathbf{F}^* \mathbf{t}_n \quad (11)$$

Substituting (6) in (11) becomes

$$\mathbf{e}^* = \mathbf{e}_0^* + \mathbf{F}^* (\mathbf{t}_{nH} + \mathbf{S}^* \boldsymbol{\alpha}^*) \quad (12)$$

To obtain axial forces ($\mathbf{t}_n$), bar elongation ($\mathbf{e}^*$) and nodal displacements of the pin-jointed nodes ($\mathbf{d}_m$), $\boldsymbol{\alpha}^*$ should be obtained. The imposition of the compatibility condition involves asserting that the condensed bar elongations ($\mathbf{e}^*$) must exhibit orthogonality concerning the left-null space ($\mathbf{B}^*$). Considering the relationship $(\mathbf{B}^*)^T = \mathbf{A}^*$, where left-null space ($\mathbf{B}^*$) is equivalent to null space ($\mathbf{A}^*$), which coincides with the states of self-stress, the compatibility condition can be succinctly expressed as the requirement that the states of self-stress must be orthogonal to the elongations. In mathematical terms, this condition can be represented as $(\mathbf{S}^*)^T \mathbf{e}^* = \mathbf{0}$, thus

$$\mathbf{S}^{*T} \mathbf{e}_0^* + \mathbf{S}^{*T} \mathbf{F}^* (\mathbf{t}_{nH} + \mathbf{S}^* \boldsymbol{\alpha}^*) = \mathbf{0} \quad (13)$$

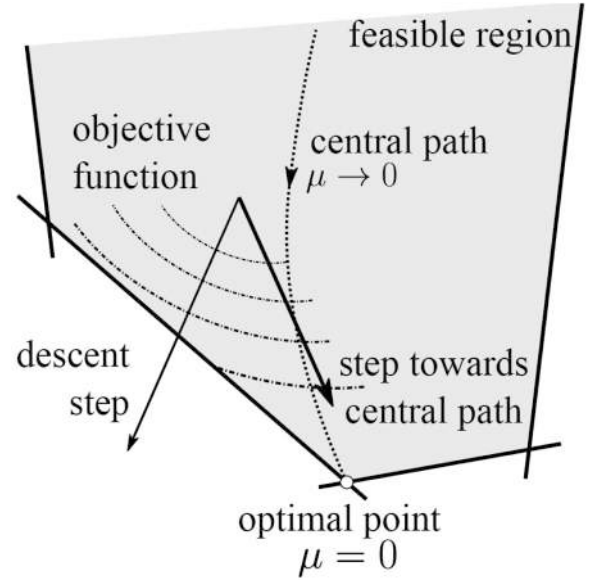


Fig. 2. The fundamental concept behind the interior point method. [60].

And thus

$$-\boldsymbol{\alpha}^* = (\mathbf{S}^{*T} \mathbf{F}^* \mathbf{S}^*)^{-1} [\mathbf{S}^{*T} \mathbf{e}_0^* + \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH}] \quad (14)$$

By substituting (12) in (6) and (11) $\mathbf{t}_n$ and $\mathbf{e}^*$ are obtained, after getting $\mathbf{e}^*$ and considering $(\mathbf{B}^*)^T = \mathbf{A}^*$ and substituting (5) in (7) $\mathbf{d}_m$ is obtained as follows

$$\mathbf{d}_m = \mathbf{B}^{*+} [\mathbf{e}_0^* + \mathbf{F}^* (\mathbf{t}_{nH} - \mathbf{S}^* (\mathbf{S}^{*T} \mathbf{F}^* \mathbf{S}^*)^{-1} [\mathbf{S}^{*T} \mathbf{e}_0^* + \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH}])] \quad (14')$$

Now, the considered nodes $\mathbf{d}_m$ to be controlled can be split out and addressed as ($\mathbf{d}_{pc}$) and related to the active bars imbedded with turnbuckles ($\mathbf{e}_{oc}$) by $\mathbf{Y}_c$.

$$\mathbf{d}_c = \mathbf{d}_{pc} + \mathbf{Y}_c \mathbf{e}_{oc} \quad (15)$$

In Eq. 15, the user must provide the exact value of the nodal displacement after actuation; in complex structures, it's almost impossible to control many nodes in the exact positions, which might be unnecessary. In this paper, the desired nodal displacement can be added as a domain; for example, instead of asking to reduce or eliminate a nodal displacement from a certain value to zero or another exact value, a domain like $[\mathbf{d}_{min} \mathbf{d}_{max}]$ can be added. Thus, Eq. 15 can be changed as follows.

$$\mathbf{d}_{min} \leq \mathbf{d}_{pc} + \mathbf{Y}_c \mathbf{e}_{oc} \leq \mathbf{d}_{max} \quad (16)$$

$\mathbf{d}_{min}$ and $\mathbf{d}_{max}$ are the minimum and maximum allowable nodal displacements after actuation, i.e., the joint displacement after changing the length of active bars is unleashed to take any value within the domain $[\mathbf{d}_{min} \mathbf{d}_{max}]$. This also makes the technique to solve the problem with less effort. In addition, the displacement domain can be added as a constraint in the numerical technique. After obtaining the optimum set of $\mathbf{e}_{oc}$, it is applied to the structure to find the nodal displacements after actuation ($\mathbf{d}_a$)

$$\mathbf{d}_a = \mathbf{d}_{pc} + \mathbf{Y}_c \mathbf{e}_{oc} \quad (17)$$

2.2. Interior point optimization algorithm

The fmincon interior point algorithm is a numerical method for solving nonlinear constrained optimization problems. It is a member of the class of interior-point methods, which work by iteratively moving the solution towards the minimum while staying within the feasible

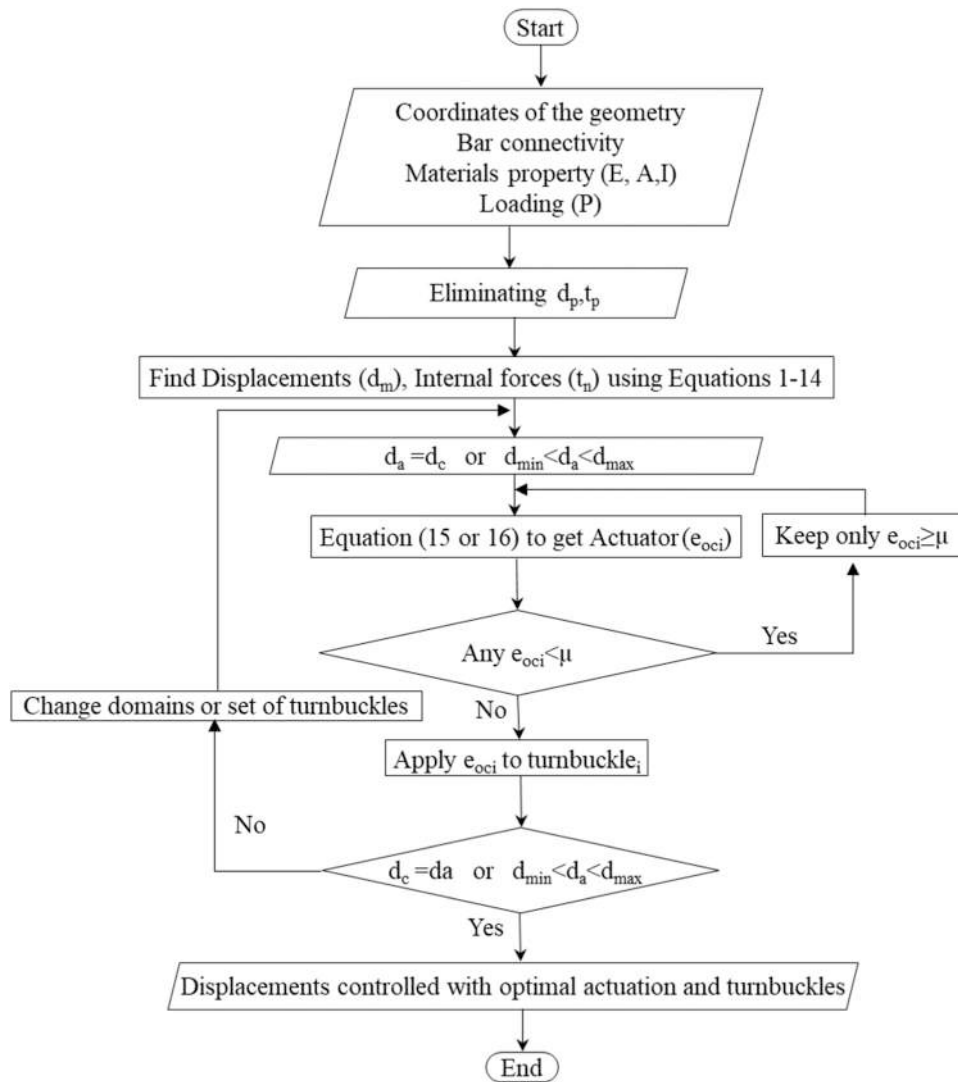


Fig. 3. Flow chart of the optimal displacement control technique.

region of the problem. The fmincon interior point algorithm is typically more efficient than other optimization algorithms for large-scale problems. An advantage is that these algorithms can solve problems without strictly feasible points. They can also be used to detect the infeasibility of certain linear programming problems [57,58]. The fmincon interior point algorithm works by first constructing a barrier function, which is a function that penalizes solutions that are outside the feasible region of the problem. The algorithm then iteratively moves the solution towards the minimum of the barrier function while ensuring that the solution remains within the feasible region. The algorithm terminates when the solution converges to a feasible point and a minimum of the objective function.

Fig. 2 shows the core idea behind the interior point method revolves around the concept of the central path, which is a trajectory traversing the inner regions of the feasible space. This trajectory is determined by a collection of equations incorporating the objective function, constraints, and barrier parameters. When the barrier parameter gradually approaches zero, the central path progressively approaches the optimal solution for the linear programming problem. In this work, the fmincon function used interior point optimization [59] to minimize the total actuation of turnbuckles (e_{oc}).

The objective function is $\min \sum_{i=1}^{n_a} e_{oci}$.
 n_a is the number of active members.

A domain can be set for actuation per actuators to stay within

$$e_{\min} \leq e_{oc} \leq e_{\max}.$$

In practical applications, various factors, including the challenges associated with performing minute actuations and considerations of cost-effectiveness, preclude the integration of turnbuckles into inactive members. In the pursuit of minimizing the total actuation of the objective function (e_{oc}), the algorithm identifies members with negligible amounts (μ), leading to the elimination of those members where the absolute value of (e_{oc}) is less than μ in the subsequent step, as illustrated in the flowchart depicted in Fig. 3. This iterative process persists until only active members, crucial to the optimization objective, remain in the system.

2.3. An illustrative example

The presented approach is exemplified through its application to a one-bay pantograph with one degree of self-stress. The objective is to demonstrate the analysis and modification of a straightforward structure using the current technique. The numerical model is designed with four pinpoints and incorporates a shear connector, as illustrated in Fig. 4. The initial two joints are strategically restrained against translation to ensure structural stability. The structural members possess a cross-section dimension of 6 * 9 mm, and their elastic modulus is specified at 70000 MPa. This configuration aims to capture the essential characteristics of the pantograph structure, providing a representative model for

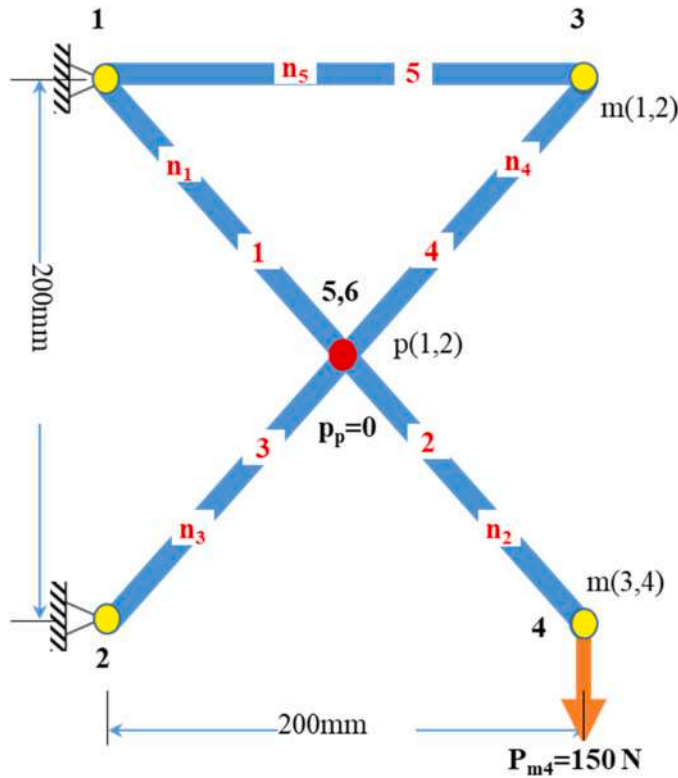


Fig. 4. The illustrative example.

the application and validation of the proposed technique.

The equilibrium matrix (A) of the structure shown in Fig. 4 is as below is condensed (A*) from 18×15 to 4×5

$$A^* t^* P^* = \begin{bmatrix} \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} P_{m1} \\ P_{m2} \\ P_{m3} \\ P_{m4} \end{bmatrix}$$

$A^* t^* P^*$

And the compatibility matrix (B) condensed (B*) from 15×18 to 5×4 .

$$B^* d^* e^* = \begin{bmatrix} \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_{3x} \\ d_{3y} \\ d_{4x} \\ d_{4y} \end{bmatrix} = \begin{bmatrix} e_{o1}^* \\ e_{o2}^* \\ e_{o3}^* \\ e_{o4}^* \\ e_{o5}^* \end{bmatrix}$$

$B^* d^* e^*$

The size of the flexibility matrix (F) reduced from 15×15 to 5×5 .

$$10^3 * \begin{bmatrix} 18.5 & -18.4 & 0 & 0 & 0 \\ -18.4 & 18.5 & 0 & 0 & 0 \\ 0 & 0 & 18.5 & -18.4 & 0 \\ 0 & 0 & -18.4 & 18.5 & 0 \\ 0 & 0 & 0 & 0 & 0.05 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} e_{o1}^* \\ e_{o2}^* \\ e_{o3}^* \\ e_{o4}^* \\ e_{o5}^* \end{bmatrix}$$

$F^* t^* e^*$

If all members considered to control all free nodes in X and Y direction the Y_c matrix would be

$$\begin{bmatrix} d_{c3x} \\ d_{c3y} \\ d_{c4x} \\ d_{c4y} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \sqrt{2} & \sqrt{2} & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{oc1} \\ e_{oc2} \\ e_{oc3} \\ e_{oc4} \\ e_{oc5} \end{bmatrix} = \begin{bmatrix} d_{pc3x} \\ d_{pc3y} \\ d_{pc4x} \\ d_{pc4y} \end{bmatrix}$$

$d_c Y_c e_{oc} d_{pc}$

Without optimization algorithms, based on Y matrices, the impact of Members 1, 2, and 3 on controlling the displacement (d_{4y}) is individually attributed to actuators s1, s2, and s3, respectively. Thus,

$$[0] = [-5.56] + \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \begin{bmatrix} e_{oc1} \\ e_{oc2} \\ e_{oc3} \end{bmatrix}$$

$d_{c3} d_{pc3} Y_c e_{oc}$

Without employing coupling optimization algorithms, the system's equilibrium can be described by Eq. 15, leading to a set of actuation of $e_o = [-1.31 \ -1.31 \ 2.62]$. Under these conditions, the total actuation would amount to 5.24 mm. However, direct application of the current technique, which involves optimizing actuation individually for each member, yields a different result. Specifically, the optimal solution dictates that only Bar 3 should be actuated, achieving a displacement of 3.93 mm.

$$[0] = [-5.56] + [\sqrt{2}] e_{oc3}$$

$d_{c3y} = d_{pc3y} + Y_c * e_{oc3}$

Applying $e_{oc3} = 3.93$ can restore node four to its original level. Employing Eq. (16) to control all nodes in X and Y directions within $[-0.5 \ 0.5]$ mm. it ended up with actuating only Members 3 and 4 with $[3.93 \ -3.92]$ mm respectively.

$$\begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix} \leq \begin{bmatrix} 0 \\ -0.1 \\ -5.55 \\ -5.56 \end{bmatrix} + \begin{bmatrix} 0.0010 & 0.0010 \\ 1.4118 & 1.4118 \\ 1.4128 & -0.0014 \\ 1.4142 & 0 \end{bmatrix} \begin{bmatrix} e_{oc3} \\ e_{oc4} \end{bmatrix} \leq \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

$d_{\min} d_{pc} Y_c e_{oc} d_{\max}$

2.4. Conducting experimental test

In this study, turnbuckles (mechanical actuators) are used to change the length of active members, the actuators are 3 mm in diameter and they are threaded at the far end (close to the pin-joint end) of the active bars. For measuring nodal displacement in structures through the integration of a camera and AutoCAD software is used. The methodology involves capturing sequential images of the structure at three critical stages: before loading, after loading, and after actuation. Each photograph is subsequently imported into AutoCAD, where the nodal

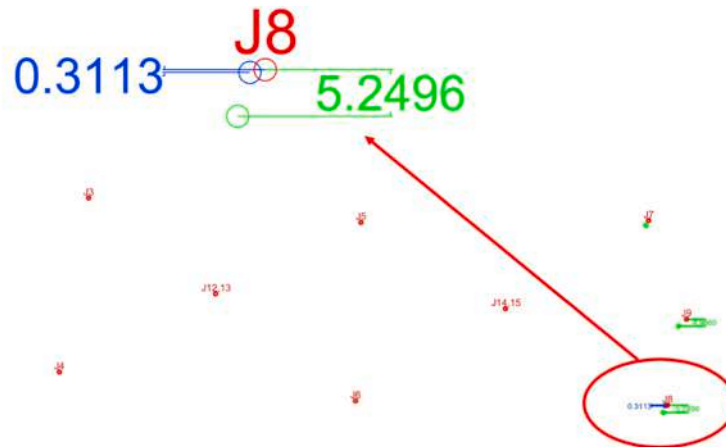


Fig. 5. Measuring nodal coordinates before and after loading and after actuation.

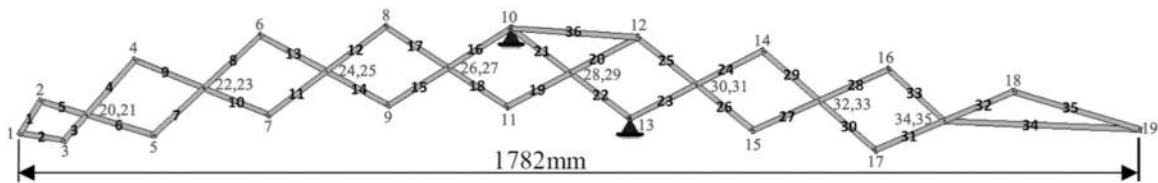


Fig. 6. The morphing (aerofoil) structure adapted from [56].

coordinates are analyzed and compared to quantify the structural deformations as presented in Fig. 5. The red, green and blues circles represent the nodes before loading, after loading, and after actuation respectively. To ensure accuracy in the scaling of the images, a right-angle ruler is strategically positioned adjacent to the structure during photography. This ruler serves as a reference for scaling within the AutoCAD environment, establishing a precise conversion where one millimeter on the physical ruler corresponds to one unit in AutoCAD. The utilization of this combined camera-AutoCAD technique provides a robust and efficient means for quantifying nodal displacements in structures, offering valuable insights into structural behavior under different loading conditions.

Given the relatively small size of the physical model and the high precision of the camera used in this study, a potential source of error arises from the recording process. The recorded images may introduce an error of up to 0.2 mm, considering that the dimensions of a pixel in the AutoCAD representation of the pictures are set at 0.2 mm. This pixel-level accuracy is crucial for maintaining the fidelity of the structural measurements within the AutoCAD environment.

3. Results and discussion

This section is subdivided into various subsections. A numerical model of a morphing structure is established to provide a rationale for

adopting the current approach. This structure has previously undergone numerical and experimental testing as part of prior research efforts. The primary motivation behind retaining the original targets from the prior work is to assess the efficacy of the current approach in comparison to established methods. Furthermore, a physical model of the structure is fabricated and subjected to examination across multiple scenarios. This examination aims to validate the inequality aspect of the formulated approach, adding an empirical dimension to its assessment.

3.1. Comparative study

This study section focuses on analyzing a morphing structure composed of multiple pantographs, as depicted in Fig. 6; the cases are available at [56]. The examination involves applying the previous technique by Saeed and Kwan [45] and the novel approach introduced in this paper across four distinct cases. The structural composition encompasses nineteen pin joints, eight shear connectors, and thirty-six bars. These bars feature a rectangular cross-section, measuring 6×9 mm, and exhibit a modulus of elasticity of 70 GPa. The nodal coordinates of the aerofoil structure are tabulated in Table 1.

3.1.1. Case 1

In this particular case, vertical loading was exclusively applied to the top joints, specifically Joints 1, 2, 4, 6, 8, 12, 14, 16, 18, and 19, with a

Table 1
Joint coordinates of the aerofoil structure.

Joint	X	Y	Joint	X	Y	Joint	X	Y
1	0	25.81	10	781.89	186.04	19	1781.9	31.724
2	31.888	74.79	11	775.78	66.191	20,21	104.9	53.8
3	69.293	15.092	12	981.89	172.33	22,23	292.06	94.431
4	181.89	137.5	13	970.82	48.548	24,25	487.32	120.25
5	212.27	22.933	14	1181.9	150.95	26,27	681.48	125.77
6	381.89	174.91	15	1167.3	30.504	28,29	875.8	117.7
7	395.69	53.926	16	1381.9	123.15	30,31	1076.6	99.877
8	581.89	188.7	17	1362.2	0	32,33	1271.9	75.641
9	586.55	68.793	18	1581.9	89.329	34,35	1471.8	44.557

Table 2

Comparison between previous study and current technique for Case 1.

Joint	direction	Load (N)	d _{pc} (mm)		d _c (mm)	[45] and [56]				Current technique			
						d _a Theo. (mm)	d _a Exp. (mm)	Members	Actuation (mm)	d _a (mm)		Members	Actuation (mm)
			Lin.	NL.						Lin.	NL.		
1	Y	-2.286	-13.44	-13.49	-9.00	-9.00	-9.45	4	0.89	-9.00	-9.05	1	0.34
2	Y	-2.286	-12.85	-12.90	-8.00	-8.00	-8.58	5	0.84	-8.05	-8.1	9	0.79
4	Y	-2.286	-9.56	-9.61	-5.00	-5.00	-5.11	12	-1.72	-5.09	-5.13	12	-0.43
6	Y	-2.286	-5.44	-5.46	-3.00	-3.00	-2.99	13	-0.82	-2.93	-2.85	16	-0.39
8	Y	-2.286	-2.13	-2.14	-1.00	-1.00	-1.13	17	1.58	-1.09	-1.1	20	-0.23
12	Y	-	0.47	0.46	0.00	0.00	0.13	20	-0.24	0.02	0.02	25	-0.49
14	Y	-2.286	-0.41	-0.41	0.00	0.00	0.23	24	1.6	0.08	0.07	33	-0.31
16	Y	-2.286	-2.14	-2.15	-1.00	-1.00	-0.96	25	-0.11	-1.05	-1.05	35	-0.23
18	Y	-2.286	-4.33	-4.35	-2.00	-2.00	-2.13	28	-0.79	-2.00	-2.00	-	-
19	Y	-2.286	-7.31	-7.34	-3.00	-3.00	-3.07	29	-1.46	-3.00	-3.00	-	-
Total Actuation (mm)									10.05	3.21			

Table 3

Comparing the results of previous method and presented approach for Case 2 (Same loading of Case 1 and lengthening bar 36 by 2 mm).

Joints	direction	Load (N)	d _{pc} (mm)		d _c (mm)	[45] and [56]			Current tech.				
						da Theo. (mm)	Members	Actuation (mm)	da (mm)		Members	Actuation (mm)	
			Lin.	NL.					Lin.	NL.			
1	Y	-2.286	-6.61	-6.9	-2.00	-2.00	4	1.43	-1.99	-1.91	4	0.49	
2	Y	-2.286	-6.45	-6.72	-2.00	-2.00	5	-0.78	-2.01	-1.91	8	-1.33	
4	Y	-2.286	-3.76	-3.9	-1.00	-1.00	12	-0.97	-1.00	0.93	13	-0.67	
6	Y	-2.286	-0.69	0.71	-2.00	-2.00	13	-2.28	-2.00	-1.97	16	-0.62	
8	Y	-2.286	0.65	0.67	0.00	0.00	17	1.06	0.00	0.00	20	1.57	
12	Y	-	-2.97	-3.04	0.00	0.00	20	2.04	0.06	0.03	29	0.44	
14	Y	-2.286	-7.42	-7.61	-4.00	-4.00	24	-0.13	-4.06	-4.09	32	0.67	
16	Y	-2.286	-13.21	-13.53	-10.00	-10.00	25	-0.08	-10.00	-10.07	33	-0.43	
18	Y	-2.286	-19.62	-20.09	-15.00	-15.00	28	-0.01	-15.00	15.08	-	-	
19	Y	-2.286	-29.72	-30.27	-24.00	-24.00	29	1.25	-24.00	-24.02	-	-	
Total Actuation (mm)													6.22

force magnitude of -2.284 N. These loads induced nodal displacements (d_c), as detailed in Table 2. The primary objective was to mitigate the vertical displacements of the aforementioned top nodes and align them with the specified target values denoted as (d_{pc}), also presented in Table 2.

The data within the table underscores that the previous method necessitated the utilization of 10 turnbuckles, with a cumulative actuation requirement of 10.05 mm, to achieve the desired displacements. In contrast, employing the current technique, which allowed for attaining the same objectives using only eight turnbuckles and a total actuation of 3.21 mm. To validate the outcomes before and after actuation, both linear and nonlinear analyses were conducted. The results obtained from these two analyses demonstrated a high degree of similarity this is because the structure behaves linearly for the given loading. The advantages of the present method over the prior technique are notable, encompassing a 20% reduction in the utilization of turnbuckles and a precise 68% decrease in the actuation requirement. These benefits are graphically illustrated in Figure 4Figure 5 for enhanced clarity and visual representation.

3.1.2. Case 2

In this scenario, the same loading conditions as Case 1 were applied to the structures, with the sole distinction being the initial lengthening of Bar 36 (the morphing bar) by 2 mm. This alteration resulted in larger induced displacements compared to those observed in Case 1, as detailed in Table 3. Both the previous and current methods successfully achieve the specified displacement targets. The table also compares the results of linear and nonlinear structural analyses after lengthening Bar 36 and after adjustment. The analyses exhibit minimal differences, while the previous method utilized ten turnbuckles, compared to only eight for the current technique. Regarding actuation, the previous method

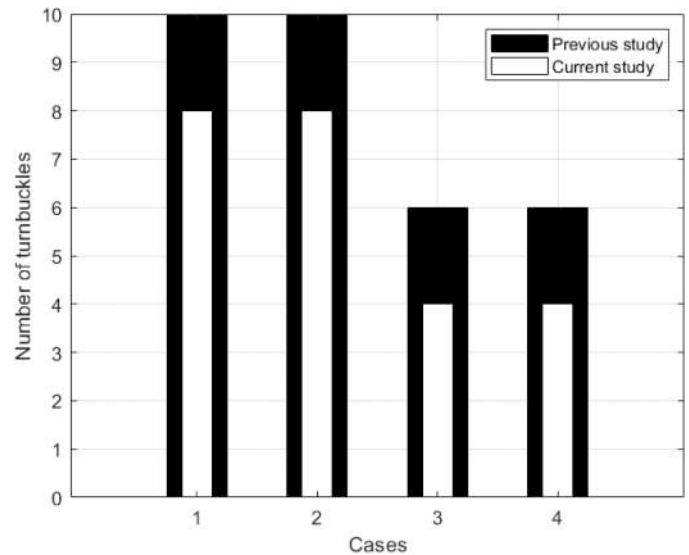


Fig. 7. The number of turnbuckles used in the previous and current methods in four cases.

required 10.04 mm of movement to achieve the desired results, whereas the current approach attained the same goals with significantly less actuation, at merely 6.22 mm. These distinctions are clearly illustrated in Figs. 7 and 8.

In terms of stress, negligible changes were observed upon altering the length of Bar 36 or during adjustment. Additionally, the tabulated data in Table 4 confirms the stability and safety of the structure before and

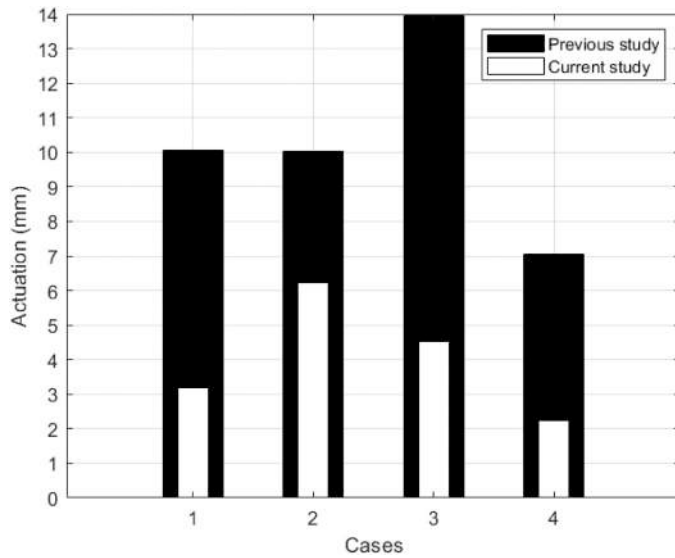


Fig. 8. Comparison of actuation amount of the previous method and the current technique.

after actuation. This is evident from the observation that the maximum stress in all cases, both pre and post-actuation, remains at a mere 1.23 MPa. This value is notably insignificant compared to the material strength, further establishing the overall stability and safety of the structure.

Table 4
Axial stress in Members in MPa for all cases before and after actuation.

Members	Case 1		Case 2		Case 3		Case 4	
	Before	After	Before	After	Before	After	Before	After
1	0.05	0.05	0.05	0.05	0.21	0.21	0.05	0.05
2	-0.02	-0.02	-0.03	-0.03	-0.11	-0.11	-0.03	-0.03
3	-0.01	-0.01	-0.01	-0.01	-0.06	-0.06	-0.01	-0.01
4	0.16	0.16	0.16	0.16	0.38	0.38	0.16	0.16
5	0.00	0.00	0.00	0.00	0.06	0.06	0.00	0.00
6	-0.10	-0.10	-0.10	-0.10	-0.26	-0.26	-0.10	-0.10
7	-0.10	-0.10	-0.10	-0.10	-0.25	-0.25	-0.10	-0.10
8	0.37	0.37	0.38	0.38	0.66	0.66	0.38	0.38
9	0.05	0.05	0.05	0.05	0.18	0.18	0.05	0.05
10	-0.26	-0.26	-0.26	-0.26	-0.49	-0.49	-0.26	-0.26
11	-0.30	-0.30	-0.31	-0.31	-0.56	-0.56	-0.31	-0.31
12	0.66	0.66	0.67	0.67	0.98	0.98	0.67	0.67
13	0.16	0.16	0.16	0.16	0.36	0.36	0.16	0.16
14	-0.47	-0.47	-0.48	-0.48	-0.72	-0.72	-0.48	-0.48
15	-0.58	-0.58	-0.59	-0.59	-0.90	-0.90	-0.59	-0.59
16	1.00	1.00	1.02	1.02	1.30	1.30	1.02	1.02
17	0.32	0.32	0.33	0.33	0.56	0.56	0.33	0.33
18	-0.73	-0.73	-0.74	-0.74	-0.93	-0.93	-0.74	-0.74
19	-0.92	-0.92	-0.94	-0.94	-1.23	-1.23	-0.94	-0.94
20	-0.09	-0.09	-0.09	-0.09	0.56	0.56	-0.09	-0.09
21	0.45	0.45	0.46	0.46	0.72	0.72	0.46	0.46
22	-0.45	-0.45	-0.45	-0.45	-0.71	-0.71	-0.45	-0.45
23	-0.52	-0.52	-0.53	-0.53	0.00	0.00	-0.53	-0.53
24	0.23	0.23	0.23	0.23	0.00	0.00	0.23	0.23
25	0.73	0.73	0.74	0.74	0.00	0.00	0.74	0.74
26	-0.40	-0.40	-0.40	-0.40	0.00	0.00	-0.40	-0.40
27	-0.31	-0.31	-0.32	-0.32	0.00	0.00	-0.32	-0.32
28	0.09	0.09	0.09	0.09	0.00	0.00	0.09	0.09
29	0.47	0.47	0.48	0.48	0.00	0.00	0.48	0.48
30	-0.20	-0.20	-0.20	-0.20	0.00	0.00	-0.20	-0.20
31	-0.14	-0.14	-0.15	-0.15	0.00	0.00	-0.15	-0.15
32	0.12	0.12	0.12	0.12	0.00	0.00	0.12	0.12
33	0.25	0.25	0.26	0.26	0.00	0.00	0.26	0.26
34	-0.17	-0.17	-0.17	-0.17	0.00	0.00	-0.17	-0.17
35	0.18	0.18	0.18	0.18	0.00	0.00	0.18	0.18
36	0.61	0.61	0.62	0.62	-0.65	-0.65	0.62	0.62

3.1.3. Case 3

In this particular case, a vertical load of 10.287 N was applied to Joint 1, resulting in the observed displacements as detailed in Table 5. The primary objective was to effectively nullify the vertical displacements exhibited by all uppermost joints within the morphing structure. This goal was achieved through the utilization of the previous method. However, from a theoretical perspective, a marginal discrepancy exists between the predicted and practical outcomes.

Remarkably, the current technique achieved the desired outcome by implementing only four turnbuckles, in contrast to the previous method, which required six. Furthermore, the previous approach demanded an actuation of 13.96 mm, while the current technique accomplished the same objectives with a significantly reduced actuation of merely 4.53 mm. Moreover, one can see that there is a negligible dissimilarity between the linear and nonlinear analysis pre and post adjustment.

3.1.4. Case 4

In this scenario, the horizontal loading of Nodes 1, 2, 4, 6, and 8 with a force of 2.286 N led to the displacements recorded and outlined in Table 6. The primary objective was to eliminate the horizontal displacements exhibited by Joints 2, 4, 6, and 8, while concurrently maintaining the displacement of Joint 1 at precisely 1 millimeter. Notably, both methods successfully achieved these objectives from a theoretical standpoint. However, a minor variance exists between the theoretical predictions and the practical outcomes when applying Saeed and Kwan [45] technique. The former method required six turnbuckles and an actuation of 7.04 mm, whereas the latter required only four turnbuckles and a 2.24 mm actuation, as depicted in Fig. 7 and Fig. 8.

Table 5

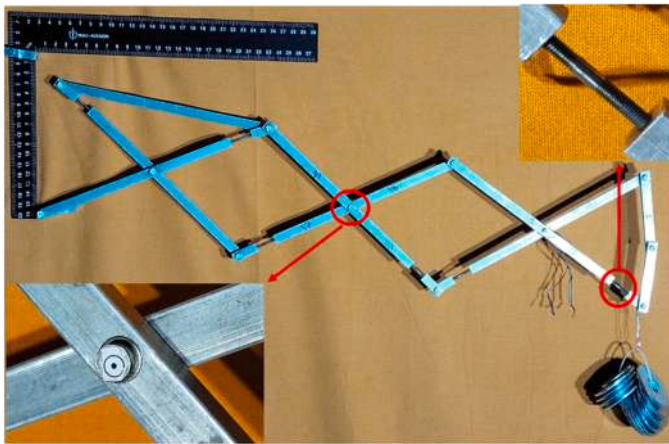
Results of the previous and current methods for displacement adjustment for Case 3.

Joints	direction	Load (N)	d _{pc} (mm)		d _c (mm)	[45] and[56]				Current tech.			
						d _a Theo. (mm)	d _a Exp. (mm)	Members	Actuation (mm)	d _a (mm)		Members	Actuation (mm)
			Lin.	NL.						Lin.	NL.		
1	Y	-10.287	-21.54	-21.62	0.00	0.00	1.36	4	-0.15	-0.03	-0.33	11	0.98
2	Y	-	-20.55	-20.64	0.00	0.00	1.39	5	-0.44	-0.04	-0.36	15	1.13
4	Y	-	-15.19	-15.25	0.00	0.00	1.23	12	-5.82	0.11	0.14	19	1.77
6	Y	-	-8.59	-8.62	0.00	0.00	0.97	13	-1.98	-0.06	-0.08	20	-0.65
8	Y	-	-3.40	-3.40	0.00	0.00	0.24	17	4.92	0.00	0.01	-	-
12	Y	-	1.25	1.25	0.00	0.00	0.13	20	-0.65	0.00	0.00	-	-
14	Y	-	1.39	1.40	0.00	0.00	0.05	-	-	0.00	0.00	-	-
16	Y	-	1.71	1.72	0.00	0.00	-0.05	-	-	0.00	0.00	-	-
18	Y	-	2.10	2.11	0.00	0.00	0.36	-	-	0.00	0.00	-	-
19	Y	-	3.60	3.61	0.00	0.00	0.11	-	-	0.00	0.00	-	-
Total Actuation (mm)									13.96	4.53			

Table 6

Displacement adjustment for horizontal loading of the morphing structure using previous and current techniques.

Joints	direction	Load (N)	d _{pc} (mm)		d _c (mm)	[45] and [56]				Current tech.			
						d _a Theo. (mm)	d _a Exp. (mm)	Members	Actuation (mm)	d _a (mm)		Members	Actuation (mm)
			Lin.	NL.						Lin.	NL.		
1	X	2.286	4.24	4.32	1.00	1.00	1.98	4	-1.27	1.01	1.01	5	0.79
2	X	2.286	4.00	4.08	0.00	0.00	0.78	5	1.06	0.00	0.00	9	0.48
4	X	2.286	2.99	3.05	0.00	0.00	0.41	12	2.25	0.00	0.00	12	0.27
6	X	2.286	1.72	1.75	0.00	0.00	0.22	13	-1.68	-0.01	0.01	36	0.70
8	X	2.286	0.65	0.66	0.00	0.00	0.13	17	0.39	-0.04	-0.04	-	-
-	-	-	-	-	-	-	-	20	-0.38	-	-	-	-
Total Actuation (mm)									7.04	2.24			

**Fig. 9.** The cantilever pantographic structure.

3.2. Experimental study

A cantilever pantographic structure was meticulously modeled, comprising a total of nine pin joints, three shear connectors, and fifteen individual members, as visually depicted in Fig. 9. The members at the joints connected with a 4 mm stainless steel the members had no friction at the joints. The members featured a consistent cross-section of 6×9 mm and exhibited a modulus of elasticity measuring 70 GPa and yield strength of 276 MPa. Detailed coordinates for the nodes to each specific test configuration are thoughtfully tabulated in Table 7 for reference and analysis. The slight differences in the nodal coordinates for each test were caused by the actuation performed during the subsequent tests and the flexibility of the structure. Typical length of the members are [114.85 118.32 117.86 116.56 113.25 122.41 124.20 115.39 111.35 130.10 120.18 118.04 74.51 62.97 195.37] mm.

3.2.1. Test 1

In this test, a downward load of 5 N was applied to Joints 4, 6, and 8,

Table 7

Nodal coordinates of the cantilever pantograph for each test.

Joint	Test 1		Test 2		Test 3		Test 4		Test 5	
	X	Y	X	Y	X	Y	X	Y	X	Y
1	23.90	181.43	23.90	181.43	23.90	181.43	23.90	181.43	23.90	181.43
2	0.00	57.33	-0.22	57.44	0.12	57.34	0.13	57.41	0.00	57.63
3	216.33	147.63	216.70	145.79	215.88	146.41	215.78	146.42	218.52	148.92
4	195.65	23.72	196.10	24.06	196.08	25.03	196.03	25.02	196.66	25.30
5	410.27	130.24	406.12	127.42	404.95	129.36	404.98	129.29	410.80	128.68
6	402.44	3.08	401.99	4.01	401.10	6.31	401.11	6.29	402.73	2.74
7	603.08	131.51	601.42	127.56	600.24	130.83	600.27	130.86	601.48	131.14
8	613.58	0.00	612.93	1.42	610.86	5.03	610.97	5.12	613.46	-1.18
9	628.03	61.30	626.20	60.18	624.81	63.68	624.79	63.56	629.63	60.50
10,11	108.39	103.63	108.86	103.31	109.14	103.67	109.18	103.35	109.54	106.38
12,13	306.70	79.36	305.31	78.21	304.64	79.48	304.37	79.54	307.66	78.83
14,15	503.11	68.74	504.38	67.17	503.05	69.95	503.11	69.85	504.48	68.10

Table 8

Adjustment of vertical displacement of Joint 8 of the cantilever pantograph.

Joint	direction	d _{pc} (mm)			d _c (mm)	d _a (mm)			Member	Actuation (mm)
		Theory		Exp.		Theory		Exp.		
		Lin	NL			Lin	NL			
8	Y	-4.99	-4.89	-5.25	0	0	0.05	-0.31	3	0.89
Total actuation (mm)										0.89

Table 9

Controlling vertical displacement of three nodes.

Joints	directions	d _{pc} (mm)		Exp.	d _c (mm)	d _a (mm)			members	Actuation (mm)
		Theory				Theory		Exp.		
		Lin.	NL.			Lin.	NL.			
7	Y	-3.95	-3.88	-4.25	0.00	-0.00	-0.03	-0.33	4	-0.51
8	Y	-6.05	-5.92	-6.85	0.00	-0.00	-0.05	-0.48	7	1.56
9	Y	-5.52	-5.38	-5.90	0.00	-0.00	-0.06	-0.43	-	-
Total actuation (mm)										2.07

Table 10

Controlling the vertical displacement of the last three nodes in a domain.

Joint	direction	d _{pc} (mm)			d _c (mm)		d _a (mm)			member	Actuation
		Theory		Exp.	Max	Min	Theory		Exp.		
		Lin.	NL.				Lin.	NL.			
7	Y	-7.15	-6.91	-7.45	2	-2	0.30	0.30	0.35	4	-0.7
8	Y	-10.86	-10.41	-11.31	2	-2	-0.08	-0.07	-0.43	7	2.85
9	Y	-9.87	-9.41	-10.29	2	-2	-0.05	-0.05	-0.39	-	-
Total actuation (mm)											3.55

with the largest displacement observed at Joint 8, measuring -4.99 mm. The primary objective of this test was to nullify the displacement at Joint 8. Utilizing Eq. 10, it was determined that this target could be achieved by lengthening the length of Member 3 by a mere 0.89 mm. Notably, the data presented in Table 8 reveals a commendable concurrence between the experimental outcomes and the theoretical predictions obtained through MATLAB. Furthermore, after applying the actuation, the structure has been analyzed with linear and nonlinear methods, there is a negligible dissimilarity between the linear, nonlinear methods and the experimental results as presented in Table 8. This is due to the fact that, the adjustment is done for small displacements thus linear methods are valid for such small movements.

3.2.2. Test 2

This experiment was specifically conducted to assess the effects of asymmetric loading applied solely to Joint 8, involving a downward force of 10 N. As detailed in Table 9, the results encompass nodal displacements attributed to the applied loading (d_c), the targeted displacement (d_{pc}), the displacement achieved through active actuation

(d_a), the corresponding values of active turnbuckle actuation, and their respective magnitudes. The primary objective entailed the elimination of vertical nodal displacements exhibited by the last three joints in the structure. Notably, the desired displacement outcomes were successfully attained through the shortening of Member 4 by 0.51 mm and the lengthening of Member 7 by 1.56 mm. The tabulated data in the table reveals that the experimental results marginally exceed the theoretical projections, possibly due to structural flexibility, experimental errors, and the precision of displacement measurement procedures. Furthermore, the table clarifies that, the structure behaves linearly since the nodal displacements before and after actuation based on linear and nonlinear analysis are almost the same.

3.2.3. Test 3

Test 3 was specifically designed to validate the applicability of Eq. 11. In this experimental trial, forces of -5 N were applied to Joints 4 and 5, while Joint 8 experienced a force of -15 N. The primary objective of this test was to ensure that the displacements denoted as (d_{pc}), as presented in Table 10, remained within the specified range of

Table 11

Controlling the vertical displacement of all free nodes.

Joint	direction	d _{pc} (mm)			d _{max}	d _{min}	d _a (mm)			member	Actuation
		Theory		Exp.			Theory		Exp.		
		Lin.	NL.				Lin.	NL.			
3	Y	-0.01	-0.01	-0.12	1	-1	-0.30	-0.33	-0.21	3	1.39
4	Y	-1.71	-1.68	-1.97	1	-1	0.57	0.50	0.42	4	-1.55
5	Y	-2.35	-2.30	-2.54	1	-1	0.23	0.15	0.07	-	-
6	Y	-4.95	-4.79	-4.45	1	-1	0.46	0.40	0.22	-	-
7	Y	-6.43	-6.26	-6.66	1	-1	-0.07	-0.18	-0.19	-	-
8	Y	-9.77	-9.46	-10.11	1	-1	-0.22	-0.31	-0.43	-	-
9	Y	-8.87	-8.55	-9.21	1	-1	-0.22	-0.32	-0.55	-	-
Total actuation (mm)											2.94

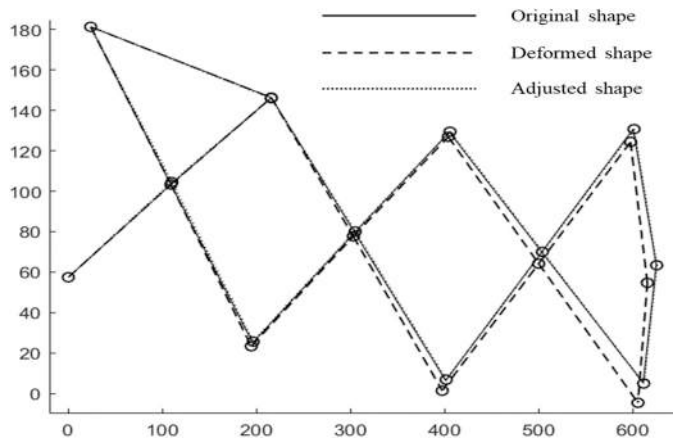


Fig. 10. Configuration of the cantilever pantograph in three various stages.

[− 2, 2] mm. The tabulated data in Table 10 demonstrates that the predetermined targets were successfully achieved, and noteworthy is the minimal disparity observed between the theoretical predictions and the empirically obtained results with linear and nonlinear analysis pre and post adjustment.

3.2.4. Test 4

In this experimental trial, the joints designated as 4, 6, and 8 were subjected to a downward load of 10 N. Suppose the identical configuration employed in §3.2.1 were to be employed. In that case, nodal displacements are anticipated to be twice as pronounced, given that the loads applied in Test 4 are twice the magnitude of those in §3.2.1. The overarching objective of this particular test was to maintain the vertical displacements of all unrestrained nodes within the prescribed domain of [− 1, 1] mm. Examination of the data presented in Table 11 reveals that the actual displacements of the joints generally approximate the theoretically predicted displacements, albeit with minor disparities. These

disparities are likely attributed to various contributing factors, including the measurement apparatus's precision and the structural system's flexibility.

Moreover, Fig. 10 shows the configuration of the cantilever pantograph before and after loading and after adjustment. A noticeable joint movement can be seen, especially for Joints 6–9, after loading the structure. One can see that the structure is almost restored to its original shape after changing the length of only two members.

3.2.5. Test 5

In this specific test, horizontal displacement was adjusted in response to applying a 15 N external load in the X-direction at Joint 8. The resulting horizontal displacements at Joints 7 and 8 are meticulously documented in Table 12. The primary objective was to effectively control the lateral movement of Joints 7 and 8 within the prescribed range of [− 1, 1] mm while employing the minimum number of turnbuckles and minimizing actuation requirements. Remarkably, it was determined that the desired outcome could be achieved by simply extending the length of Member 1 by a mere 1.8 mm. The tabulated data within the table underscores a negligible discrepancy between the theoretical results obtained from linear and nonlinear analysis methods and the experimental results, affirming the effectiveness of this approach in achieving the specified objectives.

Concerning strength and buckling stability, the applied loads were studied so that before and after actuation, the structure remained stable before implementing the experiment. The slenderness ratio of the longest members is 75.14, and the critical member can resist up to 245 MPa and 276 MPa compressive and tensile stress respectively, while in the case of the experiments, the maximum axial stress before and after actuation is up to 4.46 MPa as presented in Table 13. Meanwhile the structural deformations were small. It should be noted that increasing the compressive stress in Bar 1 in Case 5 is due to lengthening it, knowing that the single state of self-stress of the structure is [− 0.65 − 0.00 − 0.20 − 0.48 0.00 − 0.00 − 0.00 − 0.00 0.00 0.00 0.00 0.00 0.00 0.56].

Table 12

Horizontal displacement adjustment.

Joint	direction	d _{pc} (mm)			d _{max}	d _{min}	d _a (mm)			member	Actuation (mm)
		Theo.		Exp.			Theo.		Exp.		
		Lin.	NL.				Lin.	NL.			
7	X	2.19	2.13	2.54	1	-1	-0.82	-0.78	-0.71	1	1.80
8	X	3.88	3.75	4.05	1	-1	0.80	0.79	0.75	-	-
Total actuation (mm)											1.80

Table 13

Stress in members in MPa before and after actuation.

Bars	Case 1		Case 2		Case 3		Case 4		Case 5	
	Before actuation	After actuation	Before actuation	After actuation	Before actuation	After actuation	Before actuation	After actuation	Before actuation	After actuation
1	0.30	-0.27	0.18	0.99	0.48	1.60	0.61	2.15	-0.48	-4.46
2	-0.17	-0.17	-0.35	-0.35	-0.52	-0.52	-0.35	-0.35	0.05	0.05
3	-1.01	-1.18	-1.04	-0.80	-2.04	-1.71	-2.00	-1.55	0.49	-0.71
4	0.12	-0.29	0.21	0.80	0.34	1.15	0.26	1.37	-0.73	-3.69
5	0.38	0.38	0.51	0.51	0.89	0.89	0.77	0.77	0.13	0.13
6	0.00	0.00	-0.13	-0.13	-0.14	-0.14	-0.02	-0.02	0.11	0.11
7	-0.49	-0.49	-0.65	-0.65	-1.14	-1.14	-0.99	-0.99	0.52	0.52
8	0.14	0.14	0.29	0.29	0.43	0.43	0.29	0.29	-0.10	-0.10
9	0.12	0.12	0.26	0.26	0.40	0.40	0.26	0.26	0.09	0.09
10	0.05	0.05	0.10	0.10	0.14	0.14	0.10	0.10	0.23	0.23
11	-0.18	-0.18	-0.37	-0.37	-0.56	-0.56	-0.37	-0.37	0.36	0.36
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.36	0.85	0.44	-0.25	0.80	-0.15	0.71	-0.60	0.57	4.01

4. Conclusion

This paper has presented a novel method for controlling nodal displacements in pantographic structures, employing the matrix condensation of force method in tandem with an interior point optimization algorithm. The results demonstrate the superiority of this approach over the previous method, showcasing a remarkable 60% reduction in actuation requirements while simultaneously reducing the reliance on turnbuckles by up to 40% when applied to similar problem scenarios. One noteworthy advantage of the proposed method is its enhanced flexibility, as it allows for the specification of targeted displacements as a domain rather than necessitating exact values. This flexibility proves invaluable, especially in scenarios involving many adjusted nodes, where precise numerical inputs might be unattainable. To further bolster the credibility of this approach, extensive validation has been conducted, including comparisons with existing methods, experimental trials, and verification through the SAP2000 software. These comprehensive assessments collectively affirm the efficacy and reliability of the proposed technique. In summary, this research not only contributes to the advancement of control methods for pantographic structures but also highlights the potential for substantial resource savings and increased adaptability in real-world applications. As such, this innovative approach holds promise for revolutionizing the field of structural control and optimization.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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CRediT authorship contribution statement

Jankowski Robert: Conceptualization, Resources, Supervision, Writing – review & editing. **Saeed Najmadeen:** Conceptualization, Formal analysis, Methodology, Supervision, Writing – review & editing, Resources. **Manguri Ahmed:** Conceptualization, Data curation, Formal analysis, Methodology, Resources, Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Paper V

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ORIGINAL PAPER

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Deformation and bending moment mitigation through quasi-static adjustment in scissor structures

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Abstract Scissor structures are widely employed in civil, mechanical, aerospace, and other engineering applications. Although studies have been conducted on their geometrical precepts, including straight, curved folding, and unfolding, researchers have underestimated the effect of bending moment on scissor structures. In a loaded scissor structure, one of the major problems is the significant moment in shear connectors due to the difference in axial forces in the coupled macro-element members. In this study, the bending moment is mitigated to a desired limit. It is done in two ways: first, by providing cables at effective locations to mitigate bending moments and deformation, and second, by providing a novel formulation that adjusts the bending moment by changing the lengths of some effective members using turnbuckles. An optimization algorithm is used to minimize the total change in member length. Several examples are provided, including a large-span scissor structure. The study was conducted in MATLAB, and the results were verified through nonlinear analysis in SAP2000. The results show that the bending moment and deformation can be reduced by 90% by attaching cables. In addition, the presented formulation can determine the minimum member length change required to adjust the bending moment in all structural members within a specified domain.

1 Introduction

Scissor structures have numerous applications in various engineering fields, including civil, mechanical, and aerospace engineering. Scissor structures are categorized as reconfigurable frameworks, applicable in various contexts, including emergency and temporary installations, as well as deployable bridges [1] and stairs [2]. These structures are distinctive because they can be assembled compactly and expanded into a specified architectural configuration, providing a range of design and functional options and covering a large span [3] and keeping symmetry during deployment [4]. Due to their functionality and common uses, researchers have been working on such structures and extensively focused on their geometric configurations from an architectural point of view [5]. Moy et al. [6] reviewed deployable scissor structures, highlighting their ability to transform configurations and categorizing them based on structural elements, applications, strength, and flexibility. Many studies have explored the geometry and movement of scissor structures. In addition to their use in deployable systems, scissor elements have also been incorporated into reconfigurable aerofoil structures [7] and other adaptive structural systems [8], where geometric variation significantly influences structural performance. For example, Reksowardojo and Senatore [8] propose a system-level optimization framework

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for load-responsive civil structures through shape morphing, where actuator placement, geometry, and sizing are optimized to minimize whole-life energy consumption. However, this study focuses solely on deployable scissor mechanisms and does not address continuous shape morphing or load-responsive geometric adaptation.

Roovers and De Temmerman [9] introduced a general method to generate regular and irregular planar grids based on any planar circle packing. They stated that the criteria for making scissor grids foldable have been established, thereby maximizing the design potential of scissor grids composed of translational units. A design methodology for the folding and unfolding of angulated deployable units was documented, emphasizing significant geometric transformations. This methodology began with sketches and was subsequently analytically formulated to visualize various angulated deployable scissor structures [10]. Yu et al. [11] emphasized that most earlier studies focused on the geometric and kinematic properties of foldable structures, neglecting their mechanical characteristics. Fan et al. [12] described a deployable ring structure based on the Swivel Diaphragm concept, solved via group-theoretic treatment to develop constraint equations and perform symmetry-adapted kinematic analysis.

In a study by Pellegrino et al. [13], a reduction of the matrices to be analyzed for macro-elements was presented. In another study, Gantes [14] reported an analytical framework for forecasting the nonlinear responses, including the snap-through phenomenon of flat and curved deployable space structures. Sun and Li [15] established a methodology to analyze the deformation and morphological changes of structures inspired by scissor-type mechanisms. One of the significant problems with mobile scissor structures is the large deformation and bending moment in the shear connectors. These studies have, to date, underestimated the threat posed by bending moments in scissor structures under load, which can be addressed by adding cables at effective locations identified in this study. This solution is significant for reducing the total weight of the mobile bridges studied by Chikahiro et al. [16]. In such a case, the bending moment is dominant and requires large sections to resist it. Other studies have been conducted to adjust nodal displacements [17] and axial forces [7] in macro-elements the bending moment was also neglected.

Adaptive structures respond to external loads and environmental factors by adjusting their internal force distribution and geometry [18]. Previous studies have utilized actuators and prestressed members in adaptive structures to redistribute stress during reconfiguration. For example, member length adjustment has been widely applied [19, 20] to reduce or eliminate structural deformations. Axial stress adjustment in geometrically nonlinear trusses [21] and cables [22] has been achieved by redistributing stress throughout the structure via length adjustments of some members. In addition, the effects of prestressing and cable loss on deployable structures were studied [23]. Recently, studies [18, 24] advanced the field further by developing quantitatively optimized, system-level methodologies that integrate actuator placement, topology optimization, and member sizing within scalable frameworks, achieving globally optimal and energy-efficient control. This study presents a strategy that mitigates bending moment and deformation by combining the use of attaching cables with changes in member length. Turnbuckles are used to adjust the length of members, redistributing stress and reducing bending moments throughout the structure. The SQP algorithm is used to minimize the total change in member length; it removes the members that require negligible length changes to be equipped with turnbuckles. The novelty of this contribution lies in demonstrating that quasi-static, turnbuckle-based adjustments, coupled with cable attachments, provide an effective means of mitigating bending moments.

Optimization methods using genetic algorithms were introduced for barrel vault scissor structures. It aims to find optimal designs for weight and compactness, demonstrates how the framework works, and analyzes the influence of design variables and constraints to achieve efficient solutions [25, 26]. These works mainly focus on the geometric and size optimization of scissor mechanisms during the design process. They provide insights into the influence of structural factors on performance.

The multi-objective optimization of deployable bistable scissor structures was investigated by Arnouts et al. [27], who aimed to minimize the transformation force during the snap-through process while maximizing the stiffness of the deployed structure, using evolutionary algorithms. This problem concerns the nonlinear transformation of these structures.

Li and Krishnan [28] developed a geometric design and optimization approach for axisymmetric scissor-type deployable structures. The approach combines motion criteria with structural performance criteria. The parameters were optimized using genetic algorithms to meet the stiffness requirement and minimize the structure's self-weight during deployment. These approaches primarily relate to the structure's design phase. However, this research maintains the structure's geometry after deployment and seeks to reduce bending moments by varying member lengths. Topology optimization was used to design lightweight, deployable scissor elements that balance structural performance with deployment energy requirements. It investigates the trade-off among stiffness, material savings, and power savings [29]. The research combines nonlinear

deployment analysis and structural performance evaluation. It also considers the optimal trade-off between weight and energy. Although previous research has focused on improving the internal material distribution and the geometric properties of the scissor components during the design stage, the present research retains the original topology and component cross sections and focuses on internal force redistribution during deployment by modifying component length. Chikahiro et al. [30] optimized reinforcement for a scissor-type Mobile Bridge using finite element analysis and differential evolution. It significantly increased load capacity and reduced weight by adding reinforcing members, leading to a lighter, stronger, and safer deployable bridge. In certain instances, scissor structures require additional members, joint constraints, or special joints [31] to ensure stability during deployment, yet limited research has been undertaken regarding the locking of these structures in designated configurations; the self-locking mechanism in foldable structures is elucidated through an approximate evaluative expression based on the work-energy principle [32].

Incorporating additional members (struts or cables) in civil engineering structures significantly enhances structural performance by increasing stiffness and energy absorption, particularly under seismic loads, thereby reducing sway. Furthermore, this technique maintains structural integrity in systems vulnerable to component loss, as evidenced by studies demonstrating deformation mitigation [33] and weight reduction [34] in space frames through strategic member addition. However, the effect of adding cables in tension zones in scissor structures on deformation and bending moment has not been investigated. In addition, experimental and numerical studies have confirmed that the addition of steel cables to scissor structures significantly increases stiffness and load-carrying capacity, while decreasing member bending moments [35, 36]. For instance, the study in reference [35] experimentally validated the efficiency of cable-reinforced scissor bridges in improving the structures' performance under static loading. However, the aforementioned studies were based on the global stiffening of the structures upon the addition of the cables, whereas the optimization of the cable positions, as well as the utilization of the cable attachment in conjunction with the length variation of the members, for the achievement of reduced bending moments, has not been investigated.

Overall, scholars have predominantly focused on the mechanisms for deploying and retracting scissor structures to achieve various configurations. In contrast, the structural response of a loaded scissor structure has received comparatively less scrutiny. However, even fewer studies have examined how to reduce deformation in loaded scissor structures. More specifically, the threat of bending moments, which is dominant in loaded scissor structures, has been underestimated. This study has two main objectives. First, it presents a numerical sensitivity analysis of how adding cables to effective zones mitigates deformation and bending moments in deployable structures. Second, it introduces a prototype formulation to reduce maximum bending within the desired domain across the entire structure by adjusting the lengths of certain members. Additionally, the objective function minimizes the total change in member length using Sequential Quadratic Programming (SQP) with MATLAB's `fmincon` function. By minimizing the total length change, the algorithm effectively reduces the number of minor turnbuckles, thus decreasing the overall number required. However, the optimality of the number of turnbuckles is not guaranteed because the algorithm removes only turnbuckles equipped with ineffective members.

In the context of the current study, the term "adjustment" refers to the quasi-static retuning of selected member lengths using manual or predefined turnbuckles. The optimization process is performed offline to determine the required adjustments in the members' lengths, which are then applied as static geometrical changes. It should be noted that the term member length change is used throughout the manuscript in a general sense, as any structural member could theoretically be selected for length adjustment using turnbuckles. However, in all examples presented in this study, only the cables exhibited significant length changes, while the remaining members showed only minor or negligible length changes. The proposed approach does not involve sensing, feedback, or real-time actuation. Accordingly, it should not be viewed as an active or semi-active approach; rather, it is a deterministic, quasi-static mitigation approach aimed at reducing the effects of applied loads on members' bending moments.

2 Bending moment and deformation mitigation

2.1 Attaching cables

When a scissor structure is deployed and loaded, a significant moment comes from the differences in the internal forces of the two elements in the adjacent macro-element. If the axial force in Members 3 and 4 shown in Fig. 1 is different, it causes a bending moment with the amount $(t_{n3} - t_{n4}) L/4$ in Macro-element 1. t_n is the

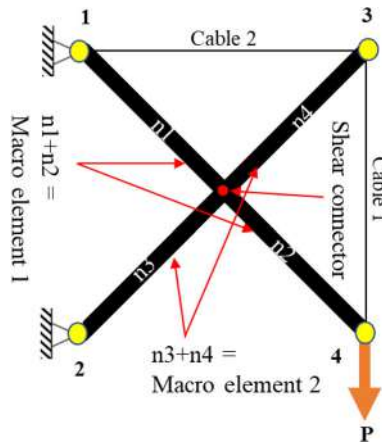


Fig. 1 The one-bay scissor structure is attached with two cables

axial force in members, and L is the length of Macro-element 2, which is the sum of the lengths of members 3 and 4. Adding cables in the tension zone of a scissor structure allows the structure to fold and unfold while significantly mitigating deformation and bending in shear connectors.

2.1.1 An illustrative example

In this study, Joint 1 is restrained after deployment, and Cable 1 is attached. However, Cable 2 also locks the deployment once the bay width equals the cable length, but the structure would not be stable with only one restrained joint. Due to the applied load ($P = 25\text{N}$), axial force (t_n) was developed in the members, leading to shear and bending moments in the macro-elements, with the maximum occurring in the shear connectors.

The structure shown in Fig. 1 has a height and width of 180 mm, and the members possess a modulus of elasticity of 69 GPa with a square cross section of 8 mm. The diameter and elastic modulus of the attaching cable are 0.89 mm and 180 GPa, respectively. These values were assumed to resist the maximum axial force induced in the cables in the worst-case scenario. The variance in axial force between two members of a macro-element results in shear and bending in its counterpart. Figure 2 shows the axial, shear forces, and bending moment diagrams of the scissor structure without cables under a $P = 25\text{ N}$ downward load at joint 4. In this loading scenario, Members 1 & 2 ($n1$ & $n2$) exhibit identical axial forces, thereby avoiding the induction of shear or moment in Macro-element 2. Conversely, Members 3 & 4 ($n3$ & $n4$) experience the axial force by -35.35 N and $1e-15\text{ N}$, respectively, generating $35.35/2\text{ N}$ shear and a maximum bending moment of 2250 N in Macro-element 1. When Cables 1 or Cables 1 & 2 are simultaneously attached to the structure (see Fig. 3), a significant reduction in bending moment occurs. The figure shows the maximum bending moment of the four cases, namely, one-bay scissor structure (a) with no cables and (b) with Cable 1, (c) Cable 2, and (d) Cables 1 & 2 simultaneously.

The detailed analysis of the four cases is presented in Fig. 4. The best-case scenario was attaching Cables 1 & 2, which significantly reduced bending moments and deformation. At the end of the free-end member of the loaded joint, maximum displacement was induced by the bending moment. In scissor structures, the macro-elements often experience maximum bending moments in shear connectors. To mitigate this behavior, cables can be introduced to transfer part of the bending demand into axial tension. For effective moment reduction, the cable should be anchored at the end joints of the macro-element, since these nodes provide stable force transfer without disrupting the overall kinematics. Importantly, the cable must be placed at the end opposite to the direction of joint displacement relative to the element's axis. This ensures that the cable develops tensile resistance as the joint displaces, thereby counteracting the induced bending moment. In other words, the cable orientation should not coincide with the macro-element's axis but rather follow an alternate direction that intercepts the displacement path. By doing so, the mid-span moment is effectively minimized, and the structural response shifts toward axial rather than flexural action, enhancing both efficiency and stability of the scissor system.

It can be seen that the maximum bending moment was -221 N.mm , and the maximum displacement was -0.07 mm . In contrast, the maximum bending moment and nodal displacement without additional cables were -2241 N.mm and 0.73 mm , respectively. It is worth noting that attaching Cables 1 & 2 simultaneously

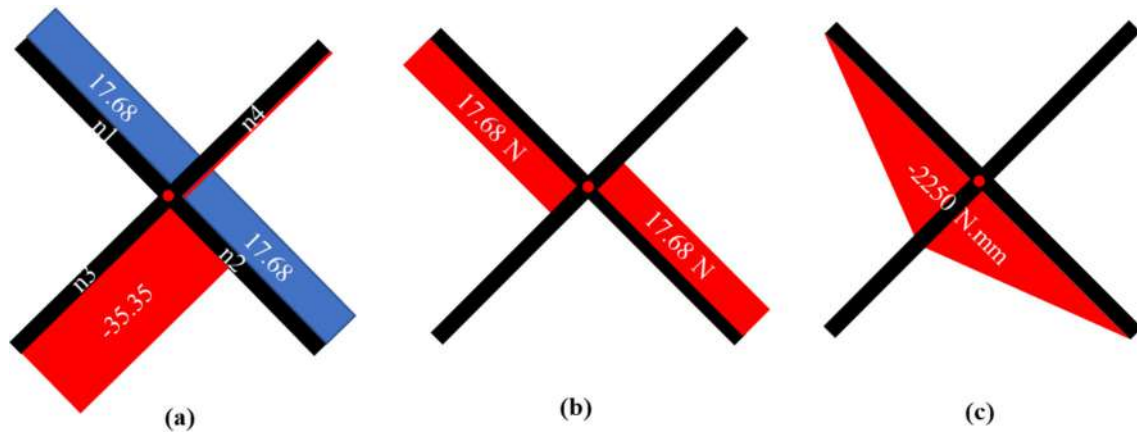


Fig. 2 Structural analysis of a one-bay scissor structure without cables **a** axial, **b** shear, and **c** moment diagrams

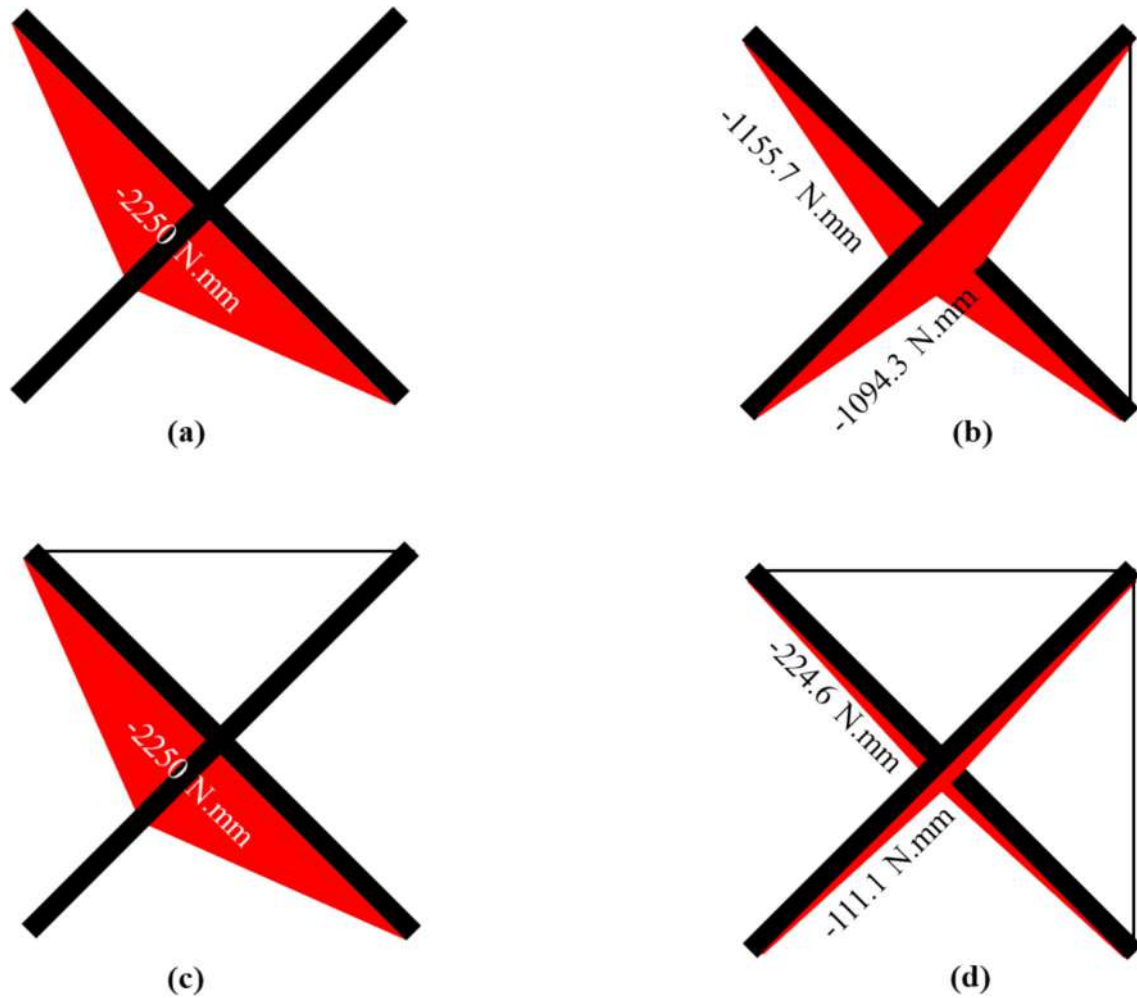


Fig. 3 Moment diagram in (N.mm) of a one-bay scissor structure **a** without cables **b** with Cable 1 **c** with Cable 2 **d** with Cables 1&2

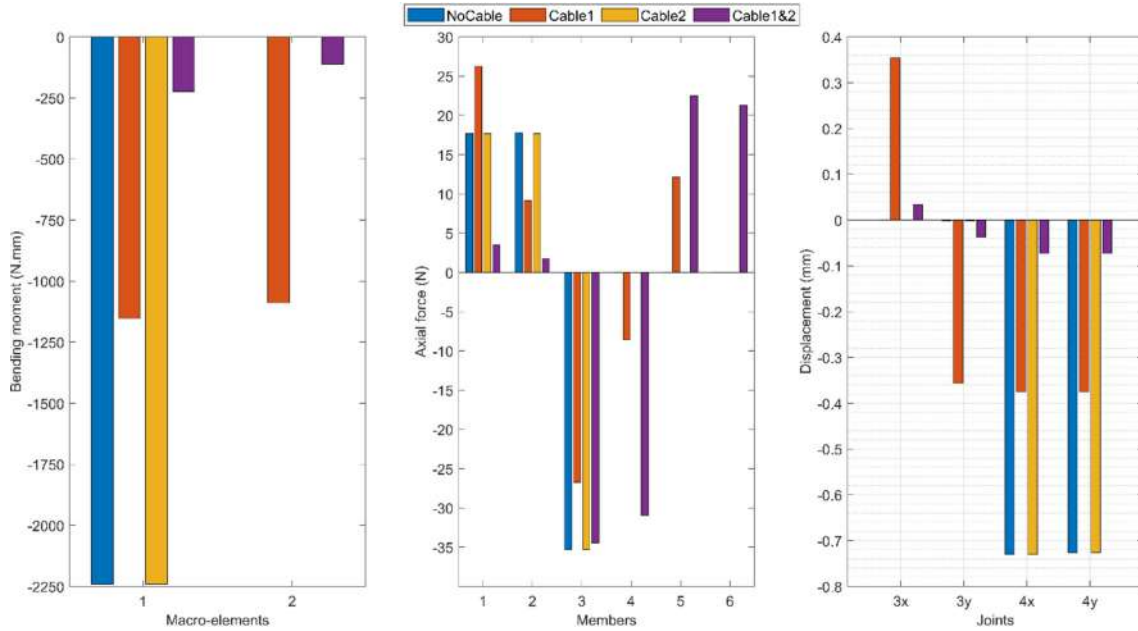


Fig. 4 Structural analysis of geometric scenarios of a one-bay scissor structure

reduced the deformation and bending moment by about 90%. On the other hand, adding Cable 1 alone reduced the bending moment and nodal displacement to around -1153 N.mm and -0.38 mm , respectively. However, Cable 2 alone did not reduce bending moment and deformation since the nodal displacement of Joint 3 was negligible even without cables.

The theoretical framework used in the present study is based on [13] and follows a quasi-static procedure for applying the load, dividing the total external load into increments and updating the nodal coordinates at each increment. The above procedure agrees with the load-discretization stiffness matrix method proposed in [37], in which the authors demonstrated that updating the nodal coordinates of a system subjected to incremental loading can be a reliable method for approximating nonlinear behavior. The theoretical results are compared with those of the nonlinear analysis performed using SAP2000. The comparison of the nonlinear analysis results for bending moments and nodal displacements of the beam is presented in Tables A1 and A2, respectively. Although the present formulation can be considered linear in the stiffness matrix, the incremental loading procedure used in the present work approximates nonlinear behavior. The discrepancies between the nonlinear analysis performed with SAP2000 and the present formulation are within acceptable limits.

2.2 Member length change

The Extended Integrated Force Method (IFME) [38] offers a unified, block-structured formulation that inherently incorporates prestress, actuation, and compatibility within a single framework, thereby eliminating redundancies and providing a more systematic and insightful treatment of prestress-stable indeterminate structures. Nevertheless, the present study is formulated using the conventional force method combined with matrix condensation, as it is easier to adapt our existing codes to this application. In some cases, attaching cables may not be sufficient to reduce the bending moment in certain members to the desired limit. This section is dedicated to formulating the method.

The displacements and axial forces have been previously derived [13]; it is available in Appendix B. In this study, the focus is on finding the bending moment and mitigating it. Substituting (B20) into (B19), then both into (B14), then all into (B15) yields the following expression that relates the member length change to the bending moment (bm) in the shear connectors.

$$\mathbf{bm} = \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{A}_{pn} \left[-\mathbf{t}_{nH} + \mathbf{S}^* \left(\mathbf{S}^{*T} \mathbf{F}^* \mathbf{S}^* \right)^{-1} \mathbf{S}^{*T} \left(\mathbf{F}^* \mathbf{t}_{nH} + \mathbf{e}_{no}^* - \left(\mathbf{A}_{pn}^T \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{e}_{po}^* \right) \right) \right] \quad (1)$$

We are now separating the internal forces due to the applied loads only as follows.

$$\mathbf{tp}_c = \begin{bmatrix} \mathbf{t}_{nH} - \mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH} \\ \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{A}_{pn} \left(-\mathbf{t}_{nH} + \mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH} \right) \end{bmatrix} \quad (2)$$

where $\mathbf{tp}_c$ contains the axial forces ($\mathbf{t}_{nt}$) and bending moments ($\mathbf{bm}_c$) due to loading only. Our focus is on the bending moment due to loading ($\mathbf{bm}_c$), which is $\left[\left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{A}_{pn} \left(-\mathbf{t}_{nH} + \mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH} \right) \right]$. The matrix ($\mathbf{Z}_c$) that relates the induced axial forces and bending moment due to loading and the targeted internal forces ($\mathbf{tp}_t$) after some member length change ($\mathbf{e}_{oc}$) is expressed as follows.

$$\mathbf{Z}_c = \begin{bmatrix} -\mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} & \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{A}_{pn} \mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \\ -\mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \mathbf{A}_{pn}^T \left(\mathbf{A}_{pp}^T \right)^{-1} & \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{A}_{pn} \mathbf{S}^*(\mathbf{S}^T \mathbf{F}^* \mathbf{S}^*)^{-1} \mathbf{S}^{*T} \mathbf{A}_{pn}^T \left(\mathbf{A}_{pp}^T \right)^{-1} \end{bmatrix}^T \quad (3)$$

And member length change and rotation are expressed as follows.

$$\mathbf{e}_{oc} = [\mathbf{e}_{no} \mathbf{e}_{po}]^T \quad (4)$$

where $\mathbf{e}_{no}$ is the axial member length change, while $\mathbf{e}_{po}$ is the imposed member rotation, which is assumed to be zero, and it is not considered in this study. The relationship between the induced ($\mathbf{tp}_c$) and targeted internal forces ($\mathbf{tp}_t$) is given by the following. $\mathbf{tp}_t$ contains targeted axial forces ($\mathbf{t}_{nt}$), and the targeted bending moment ($\mathbf{bm}_t$) after some member axial length change ($\mathbf{e}_{oc}$) is expressed as follows.

$$\mathbf{tp}_t = \mathbf{tp}_c + \mathbf{Z}_c \mathbf{e}_{oc} \quad (5)$$

However, $\mathbf{t}_{nt}$ is zero, because we are targeting only $\mathbf{bm}_t$. Most of the time, when only one or a few members exceed the limit, the goal is to reduce their bending moment. However, the target is achievable; some other members may exceed the limit after applying $\mathbf{e}_{oc}$. Thus, keeping all members' bending moments within the limit in one step is necessary. To overcome this problem, (5) is expressed as an inequality equation as follows

$$\mathbf{bm}_{\min} \leq \mathbf{bm}_c + \mathbf{Z}_c \mathbf{e}_{oc} \leq \mathbf{bm}_{\max} \quad (6)$$

$\mathbf{bm}_{\min}$ and $\mathbf{bm}_{\max}$ are all structure members' minimum and maximum allowable bending moments. All members are guaranteed to stay within the allowable limit and are free to take any value within the given domain. Applying (6) gives the necessary length changes of the members; the internal forces after the member length changes ($\mathbf{tp}_a$) can be found as follows.

$$\mathbf{tp}_a = \mathbf{tp}_c + \mathbf{Z}_c \mathbf{e}_{oc} \quad (7)$$

$\mathbf{tp}_a$ includes axial forces ($\mathbf{t}_{na}$) and bending moments ($\mathbf{bm}_a$) after the member length change is applied. In this research, the SQP optimization algorithm was implemented using `fmincon` in MATLAB and evaluated using the objective function, the total change in member length.

$$\sum_{i=1}^n \mathbf{e}_{oci} \quad (8)$$

subjected to

$$\begin{aligned} \mathbf{bm}_{\min} &\leq \mathbf{bm}_a(e_{oc}) \leq \mathbf{bm}_{\max}, i = 1, \dots, n \\ -10 \text{ mm} &< e_{oci} < 10 \text{ mm} \\ e_{oci} &= 0 \text{ for members removed by } |e_{oci}| = \varepsilon \end{aligned}$$

where i is the index of the current member and n is the total number of members requiring length modification. The constraints are bending moments in shear connectors. While the optimizer seeks to minimize the total change in members' lengths, the algorithm removes members that require negligible length change to be equipped with turnbuckles. $\mathbf{e}_{oci}$ is bounded between the upper bound (Ub) and the lower bound (Lb); in this study, these bounds are 10 mm and -10 mm, respectively. Despite the bounds, the algorithm iteratively

removes members whose absolute change in length is less than ε for embedding with turnbuckles, where ε is 0.03 mm in most cases in this study, unless otherwise noted. The problem converges where the minimum of the objective function among feasible solutions is found; the allowable tolerance is assumed to be $e-6$. Regarding computational time, the algorithm removes members that require a negligible length change in the first iteration, leaving only cables to be embedded with turnbuckles. In most problems, the solution is obtained in 3 iterations and in about 1 s, and it does not change with different initial guesses, so it is unique.

In this work, the cable placement strategy relies on a geometry-driven heuristic that minimizes the overall change in member length. At each step, turnbuckles with negligible or no length change are removed from consideration, gradually reducing the number of length-adjusting members. The user sets acceptable upper and lower bounds for member length variation, and any turnbuckles with predicted adjustments outside this range are discarded. This approach offers a practical selection of cable locations; however, it does not guarantee a globally minimal or optimal set of turnbuckles. Instead, it provides a simplified approximation that prioritizes feasibility and physical clarity over a comprehensive global solution search.

2.2.1 Computational procedure

Figure 5 shows the procedure of analysis, bending moment, and deformation mitigation of scissor structures using the coordinate update of condensed matrices of the force method. After inputting the structural geometry and properties, the matrices are condensed. The load is applied incrementally, and nodal displacements are computed and added to the previous coordinate step to update the coordinates. After the number of iterations (I) ended, all steps' nodal displacements and internal forces $\mathbf{t}_n$ and $\mathbf{bm}$ are summed to obtain the number corresponding to the total load.

The desired domain for the bending moment in all members and the members to be equipped with turnbuckles is introduced; then, the differences of the domain and the current moment are divided by I to obtain (Mdif1 and Mdif2), to find the necessary incremental e_{oi} to keep $\mathbf{bm}$ within the domain. After applying e_{oi} to the structure in each iteration, $\mathbf{bm}_a$ and $\mathbf{d}_a$ are determined, and then the structural geometry is updated again. Since the obtained M will be in the given incremental domain, in each iteration, the difference between M and the domain should be calculated and divided by the number of remaining iterations to get (Mdif11 and Mdif22). When the iterations end, the member length is calculated by summing the lengths of all iterations, and then the check should be performed to see whether $\mathbf{bm}_a$ for all members is within the domain. If not, the set of members to be equipped with the turnbuckles or the domain of the $\mathbf{bm}$ should be changed.

In this study, the proposed procedure is used as an approximate nonlinear solution strategy that updates coordinates iteratively under quasi-static loading. Although the formulation is not fully nonlinear, the successive updates allow the method to accurately capture the dominant geometric nonlinear effects. A more comprehensive nonlinear formulation is not intended here; instead, the aim is to provide an efficient approximation whose reliability is supported by comparison with SAP2000 nonlinear analyses.

3 Numerical examples

3.1 Attaching cables

This section investigates two other scissor structures of different sizes to assess the significance of adding tension cables in the tension zones in reducing deformation and bending moments.

3.1.1 Three-bay scissor structure

In this example, a three-bay scissor structure is shown in Fig. 6 made of the same material properties as the illustrative example and is vertically loaded at Node 8 with a downward force of 25 N. This study aims to determine the most effective locations for attaching cables to reduce deformation and bending moments in scissors of different sizes. As expected, deformation and bending moments increase with the span length and the number of bays. This research conducts a sensitivity analysis to identify effective cable attachment points that can be generalized for scissor structures of any scale.

Figure 7 illustrates the structural analysis conducted on the numerical model of the three-bay scissors with seven different scenarios. For all cases, the maximum bending moment and displacements occur in the first macro-element and at Node 8, respectively. The figure clearly shows that attaching Cables 1 and 2 is the most

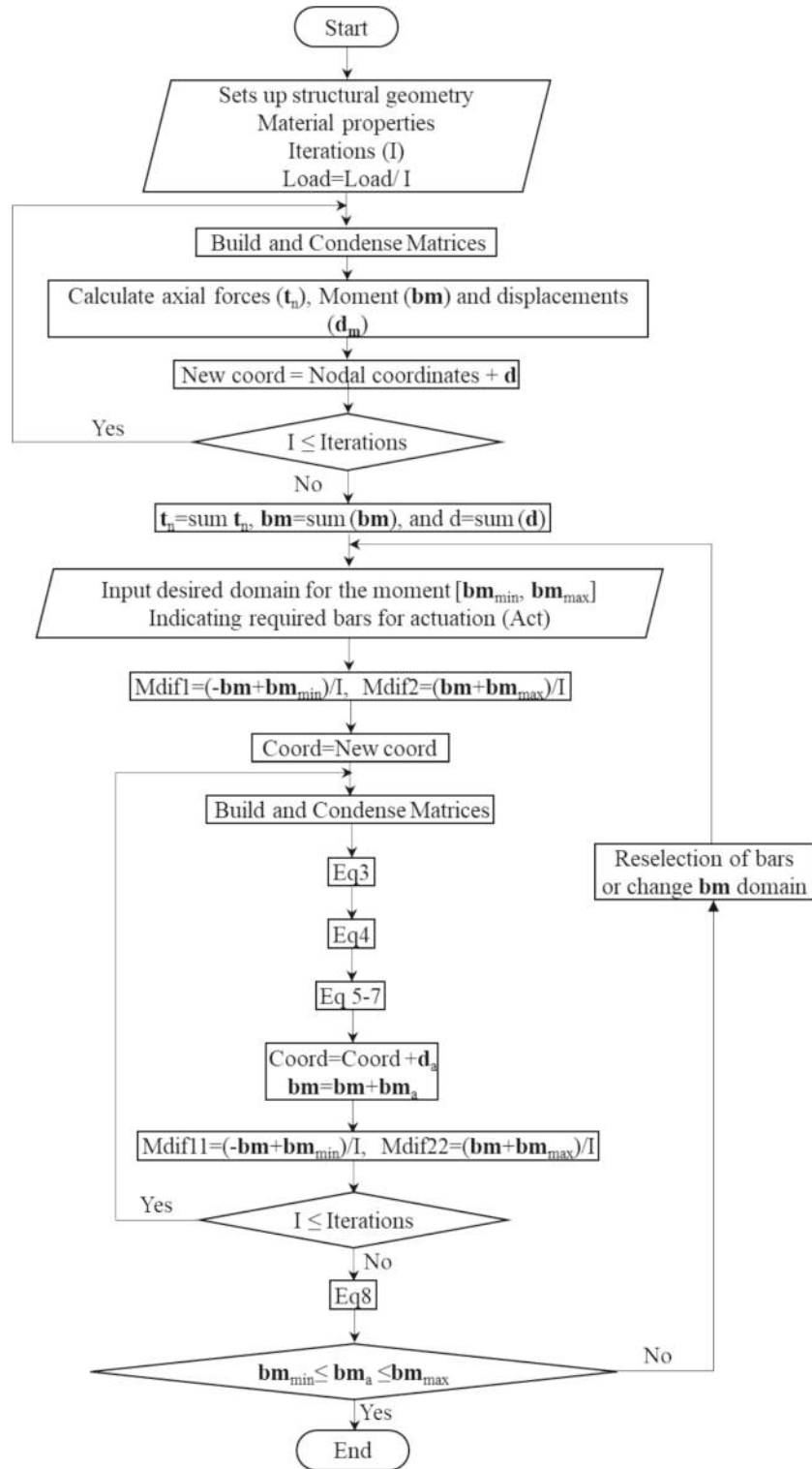


Fig. 5 The strategy of the quasi-static adjustment technique

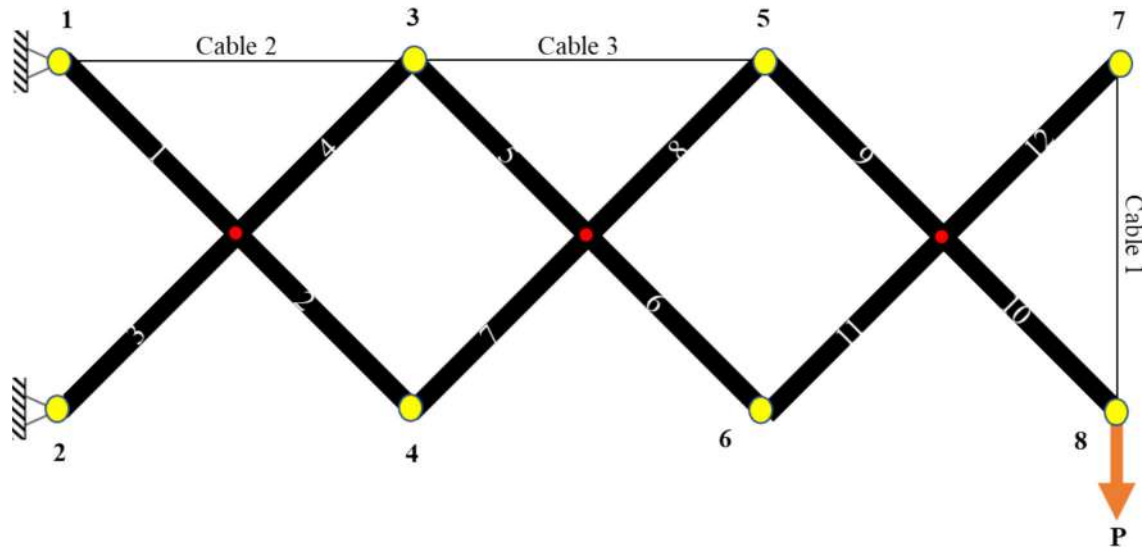


Fig. 6 The three-bay scissor structure

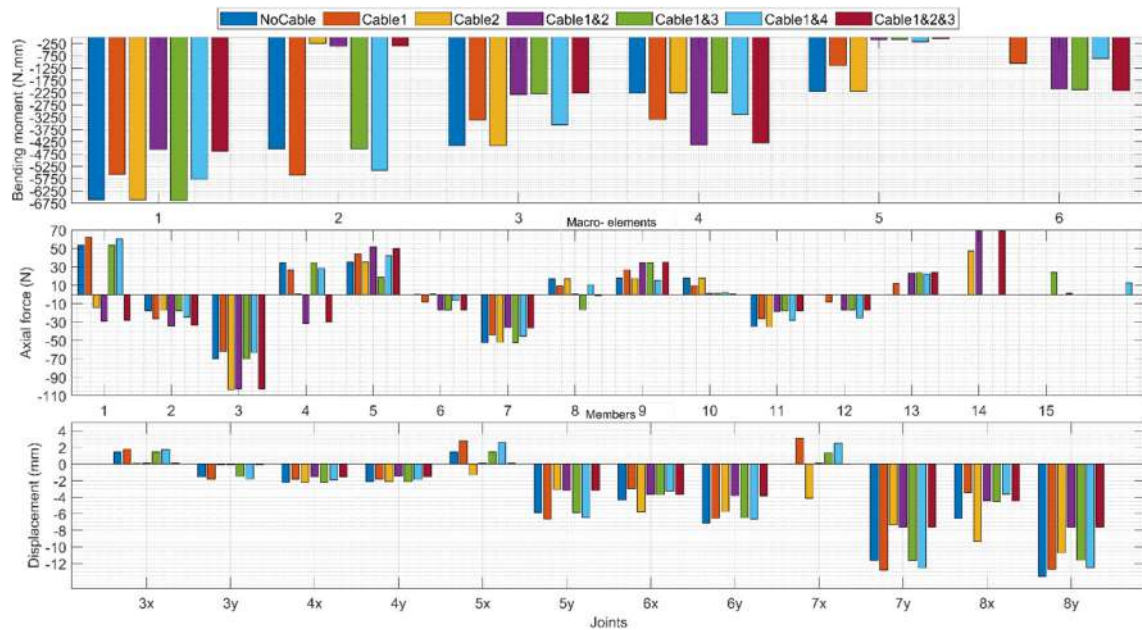


Fig. 7 Structural analysis of the numerical model of the three-bay scissor structure with seven various scenarios of extra cables

effective scenario for mitigating bending moments and structural deformation. It should be noted that Cables 1, 2, 3, and 4 correspond to Members 13, 14, 15, and 16, respectively. The analysis indicates that Cable 2 experiences the highest axial force among the cables in the scenarios involving Cable 1 & 2, Cable 1,2 & 3, and Cable 2, respectively. The comparison between the analysis of the theory and NL-SAP is tabulated in Tables A3 and A4. It can be concluded that the cable should be positioned at the end opposite to the joint's displacement direction along the element's axis. This placement allows the cable to engage in tension as the joint moves, thereby resisting the resulting bending moment.

3.1.2 Simply supported three-bay scissor structure

In this example, a simply supported three-bay scissor structure is shown in Fig. 8, with the same material properties and geometry as the previous example, and is vertically loaded at Joints 4 and 6 with $P = 25 \text{ N}$. Three

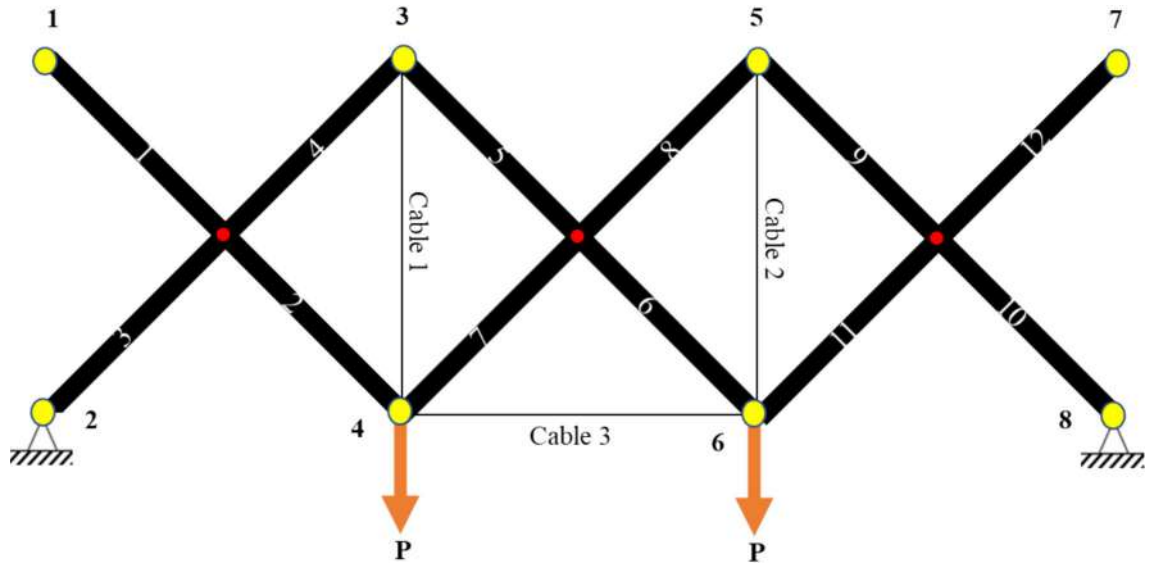


Fig. 8 The simply supported three-bay scissor structure

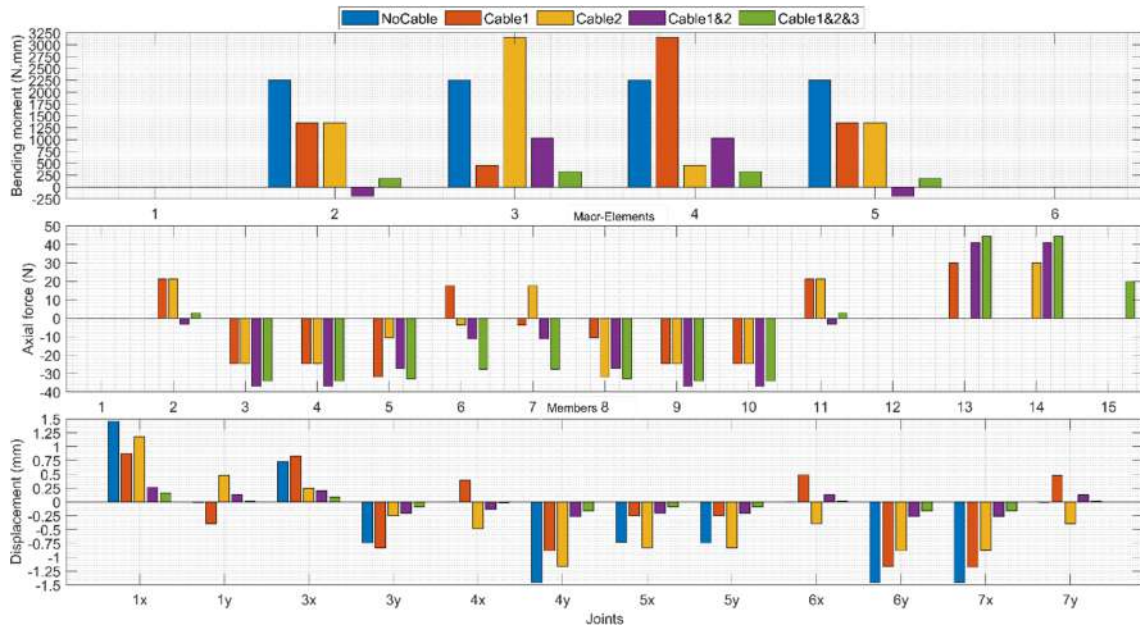


Fig. 9 Structural analysis of the numerical model of the simply supported three-bay scissor structure

locations are suggested for adding to the structure to reduce the bending moment and deformation. As shown in Fig. 9, attaching the three cables significantly reduces deformation and bending moment. The maximum bending moment without cables is 2253 N.mm, while attaching three cables at the locations shown in Fig. 8 lowers the maximum bending moment to 325 N.mm. Regarding deformation, the maximum displacement decreases from 1.46 to 0.16 mm. It should be noted that the axial forces in Cables 1, 2, and 3 are 44.4, 44.4, and 19.7 N, respectively.

3.1.3 Large-span reconfigurable structure

A large-span steel scissor structure in Fig. 10 was previously studied as an aerofoil system [7]. In the present study, it is examined in its fully deployed configuration. No geometric transformation behavior is considered in the present analysis. Its length is 7128 mm, and a vertical load of 1N downward has been applied to Joints

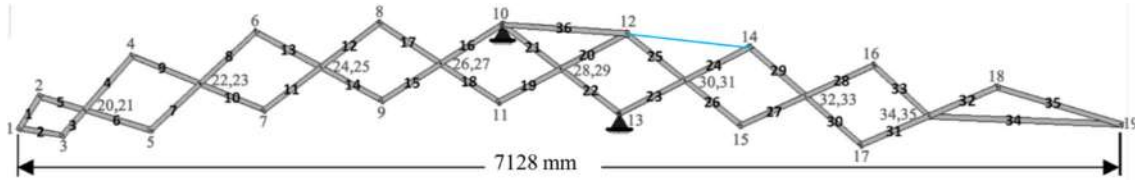


Fig. 10 The large-span steel scissor structure

Table 1 Nodal coordinates of the large-span steel scissor structure

Joint	X	Y	Joint	X	Y	Joint	X	Y
1	0	103.24	10	3127.6	744.14	19	7127.6	126.9
2	127.55	299.16	11	3103.1	264.76	20,21	419.59	215.2
3	277.17	60.368	12	3927.6	689.31	22,23	1168.3	377.72
4	727.55	549.98	13	3883.3	194.19	24,25	1949.3	480.99
5	849.06	91.732	14	4727.6	603.81	26,27	2725.9	503.09
6	1527.6	699.65	15	4669.3	122.02	28,29	3503.2	470.78
7	1582.8	215.7	16	5527.6	492.59	30,31	4306.5	399.51
8	2327.6	754.81	17	5448.9	0	32,33	5087.4	302.56
9	2346.2	275.17	18	6327.6	357.32	34,35	5887.2	178.23

Table 2 Structural analysis before and after attaching the cable

Joint	Displacement (mm)				Moment (N.mm)		Macro-element
	Before		After		Before	After	
	X	Y	X	Y			
1	0.024	- 5.114	0.752	- 8.619	- 104.42	- 104.33	1
2	- 0.359	- 4.865	0.149	- 8.228	- 315.12	- 315.14	2
3	0.171	- 4.150	0.946	- 7.345	- 335.1	- 334.85	3
4	- 0.626	- 3.419	- 0.401	- 6.110	- 833.08	- 833.17	4
5	0.290	- 2.595	1.027	- 5.149	- 514.69	- 514.3	5
6	- 0.514	- 1.698	- 0.461	- 3.492	- 1434.5	- 1434.7	6
7	0.249	- 1.173	0.844	- 2.905	- 670.46	- 669.95	7
8	- 0.232	- 0.510	- 0.242	- 1.407	- 2454	- 2454.4	8
9	0.195	- 0.326	0.723	- 1.202	- 1186.5	- 1185.7	9
11	- 0.119	- 0.331	0.419	- 0.359	1672.3	- 1343.2	10
12	- 0.033	- 0.499	0.028	0.399	- 4975	- 1401.5	11
14	1.019	- 2.112	- 0.003	0.005	- 642.02	469.26	12
15	0.165	- 0.240	- 0.142	0.178	- 3061.5	- 3060.1	13
16	2.273	- 5.164	0.485	- 1.277	- 207.42	378.45	14
17	0.904	- 2.250	- 0.002	0.005	- 1681.3	- 1681	15
18	3.601	- 8.957	1.242	- 3.071			
19	1.673	- 15.550	0.284	- 6.372			

1, 8, 12, 14, 16, 18, and 19. The members have $E = 200$ GPa and a cross section of $7.5 \text{ mm} \times 15 \text{ mm}$, and the cable has $E = 140$ GPa and 3 mm diameter. Table 1 presents the nodal coordinates of the structure, and Table 2 shows the nodal displacements before and after attaching Cable 1 between Joints 12 and 14. The displacement of Joint 19 reduced from -15.55 mm to -6.372 mm, and the maximum bending moment reduced from just under 5000 N.mm to just over 3000 N.mm. The details of the structural analysis before and after cable attachment are presented in Table 2.

To quantify and demonstrate the effectiveness of attaching cables in various structural geometries, the maximum bending moments, displacements, and axial forces in the cables, along with the percentage changes before and after attachment, are presented in Table 3. The table shows that reductions of 31.2% and 43.9% in bending moment and displacement were achieved when cables 1 and 2 were attached simultaneously in the three-bay scissor cantilever structure. For the three-bay simply supported scissor system, reductions of 85.6% and 89% in bending moments and displacements were achieved when cables 1, 2, and 3 were attached. For the large reconfigurable structure, reductions of 39.5% and 59.4% in the maximum bending moment

Table 3 Maximum bending moment, displacement, cable forces, and percentage of change before and after attaching cables

Example	Case	Max M before (N.mm)	Max M after (N.mm)	Change %	Max disp before (mm)	Max disp after (mm)	Change %	Max cable force (N)
Three-bay cantilever	Cable1	− 6622	− 5586	− 15.6	− 13.55	− 12.77	− 5.7	11.8
	Cable2	− 6622	− 6622.3	0.0	− 13.55	− 10.72	− 20.9	47.4
	Cable1,2	− 6622	− 4557.1	− 31.2	− 13.55	− 7.60	− 43.9	69.8
	Cable1,3	− 6622	− 6665.9	0.7	− 13.55	− 11.66	− 14.0	23.89
	Cable1,4	− 6622	− 5789.2	− 12.6	− 13.55	− 12.55	− 7.4	13
	Cable1,2,3	− 6622	− 4644.1	− 29.9	− 13.55	− 7.59	− 43.9	68.8
Three-bay simply support	Cable1	2253.1	3148	39.7	− 1.46	− 1.17	− 19.9	29.9
	Cable2	2253.1	3148	39.7	− 1.46	− 1.17	− 19.9	29.9
	Cable1,2	2253.1	1025	− 54.5	− 1.46	− 0.26	− 82.2	40.86
	Cable1,2,3	2253.1	325	− 85.6	− 1.46	− 0.16	− 89.0	44.45
Large-span reconfigurable structure	Cable 1	− 4975	− 3060.1	− 38.5	− 15.55	− 6.32	− 59.4	14.33

Table 4 Theoretical and NL-SAP bending moment in (N.mm) before and after member length change for a single bay scissor structure attached with two cables

	Pre-adjustment		Post-adjustment		Change %
	Theory	SAP	Theory	SAP	
Maximum moment (N.mm)	− 884	− 884	− 80	− 81	− 90
Maximum displacement (mm)	− 0.29	− 0.29	− 0.03	− 0.03	− 89
Maximum axial force in cables (N)	90	90	99	99	10

and displacements were recorded after attaching only a cable between joints 12 and 14. The table shows the maximum induced axial forces in cables for each case.

3.2 Member length change

3.2.1 One-bay scissor structure

The one-bay scissor structure shown in Fig. 1 is tested under $P = 100$ N to verify the effectiveness of the proposed formulation. In this case, joints 3–4 and 1–3 were connected with two cables (Cables 1 and 2). When the load is applied to the structure without cables, the maximum bending moment reaches 8,854 N.mm. However, under the same loading conditions with the two cables attached, the maximum bending moment is significantly reduced to 893 N.mm. The design objective is to further reduce bending moments by adjusting cable lengths, achieving a maximum bending moment that is about 100 times lower than that of the cable-free structure.

The goal is to mitigate the bending moment in both macro-elements within 80 N.mm, and the minimum possible e_{oc} is determined to achieve the goal. The target was achieved by changing the length of Cables 1 and 2 by 0.1592 and 0.1374 mm, respectively. It can be seen that a slight change in the length of two cables results in a 90% reduction in bending moment, as shown in Table 4. The table also reveals that the maximum displacement is reduced by 89% and the cable's axial force increased by 10%. The maximum axial stress is 159 MPa, while F_y is 400 MPa. Figure 11 shows the value of the bending moment results obtained from MATLAB for the structure with no cable (a), with two cables without length changing (b) and with the two cables with length changing. It can be seen that the maximum bending moment of (a) is ten times higher than that of (b) and that of (b) is ten times higher than that of (c). The structure was also modeled in SAP2000 and analyzed to account for structural nonlinearity. There was a negligible dissimilarity between the results obtained from MATLAB and the commercial software (see Table 4).

Sensitivity analysis for the cable length change has been conducted to observe its effects on bending moments and displacements across various scenarios shown in Table 5. The table shows that the optimal solution was achieved when the constraint variables were met, and the objective function was minimized.

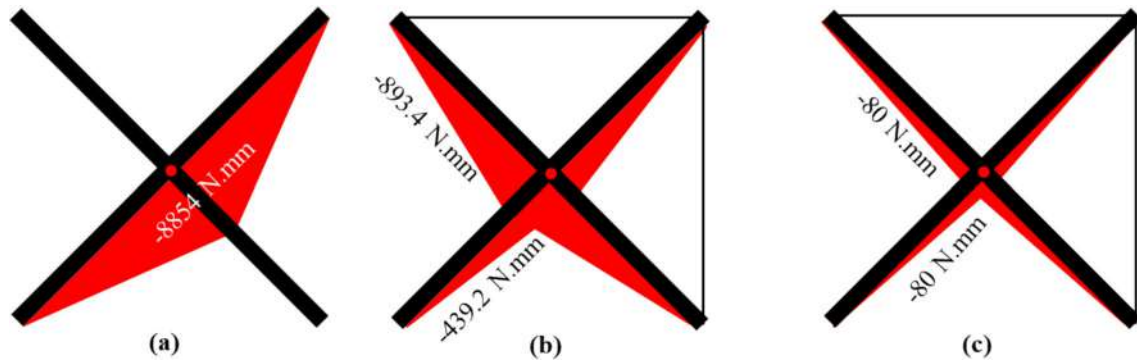


Fig. 11 Bending moment in $\text{N} \cdot \text{mm}$ **a** no cable, **b** two cables attached with no length change, **c** Cables 1 and 2 attached with length change using Eq. 6

Table 5 Sensitivity analysis of cable length change, the maximum bending moment, and deformation

Cases	e_{oc} (mm)		Total e_{oc}	Max moment	Max disp.
	Cable 1 (C1)	Cable 2 (C2)			
Optimum (C1, C2)	-0.1592	-0.1397	0.2989	-80	-0.031
C1-10%, C2	-0.1752	-0.1375	0.3127	-82.5	-0.032
C1, C2-10%	-0.1592	-0.1513	0.3105	-44	-0.02
C1-10%, C2-10%	-0.1752	-0.1513	0.3265	-44	-0.04
C1-10%, C2 + 10%	-0.1752	-0.12573	0.30093	-114	-0.04
C1 + 10%, C2-10%	-0.15768	-0.1513	0.30898	-48.5	-0.021
C1 + 10%, C2 + 10%	-0.15768	-0.12573	0.28341	-115.7	-0.043

Table 6 Structural analysis pre- and post-adjustment, and the percentage of the changes

	Pre-adjustment	Post-adjustment	Change %
Maximum moment (N.mm)	325	80	-75
Maximum displacement (mm)	-0.16	-	-
Maximum axial force in cables (N)	44.4	48.2	7.8

However, the option (C1 + 10%, C2-10%) satisfies the constraints and has a slightly higher objective function than the optimal solution.

3.2.2 Three-bay simply supported scissor structure

The structure shown in Fig. 8, with the same geometry, properties, and loading, has been tested to further reduce the bending moment from 325 N·mm to 80 N·mm. The cable length change resulted in a 90% and 66.8% reduction in maximum displacement, respectively, and a 10% increase in the cable axial force (see Table 6). In this test, it is assumed that all members and cables are included in the process, but only members with a length change greater than 0.03 mm are retained. The algorithm excluded all members and kept only the three cables. The targeted bending moment is achieved by shortening Cables 1 and 2 by 0.0535 mm and Cable 3 by 0.0397 mm. The results were verified using SAP2000 software. Figure 12 shows the bending moment diagram of the structure after applying the cable length change. It can be seen that the target has been obtained precisely. The optimal solution converged to the minimum possible total change in member length and satisfied the constraints. A sensitivity analysis was conducted for $\pm 10\%$ of Cables 1–3 (C1, C2, and C3) length changes and their effects on the maximum bending moment, displacement, and cable axial forces, as shown in Table 7. However, the efficiency of the proposed method depends on the structural configuration; using a combination of horizontal and vertical cables improves the effectiveness of adjusting member lengths to reduce bending moment.

Table 7 Sensitivity analysis of cable length variation and its impact on structural analysis parameters

	Opt (C1, C2, C3)	C1, - 10%C2, - 10%C3	C1, C2, - 10%C3	C1, - 10%C2, C3	- 10%C1, - 10%C2, - 10%C3	- 10%C1, C2, - C3 - 10%C3	- 10%C1, C2, - 10%C2, C3	C1, + 10%C2, - 10%C3	C1, C2, + 10%C3	C1, + 10%C2, C3	+ 0.1C1, + 10%C2, + 10%C3	. + 0.1C1, C2, - + 10%C3	+ 0.1C1, + 10%C2, C3
eoC1 (mm)	- 0.05	- 0.05	- 0.05	- 0.05	- 0.06	- 0.06	- 0.06	- 0.05	- 0.05	- 0.05	- 0.05	- 0.05	- 0.05
eoC2 (mm)	- 0.05	- 0.06	- 0.05	- 0.06	- 0.06	- 0.05	- 0.06	- 0.05	- 0.05	- 0.05	- 0.05	- 0.05	- 0.05
eoC3 (mm)	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04	- 0.04
Max Mom (N.mm)	80	77.8	85	80.4	70.0	77.7	68.5	105.0	90	94.5	104	105	95
Max disp (mm)	- 0.05	0.05	- 0.05	- 0.05	- 0.04	- 0.05	- 0.04	- 0.06	- 0.05	- 0.06	- 0.06	- 0.06	- 0.06
Max axial (N)	48	48	48	84.2	48.6	48.5	48.5	48	48.2	48.2	47.8	48.1	97.9
Sum ϵ_{ec} (mm)	0.15	0.16	0.15	0.15	0.16	0.16	0.16	0.14	0.14	0.14	0.13	0.14	0.14

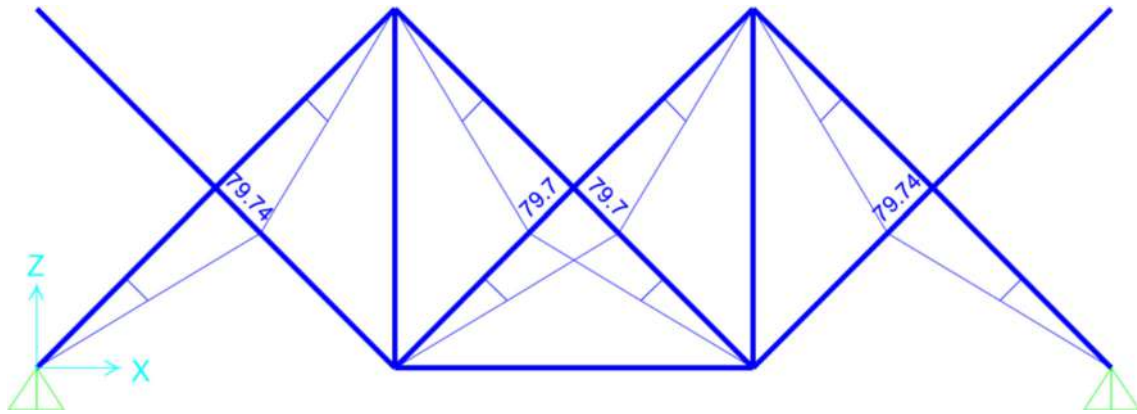


Fig. 12 Bending moment diagram of the simply supported scissor structure after applying the member length change

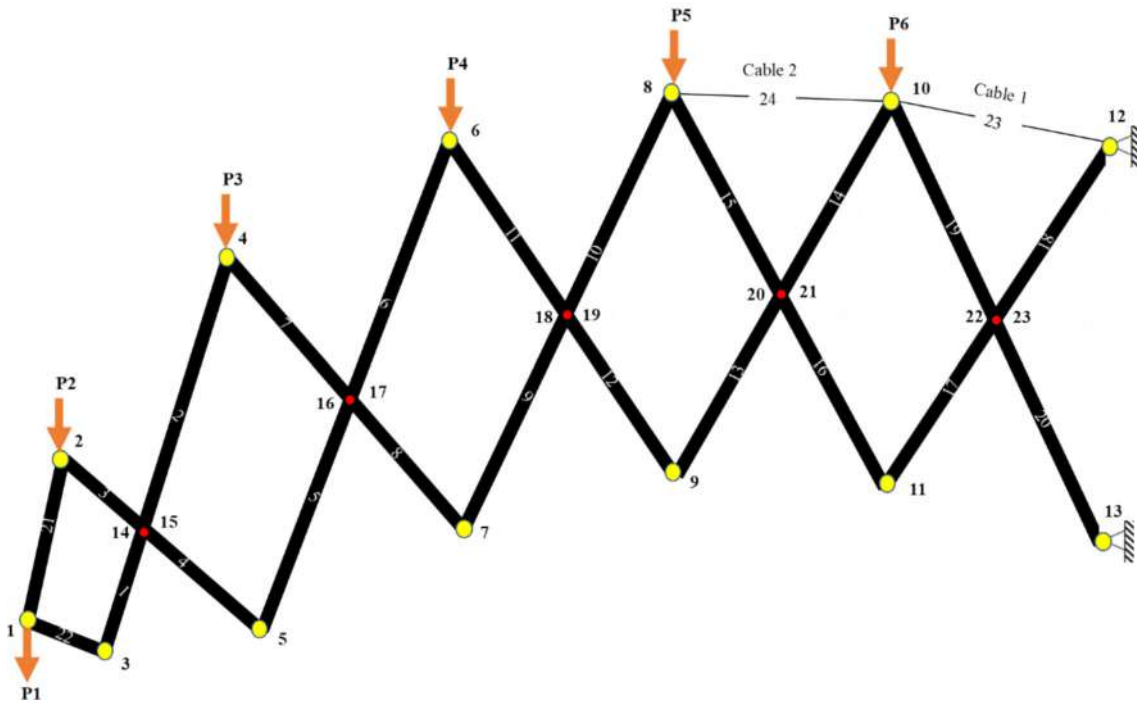


Fig. 13 Large-span deployable configuration with pinned supports at nodes 12 and 13

3.2.3 Large-span structure

Symmetric loading In this example, a 3927-mm-long structure, as indicated in Fig. 13 under the load $P1$ to $P6 = 3$ N, is examined. The nodal coordinates of the structure are presented in Table 8. The structure consists of five scissor assemblies (Members 1–20) connected together, two truss members (Members 21 and 22), and two cables (Members 23 and 24). The members are all constructed of steel with a modulus of elasticity of 200 GPa. The structural members have a rectangular cross section with a width of 10 mm and a height of 20 mm, whereas the cables have the same elastic modulus but a circular cross section with a diameter of 2 mm, its yield strength is 400 MPa. Nodes 12 and 13 are modeled as pinned supports, restrained in translation in both horizontal and vertical directions, while all other nodes remain free. These boundary conditions are consistently maintained in all loading scenarios presented in this example.

The large-span configuration examined in the above example represents an idealized, multi-bay deployable scissor assembly designed for use as an academic demonstrator. While it does not relate to a specific structure under construction, it represents the structural attributes of lightweight roofs, canopies, or aerofoil-type

Table 8 Nodal coordinates of the large-span deployable structure shown in Fig. 13

Joint	X	Y	Joint	X	Y
1	0	103.24	10	3127.6	744.14
2	127.55	299.16	11	3103.1	264.76
3	277.17	60.368	12	3927.6	689.31
4	727.55	549.98	13	3883.3	194.19
5	849.06	91.732	14,15	419.59	215.2
6	1527.6	699.65	16,17	1168.3	377.72
7	1582.8	215.7	18,19	1949.3	480.99
8	2327.6	754.81	20,21	2725.9	503.09
9	2346.2	275.17	22,23	3503.2	470.78

Table 9 Bending moment in macro-elements in N.mm pre- and post-adjustment

Macro-element	No cable		With the two cables			
			Pre-adjustment		Post-adjustment	
	Theory	SAP	Theory	SAP	Theory	SAP
1	- 310.73	- 308	- 312.12	- 310	- 312.12	310
2	- 1826	- 1830	- 1822.9	- 1825	- 1822.9	1824
3	- 1726.2	- 1711	- 1732.8	- 1725	- 1732.8	- 1724
4	- 6230	- 6253	- 6216	- 6224	- 6216	- 6225
5	- 3626.2	- 3600	- 3645.1	- 3628	- 3645.1	- 3628
6	- 12,635	- 12,689	- 12,598	- 12,615	- 12,598	- 12,617
7	- 6158.5	- 6100	- 6194	- 6173	- 6194	- 6171
8	- 20,632	- 20,722	- 14,556	- 14,559	- 15,187	- 15,190
9	- 9768.2	- 9696	- 15,831	- 15,829	- 15,200	- 15,199
10	- 29,820	- 29,938	- 1962.2	- 1958	- 2015	- 2011

structures in their deployed configurations. This example was selected to test the scalability of the proposed quasi-static adjustment method on a system with greater geometric complexity and higher degrees of indeterminacy than those in the single- or three-bay systems.

Before cable integration, the structure had a maximum bending moment of 29,820 N·mm and a maximum displacement of -37.28 mm. Following the attachment of the two cables, these values were significantly decreased to 15,831 N·mm and -18.18 mm, respectively, see Tables 9 and 10. Table 11 shows the percentage changes in the maximum bending moment and displacements before and after cable attachment, pre- and post-adjustment. The objective is to constrain the bending moment on every macro-element within 15,200/ N·mm. As stipulated by Eq. 6, this objective is attained through the lengthening of Cable 2 (Member 24) by 0.07808 mm.

One can see from Table 9 that none of the macro-elements exceed the limit after changing the length of a single cable. It is critical to ensure that both cables remain under tension throughout the adjustment process, both before and after length modification. The MATLAB simulations show a strong correlation with those from the nonlinear analyses conducted in SAP2000, thereby confirming the computational strategy. The bending moment of structures with a higher degree of self-stress can be reduced more than that of structures with a lower degree of self-stress.

Figure 14 shows the sensitivity analysis and the effect of changes in the lengths of Members 23 & 24 (Cables 1 & 2) on the bending moment in the macro-elements. The optimization algorithm identified Member 24 (Cable 2) as the most effective member. It can be seen that changing the cable length significantly affects Macro-elements 8 and 9. The figure also shows that the algorithm found the optimal solution satisfying the constraints: lengthen Cable 2 by just under 0.078 mm. Figure 14 (B) shows that changing the length of Member 23 (Cable 1) by 0.09 mm can achieve the maximum bending moment of 15,221 N · mm, which is larger than 15,200 N · mm. If both cables are involved with the same amount shown in Fig. 14 (C), no significant reduction will occur with the macro-element with high bending moment (Macro-element 9). When both cables' lengths change by the same amount, but one lengthens, and the other shortens (see Fig. 14 (D)). Shortening Member 23 and lengthening Member 24 by 0.04 each (0.08 in total) achieve a maximum bending moment of 15,212 N.mm (Table 11).

Table 10 Nodal displacement of the first free eleven nodes in mm, pre- and post-adjustment

Joints	Directions	No cable		With the two cables			
				Pre-adjustment		Post-adjustment	
		Theory	SAP	Theory	SAP	Theory	SAP
1	X	− 3.94	− 4.05	2.93	2.93	2.69	2.68
	Y	− 37.29	− 37.46	− 18.18	− 18.21	− 18.38	− 18.41
2	X	− 6.01	− 6.09	1.62	1.61	1.37	1.37
	Y	− 35.95	− 36.12	− 17.33	− 17.36	− 17.53	− 18.56
3	X	− 3.08	− 3.15	3.29	3.29	3.06	3.06
	Y	− 31.33	− 31.46	− 15.77	− 15.79	− 15.89	− 15.91
4	X	− 7.23	− 7.31	0.22	0.21	0.00	0.00
	Y	− 27.55	− 27.67	− 12.95	− 12.97	− 13.09	− 13.11
5	X	− 2.07	− 2.13	3.30	3.30	3.10	3.10
	Y	− 21.71	− 21.78	− 11.40	− 11.42	− 11.42	− 11.43
6	X	− 6.34	− 6.39	− 0.27	− 0.27	− 0.42	− 0.42
	Y	− 16.98	− 17.05	− 7.45	− 7.46	− 7.51	− 7.52
7	X	− 1.45	− 1.49	2.59	2.60	2.46	2.45
	Y	− 12.37	− 12.39	− 6.80	− 6.81	− 6.74	− 6.75
8	X	− 4.27	− 4.30	− 0.19	− 0.19	− 0.27	− 0.27
	Y	− 8.50	− 8.54	− 2.98	− 2.98	− 3.00	− 3.00
9	X	− 0.35	− 0.36	1.82	1.82	1.75	1.75
	Y	− 5.20	− 5.20	− 3.39	− 3.40	− 3.29	− 3.30
10	X	− 1.90	− 1.91	− 0.12	− 0.12	− 0.13	− 0.13
	Y	− 2.63	− 2.64	− 0.17	− 0.17	− 0.18	− 1.20
11	X	0.47	0.47	0.76	0.76	0.73	0.73
	Y	− 0.91	− 0.90	− 1.47	− 1.47	− 1.41	− 1.41

Table 11 The maximum bending moment, displacement, and cable forces before and after attaching cables and member length changes

	Before cable placement	After cables placement				Total member length change (mm)
		Pre-adjustment	Change %	Post-adjustment	Change %	
Maximum moment (N.mm)	− 29,820	− 15,831	− 46.9	− 15,200	− 3.9	0.078
Maximum displacement (mm)	− 37.29	− 18.18	− 51.3	− 18.38	1.1	
Maximum axial force in cables (N)	0	88	—	90.1	2.4	

Asymmetric loading

A. Two adjacent loading

Three different asymmetric load scenarios were tested to validate the proposed method. In each case, the two top adjacent joints were loaded with the magnitude shown in Table 12. The table illustrates the reduction in the bending moment achieved by changing only the length of Cable 2 (Member 24) across the three loading cases. However, although the bending moment decreased, the maximum nodal displacement increased. It is noteworthy that attaching cables reduces both bending moment and displacement, whereas the proposed formulation focuses solely on bending moment mitigation, disregarding displacement. Regarding the axial force in the cables, they experience a significant force during initial attachment, with only a slight increase after the adjustment. The cable yield stress is 400 MPa, but the maximum demand is 60 MPa, which is 15% of the yield stress.

In addition, sensitivity analyses for $\pm 10\%$ changes in Cable 2 (Member 24) length and their effects on the maximum bending moment for Case 1 are shown in

Table 12 Maximum internal forces and nodal displacements before and after applying the member length change for three various asymmetric load cases

Cases	Loaded joints	Loads * - 1 (N)	Max Moment (N.mm)	Max disp. (mm)			Max cable axial force (N)			Total length change (mm)	
				Pre-adjustment	Post-adjustment	Change %	Pre-adjustment	Post-adjustment	Change %		
1	1, 2	4,2, 6,5	- 15,606	- 15,200	- 2,6	- 0,33	- 0,20	78,1	79,6	1,92	0,05
2	4, 6	7, 10	- 15,872	- 15,200	- 4,2	- 0,16	- 0,38	86,5	89,1	3,00	0,082
3	8, 10	40, 50	- 16,680	- 15,200	- 8,8	- 11,9	- 12,4	182,4	189,8	4,02	0,18

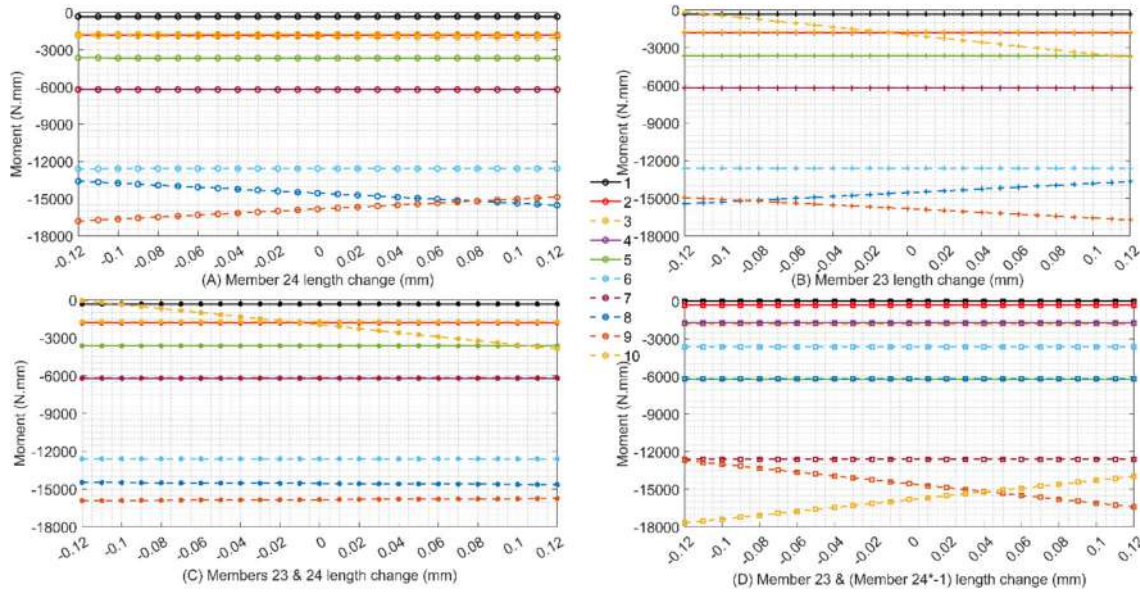


Fig. 14 Sensitivity analysis of Members 23 and 24 length change and their response to the bending moment in all macro-elements

Table 13 Sensitivity analysis of Cable 2 length change

e_{oc} (mm)	0.045	0.050	0.055
Maximum bending moment (N.mm)	− 15,246	− 15,200	− 15,165

Table 14 Maximum internal forces and nodal displacements before and after applying the member length change for the asymmetric loading case scenario

	Max moment (N.mm)			Max disp (mm)			Change %	Max cable force (N)		Change %	Total member length change (mm)
	Pre-adjustment	Post-adjustment	Change %	Pre-adjustment	Post-adjustment	Change %		Pre-adjustment	Post-adjustment		
Case 1	− 17,568	− 16,898	− 3.81	− 20.58	− 19.9	− 3.28	95.25	103.58	8.74	0.090	
Case 2	− 15,811	− 15,208	− 3.81	− 18.52	− 17.91	− 3.29	85.28	93.21	9.29	0.0815	

Table 13. It can be seen that increasing the length change by 10% results in a greater reduction in bending moment, but since the algorithm stops when the constraint functions meet, the objective function is at a minimum.

B. Asymmetric load across all top nodes

In this load scenario, two load cases were considered. First, the total loading applied in the symmetric load scenario has been divided to 60% to the left half and 40% to the right half of the structure shown in Fig. 13. In other words, joints 1, 2, and 4 were loaded with 3.6 N downward, while joints 6, 8, and 10 were loaded with 2.4 N downward. The maximum bending moments, displacements, and axial forces induced due to the load are reported in Table 14. It was planned to keep the bending moment within /15200/ N.mm in all macro-elements, similar to the other cases reported above. It was found out that the maximum bending moment can be reduced from − 17,568 N · mm to − 16,898 by shortening Cable 1 (Member 23) by 0.0907 mm. However, the goal is not achievable; the maximum bending moment is reduced by 3.81%. The table reports 3.28% reduction in the maximum displacement and 8.74% increase in the maximum axial force in cables after applying the member length change.

For the second case, the loads decreased by 10%, i.e., joints 1, 2, and 4 loaded with -3.24 N downward and joints 6, 8, and 10 loaded with 2.16 N downward. The maximum bending moment is -15,811 N.mm, and it was planned to keep the bending moment within /15200/ N.mm. The goal is almost achieved, Cable 1 (Member 23) needed to be shortened by 0.0815 mm, and thus, the maximum bending moment drops to -15,208

Table 15 Sensitivity analysis of Cable 1 length change

e_{oc} (mm)	– 0.0816	– 0.0907	– 0.0998
Maximum bending moment (N.mm)	– 16,965	– 16,898	– 16,965

N.mm. Table 14 shows that the percentage reductions in maximum bending moment and displacement, pre- and post-adjustment, are almost the same for both cases. It can be concluded that the efficiency of the proposed method depends on the geometry of the structure, the level of indeterminacy, and the level of bending demand. Under conditions of highly concentrated bending or when the freedom of degrees of adjustments is limited, the given task may not be fully attainable within the limits of the adjustments. Such a result simply indicates the controllability of the system.

Sensitivity analyses for case 1 for $\pm 10\%$ changes in Cable 1 (Member 23) length and their effects on the maximum bending moment for the asymmetric loading are presented in Table 15. The table shows that increasing or decreasing the length by 10% results in a 0.39% increase in the maximum bending moment. It indicates that the member length change found by the algorithm is the closest point to the optimal solution; however, the constraints were not met.

4 Conclusions

Loaded scissor structures experience significant moments at the shear connectors and substantial bending deformation. This paper presents two effective solutions to mitigate deformation and bending moment through quasi-static adjustment in scissor structures.

- The study shows how to mitigate deformation and bending moments by adding cables in tension zones.
- The cable should be positioned at the end opposite to the joint's displacement direction along the element's axis. This placement allows the cable to engage in tension as the joint moves, thereby resisting the resulting bending moment.
- The results showed a 90% reduction in deformation and bending moment by attaching cables.
- Similar significant reductions in the amount of bending moment are possible by making adjustments in the length of the members, to the extent of up to 90%. However, the amount of possible reduction varies across cases, as in some cases only marginal reductions, i.e., 4%, can be achieved. This depends on the amount of bending to be reduced, the structure's geometry, the type of load applied, and the system's degree of static indeterminacy.
- The study presents a new formulation to mitigate the bending moment within a desired limit by changing the length of some members.
- Bending moment is sensitive to member length change, i.e., a slight change in member length causes a significant change in bending moment.
- The bending moment can be further reduced by modifying the lengths of the attached cables. In some instances, this adjustment can reduce by up to 90% compared to configurations where the cables are present, but their lengths remain unaltered.
- The proposed techniques are applied to small- to large-scale scissor structures.
- The bending moment of structures with a higher degree of self-stress states can be reduced more than those with fewer self-stress states. In addition, attaching cables horizontally and vertically simultaneously will make the member length change more effective in reducing the bending moment.

5 Future work recommendations

Different aspects related to bending moment mitigation and optimization in scissor structures can be further investigated in future studies, as highlighted below:

- A promising extension of the proposed method would be its application to an active or real-time system through the integration of smart actuators. This would enable adaptive control of member length variations, which is crucial, as even small changes can substantially affect bending moments in the scissor system.
- Future research could explore the optimization of material properties and member sizes, particularly those of the attached cables, to achieve further improvements in the overall structural performance.

- An interesting direction for future research is the optimization of compressed members located at the bottom of the cantilever structure, combined with tensioned cables at the top. This strategy has the potential to significantly reduce bending moments both before and after applying member length adjustments. However, further investigation is needed to address the limitation that struts can only be introduced once the structure is fully deployed, whereas cables can be integrated without affecting deployability.
- This paper presents an engineering approach that utilizes cables and member length adjustments to mitigate bending moments. While actuator-based optimization methods offer globally optimal solutions, the proposed method provides a simpler, more cost-effective alternative. Future research will involve a comparative analysis of these methods, focusing on their effectiveness, computational efficiency, and practical applicability.
- Future studies are encouraged to incorporate the Extended Integrated Force Method (EIFM), given its unified formulation and systematic treatment of compatibility and prestress.

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Declarations

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Appendix A: Tables

See Tables A1, A2, A3, and A4.

Table A1 Theoretical results and those obtained from nonlinear analysis in SAP2000 software (NL-SAP) for the one-bay scissor structure

Macro-elements	Cases							
	No cable (a)		Cable1 (b)		Cable2 (c)		Cables1&2 (d)	
	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP
1	− 2241	− 2232	− 1152	− 1155	− 2241	− 2232	− 221	− 221
2	0	0	− 1089	− 1095	0	0	− 109	− 109

Table A2 Theoretical and NL-SAP results of the nodal displacements in (mm) for the one-bay scissor structure

Joints	Cases							
	No cable (a)		Cable1 (b)		Cable2 (c)		Cables1&2 (d)	
	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP
3x	0.00	0.00	0.35	0.35	0.00	0.00	0.03	0.03
3y	0.00	0.00	− 0.36	− 0.36	0.00	0.00	− 0.04	− 0.38
4x	− 0.73	− 0.73	− 0.37	− 0.37	− 0.73	− 0.73	− 0.07	− 0.07
4y	− 0.73	− 0.72	− 0.37	− 0.37	− 0.73	− 0.72	− 0.07	− 0.07

Table A3 Theoretical and NL-SAP results of bending moment in (N.mm) for three-bay scissor structure

Macro-elements	Cases													
	No Cable (a)		Cable1 (b)		Cable2 (c)		Cables1&2 (d)		Cables1&3 (e)		Cables1&4 (f)		Cables1,2&3 (g)	
	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP	Theory	SAP
1	- 6622	- 6500	- 5586	- 5540	- 6622	- 6500	- 4557	- 4516	- 6666	- 6627	- 5789	- 5745	- 4644	- 4613
2	- 4546	- 4592	- 5621	- 5625	- 243	- 244	- 364	- 357	- 4528	- 4512	- 5416	- 5415	- 354	- 351
3	- 4400	- 4303	- 3359	- 3336	- 4400	- 4303	- 2326	- 2303	- 2293	- 2257	- 3563	- 3542	- 2264	- 2237
4	- 2277	- 2305	- 3349	- 3330	- 2260	- 2270	- 4387	- 4372	- 2260	- 2226	- 3144	- 3121	- 4299	- 4273
5	- 2195	- 2143	- 1146	- 1159	- 2195	- 2143	- 104	- 109	- 80	- 80	- 179	- 178	- 43	- 44
6	0	0	- 1063	- 1008	0	0	- 2117	- 2084	- 2133	- 2096	- 859	- 803	- 2179	- 2150

Table A4 Theoretical and NL–SAP of nodal displacements in (mm) for three-bay scissor structure

Joints	Cases																			
	No Cable (a)																			
	Theory		SAP		Theory		SAP		Theory		SAP		Theory		SAP		Theory		SAP	
3x	1.46	1.48	1.81	1.81	0.08	0.08	0.11	0.11	1.46	1.44	1.74	1.74	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
3y	-1.49	-1.51	-1.84	-1.85	-0.08	-0.08	-0.12	-0.12	-1.48	-1.48	-1.78	-1.78	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
4x	-2.17	-2.13	-1.83	-1.81	-2.18	-2.14	-1.49	-1.48	-2.19	-2.17	-1.88	-1.88	-1.52	-1.52	-1.52	-1.52	-1.52	-1.52	-1.52	-1.52
4y	-2.13	-2.08	-1.80	-1.78	-2.13	-2.09	-1.47	-1.46	-2.14	-2.12	-1.86	-1.85	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.49
5x	1.48	1.56	2.84	2.86	-1.29	-1.25	0.15	0.16	1.44	1.44	2.58	2.59	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
5y	-5.86	-5.86	-6.60	-6.58	-3.02	-2.99	-3.14	-3.12	-5.85	-5.82	-6.46	-6.44	-3.13	-3.13	-3.13	-3.13	-3.13	-3.13	-3.12	-3.12
6x	-4.36	-4.22	-2.97	-2.93	-5.75	-5.63	-3.64	-3.61	-3.69	-3.65	-3.24	-3.20	-3.68	-3.68	-3.65	-3.65	-3.65	-3.65	-3.65	-3.65
6y	-7.12	-7.02	-6.50	-6.46	-5.71	-5.61	-3.79	-3.76	-6.48	-6.44	-6.62	-6.58	-3.83	-3.83	-3.80	-3.80	-3.80	-3.80	-3.80	-3.80
7x	0.02	0.20	3.09	3.11	-4.13	-4.01	0.09	0.10	1.34	1.34	2.49	2.51	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
7y	-11.60	-11.56	-12.77	-12.71	-7.34	-7.25	-7.61	-7.55	-11.66	-11.57	-12.55	-12.48	-7.59	-7.59	-7.55	-7.55	-7.55	-7.55	-7.55	-7.55
8x	-6.56	-6.30	-3.45	-3.38	-9.33	-9.12	-4.41	-4.36	-4.49	-4.44	-3.66	-3.59	-4.47	-4.47	-4.43	-4.43	-4.43	-4.43	-4.43	-4.43
8y	-13.55	-13.39	-12.67	-12.61	-10.72	-10.53	-7.58	-7.53	-11.60	-11.52	-12.48	-12.42	-7.58	-7.58	-7.53	-7.53	-7.53	-7.53	-7.53	-7.53

$$\begin{array}{ccc}
 \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{1cm}}^n & \overbrace{\hspace{1cm}}^p \\ \left[\begin{array}{cc} \mathbf{A}_{mn} & \mathbf{A}_{mp} \\ \mathbf{A}_{pn} & \mathbf{A}_{pp} \end{array} \right] \begin{bmatrix} \mathbf{t}_n \\ \mathbf{t}_p \end{bmatrix} = \begin{bmatrix} \mathbf{p}_m \\ \mathbf{p}_p=0 \end{bmatrix} \end{array} \\ (a) \end{array} &
 \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{1cm}}^m & \overbrace{\hspace{1cm}}^p \\ \left[\begin{array}{cc} \mathbf{B}_{nm} & \mathbf{B}_{np} \\ \mathbf{B}_{pm} & \mathbf{B}_{pp} \end{array} \right] \begin{bmatrix} \mathbf{d}_m \\ \mathbf{d}_p \end{bmatrix} = \begin{bmatrix} \mathbf{e}_n \\ \mathbf{e}_p \end{bmatrix} \end{array} \\ (b) \end{array} &
 \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{1cm}}^n & \overbrace{\hspace{1cm}}^p \\ \left[\begin{array}{cc} \mathbf{F}_{nn} & \mathbf{F}_{np} \\ \mathbf{F}_{pn} & \mathbf{F}_{pp} \end{array} \right] \begin{bmatrix} \mathbf{t}_n \\ \mathbf{t}_p \end{bmatrix} = \begin{bmatrix} \mathbf{e}_n \\ \mathbf{e}_p \end{bmatrix} \end{array} \\ (c) \end{array}
 \end{array}$$

Fig. B1 Different arrangements of (a) equilibrium, (b) compatibility, and (c) flexibility matrices [13]

Appendix B: Formulations

For an i -dimensional structural assembly, b bars and j joints with c constraint joints. $\mathbf{A}$ is a $(ij-c) \times b$ matrix

$$\mathbf{A}\mathbf{t} = \mathbf{P} \quad (\text{B } 1)$$

$\mathbf{A}$ is the matrix of equilibrium, $\mathbf{t}$ is the internal forces, and $\mathbf{p}$ is the applied force. $\mathbf{B}$, which is the compatibility matrix and is the transpose of $\mathbf{A}$, is expressed as follows, and has the size of $b \times (ij-c)$

$$\mathbf{B}\mathbf{d} = \mathbf{e} \quad (\text{B } 2)$$

$\mathbf{d}$ is the vector of displacements of the free joints ($ij-c$), and $\mathbf{e}$ is a vector of b daemons, which is the member length change. The flexibility matrix that relates internal forces and internal deformation and has $b \times b$ dimensions is expressed as follows

$$\mathbf{F}\mathbf{t} = \mathbf{e} \quad (\text{B } 3)$$

Pellegrino et al. [13] partitioned the main matrices of the Force method as presented in Fig. B1. The condensation has been made so that the shear connectors that joined two macro-elements cannot be loaded; in practice, only the pin joints are loadable.

The part of $\mathbf{A}$ related to zero load components can be simplified by excluding them from immediate consideration. Similarly, the corresponding displacement components and compatibility equations can also be simplified, leaving a smaller system of equations. When macro-elements are built from elementary elements, the unloaded “internal” joints within these macro-elements generate equations for condensation. Consider vectors of external joint load $\mathbf{p}$ and displacement $\mathbf{d}$ with $(m+p)$ dimensions, and vectors of internal bar force $\mathbf{t}$ and bar elongation $\mathbf{e}$ with $(n+p)$ dimensions. The load vector $\mathbf{p}$ is divided into two sub-vectors, $\mathbf{p}_m$ with m nonzero components and $\mathbf{p}_p$ with p zero components. Likewise, the displacement vector $\mathbf{d}$ can be partitioned into $\mathbf{d}_m$ (nodal displacements of the joints other than shear connectors) and $\mathbf{d}_p$ (the displacements of the shear connectors) sub-vectors, where $\mathbf{d}_m$ and $\mathbf{d}_p$ correspond directly to $\mathbf{p}_m$ (possible applied loads at joints) and $\mathbf{p}_p$ (zero applied loads at shear connectors), respectively.

$$\mathbf{A}_{mn}\mathbf{t}_n + \mathbf{A}_{mp}\mathbf{t}_p = \mathbf{p}_m \quad (\text{B } 4)$$

And

$$\mathbf{A}_{pn}\mathbf{t}_n + \mathbf{A}_{pp}\mathbf{t}_p = \mathbf{0} \quad (\text{B } 5)$$

From the above expression, $\mathbf{t}_p$ can be obtained as $\mathbf{t}_p = -\mathbf{A}_{pp}^{-1}\mathbf{A}_{pn}\mathbf{t}_n$ and substituted in (B 4) to get the reduced equilibrium matrix $\mathbf{A}_{mn}\mathbf{t}_n - \mathbf{A}_{mp}\mathbf{A}_{pp}^{-1}\mathbf{A}_{pn}\mathbf{t}_n = \mathbf{p}_m$. The expression can also be written as $(\mathbf{A}_{mn} - \mathbf{A}_{mp}\mathbf{A}_{pp}^{-1}\mathbf{A}_{pn})\mathbf{t}_n = \mathbf{p}_m$ and then as follows.

$$\mathbf{A}^* = \mathbf{A}_{mn} - \mathbf{A}_{mp}\mathbf{A}_{pp}^{-1}\mathbf{A}_{pn} \quad (\text{B } 6)$$

The condensed equilibrium matrix is expressed as

$$\mathbf{A}^*\mathbf{t}^* = \mathbf{p}^* \quad (\text{B } 7)$$

$\mathbf{t}^*$ and $\mathbf{p}^*$ are axial forces ($\mathbf{t}_n$) and the non-trivial applied loads ($\mathbf{p}_m$), respectively. The reduced compatibility matrix ($\mathbf{B}^*$), which is the transpose of ($\mathbf{A}^*$), i.e., $\mathbf{B}^* = \mathbf{A}_{mn}^T - \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{A}_{mp}^T$, is written as follows

$$\mathbf{B}^* \mathbf{d}^* = \mathbf{e}^* \quad (\text{B } 8)$$

The reduced nodal displacements of the pin joints ($\mathbf{d}^*$) are the same as ($\mathbf{d}_m$), whereas $\mathbf{e}^*$ is a vector of the condensed member length change that can be written as follows.

$$\mathbf{e}^* = \mathbf{e}_n - \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{e}_p \quad (\text{B } 9)$$

From Fig. B1 $\mathbf{e}_n$ and $\mathbf{e}_p$ are presented as follows

$$\mathbf{e}_n = (\mathbf{F}_{nn} - \mathbf{F}_{np} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn}) \mathbf{t}_n \quad (\text{B } 10)$$

$$\mathbf{e}_p = (\mathbf{F}_{pn} - \mathbf{F}_{pp} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn}) \mathbf{t}_n \quad (\text{B } 11)$$

The condensed flexibility matrix is

$$\mathbf{e}^* = \mathbf{F}^* \mathbf{t}^* \quad (\text{B } 12)$$

where $\mathbf{F}^*$ is determined from

$$\mathbf{F}^* = \mathbf{F}_{nn} - \mathbf{F}_{np} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn} - \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{F}_{pn} + \mathbf{A}_{pn}^T (\mathbf{A}_{pp}^T)^{-1} \mathbf{F}_{pp} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn} \quad (\text{B } 13)$$

We have just condensed the system's main matrices to analyze scissor structures; axial forces, bending moments, and nodal displacements must be determined. $\mathbf{t}^*$ has a particular solution that satisfies equilibrium ($\mathbf{A}^* \mathbf{t}^* = \mathbf{p}^*$) but may not satisfy compatibility. This vector is $\mathbf{t}_{nH}$, which is calculated from $\mathbf{t}_{nH} = (\mathbf{A}^*)^+ \mathbf{p}_m$, ($\mathbf{A}^*)^+$ is the pseudo-inverse of the condensed equilibrium matrix. $\mathbf{t}^*$ also has a complementary homogeneous solution that satisfies with $\mathbf{A}^* \mathbf{t}_n = 0$; this is the axial force that equilibrates with no external loads. This is obtained by $\mathbf{S}^* \alpha$, where $\mathbf{S}^*$ is the condensed states of the self-stress matrix calculated from null space ($\mathbf{A}^*$). Moreover, α is a vector consisting of s^* coefficients representing a series of combinatorial constants, which must be established by ensuring compatibility. The total generalized solution is the sum of the particular and homogeneous solutions.

$$\mathbf{t}_n = \mathbf{t}_{nH} + \mathbf{S}^* \alpha^* \quad (\text{B } 14)$$

From (B 5), the moment in shear connectors can be determined as follows

$$\mathbf{t}_p = -(\mathbf{A}_{pp})^{-1} \mathbf{A}_{pn} \mathbf{t}_n \quad (\text{B } 15)$$

This study focuses on the effect of changes in member length on internal forces. In general, strain comes from axial forces $\mathbf{F}^* \mathbf{t}_n$ and imposed member length change ($\mathbf{e}_o^*$), after subbing (B 14) member length change is obtained from

$$\mathbf{e}^* = \mathbf{e}_o^* + \mathbf{F}^* (\mathbf{t}_{nH} + \mathbf{S}^* \alpha^*) \quad (\text{B } 16)$$

In the same fashion, (B 10) and (B 11) can be rewritten as follows, with the effect of imposed member length change.

$$\mathbf{e}_n = \mathbf{e}_{no} + (\mathbf{F}_{nn} - \mathbf{F}_{np} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn}) \mathbf{t}_n \quad (\text{B } 17)$$

$$\mathbf{e}_p = \mathbf{e}_{po} + (\mathbf{F}_{pn} - \mathbf{F}_{pp} \mathbf{A}_{pp}^{-1} \mathbf{A}_{pn}) \mathbf{t}_n \quad (\text{B } 18)$$

To determine axial forces ($\mathbf{t}_n$), then bending moment ($\mathbf{t}_p$), changing member length ($\mathbf{e}^*$), and nodal displacements of the pin-jointed nodes ($\mathbf{d}_m$), it is essential to obtain α^* . Enforcing the compatibility condition requires ensuring that the $\mathbf{e}^*$ is orthogonal to the left-null space ($\mathbf{B}^*$). Noting that the condensed equilibrium and compatibility matrices are transposed to each other, the left-null space ($\mathbf{B}^*$) corresponds to the null space ($\mathbf{A}^*$), which represents the states of self-stress ($\mathbf{S}^*$). The compatibility condition can be succinctly described

as requiring the orthogonality of the states of self-stress to the member length changes. Mathematically, this condition is expressed as $(\mathbf{S}^*)^T \mathbf{e}^* = 0$, thus

$$\mathbf{S}^{*T} \mathbf{e}_o^* + \mathbf{S}^{*T} \mathbf{F}^* (\mathbf{t}_{nH} + \mathbf{S}^* \alpha^*) = 0 \quad (\text{B } 19)$$

now, α can be determined.

$$-\alpha^* = \left(\mathbf{S}^{*T} \mathbf{F}^* \mathbf{S}^* \right)^{-1} \left[\mathbf{S}^{*T} \mathbf{e}_o^* + \mathbf{S}^{*T} \mathbf{F}^* \mathbf{t}_{nH} \right] \quad (\text{B } 20)$$

subbing α , $\mathbf{t}_n$, then $\mathbf{t}_p^*$, and $\mathbf{d}_m$, can be determined.

Now axial forces ($\mathbf{t}_n$) and bending moment ($\mathbf{bm}$) can be found

$$\mathbf{t}_n = \mathbf{t}_{nH} - \mathbf{S}^* \left(\mathbf{S}^{*T} \mathbf{F}^* \mathbf{S}^* \right)^{-1} \mathbf{S}^{*T} \left(\mathbf{e}_{no}^* - \left(\mathbf{A}_{pn}^T \left(\mathbf{A}_{pp}^T \right)^{-1} \mathbf{e}_{po}^* + \mathbf{F}^* \mathbf{t}_{nH} \right) \right) \quad (\text{B } 21)$$

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Paper VI

Paper VI: Manguri, A., D. Magisano, and R. Jankowski, *Optimal shape and stress control of geometrically nonlinear structures with exact gradient with respect to the actuation inputs*. Structures, 2024. **70:** p. 107738. <https://doi.org/10.1016/j.istruc.2024.107738>



Optimal shape and stress control of geometrically nonlinear structures with exact gradient with respect to the actuation inputs

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ABSTRACT

This paper presents an efficient and robust optimization methodology for stress and shape control of actuated geometrically nonlinear elastic structures, applied to 3D trusses. The actuation inputs, modeled as prescribed strains, serve as the optimization variables. The objective is to minimize total actuation while satisfying several constraints: (i) actuation bounds in each actuated element and (ii) target ranges for nodal displacements and element stresses. Optimizing large nonlinear structures is computationally intensive. While gradient-based methods typically converge faster than gradient-free ones, their main bottleneck lies in numerical gradient evaluation, requiring multiple time-consuming nonlinear structural analyses (finite differences) with inaccuracies that may slow down convergence. The novelty of the proposal is an implicit differentiation approach to quickly compute the exact gradient of the nonlinear finite element solution with respect to the actuation inputs. This is implemented within the structural solver and leverages the already factorized tangent stiffness matrix to make the gradient cost negligible. As a result, the number of structural analyses and overall optimization time are significantly reduced.

1. Introduction

Geometrically nonlinear structures are characterized by significant deformations, and the deformed geometry considerably influences their stiffness and strength. They are becoming increasingly common in various engineering applications, including aerospace, civil, naval, and mechanical engineering.

Structures, particularly those supporting sensitive equipment for communication and scientific purposes, often undergo large deformations when subjected to unexpected loading or harsh environments. In these cases, shape control becomes paramount [1,2]. Active shape control is essential for flexible space structures, such as antenna reflectors and solar sails, to meet specific mission objectives, including shape accuracy and adaptability to different operational conditions [3]. On the other hand, structures like cable bridges and tensegrities require heightened emphasis on regulating internal forces within their components, rather than focusing solely on deformations. This is particularly evident in configurations featuring cable elements, which may experience slackness under specific loads, necessitating adjustments to maintain their structural integrity [4]. In real-world applications, manipulating a single variable without concomitant effects on other variables poses a considerable challenge. For instance, when restoring

an antenna to a predetermined shape by adjusting the lengths of specific members, caution must be exercised to prevent the generation of hazardous levels of strut forces or the slackening of cables due to diminished internal force. Achieving this delicate balance requires controlling nodal displacements and bar forces, a task that is often inherently intricate [5]. Shape and stress control are achieved by changing the length of some active bars using a device called an actuator [6]. Various types of actuators, such as mechanical [7], shape memory alloy [8], and piezoelectric actuators [9–11], are employed.

The idea of structural control was presented on a flexible beam [12] and a large space antenna [13], which then further developed analytically [14]. In a comprehensive review [15], the control of structural shape and stress and the optimization of various actuators were extensively discussed, covering key advancements and applications in the field. The shape and stress control of geometrically nonlinear structures pose a challenging problem due to the complex nonlinear relationships between geometry, loading, and deformation. Traditional design and analysis methods based on linear elasticity are often inadequate. In recent years, there has been a growing interest in developing new methods for the shape and stress control of flexible structures. Some of these methods are based on linear methods but are not valid for

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large displacements. In an experimental study, the displacement of a specific nod was controlled [16]. Then the study was extended to simultaneously control nodal displacement and internal bar forces of linear truss structures [17,18]. A study compared two equivalent methods, the integrated force method (IFM) and the singular value decomposition approach (SVD-FM) [19]. On the other hand, techniques based on nonlinear methods [20,21] may require significant computational time and may be affected by a lack of robustness. Another limitation of previous studies on shape and stress control is the requirement for a significant amount of actuation which this study also aims to address.

Optimization is defined as the process of finding the best possible solution for a given problem [22], opening a new chapter in structural engineering [23]. In structural engineering, optimization is implemented for one or more purposes, such as minimizing material usage and overall cost [24], or improving structural resistance [25]. Additionally, structural optimization has been implemented for topology optimization, considering thermal effects and initial imperfections [26] and maintaining the structural integrity and performance [27]. A constructability-based optimization method for steel trusses is proposed. Using a penalty approach, complexity is minimized, and the Howe truss is identified as the optimal solution for a real 3D roof structure [28]. Optimization algorithms can minimize the number of actuators and actuation in adaptive structures [29,30]. Adaptive structures can achieve significant whole-life energy savings by using controlled shape changes to redistribute stress, optimize material usage, and reduce embodied energy, as demonstrated through experimental testing of a small-scale planar truss prototype by actuating active actuators [31].

Structural geometry and properties can be changed in real-time as an actuation system to maintain the desired shape and stress distribution. For geometrically linear and nonlinear trusses, a weight optimization method based on the gradient and two sub-problems is proposed in [32] by reformulating the optimization problem in displacement variables. Gradient-based optimizations of shells prone to buckling are also developed in [33,34], exploiting a reduced order model of the structure to reduce the computational time. The computation of the gradients, generally possible only by numerical differentiation, is the bottleneck in nonlinear design optimization, as addressed in [35] exploiting parallel computing. Alternative metaheuristic approaches based, for example, on the genetic algorithm and the mine blast algorithm are also available [36,37]. The algorithm was used to optimize the number, spatial arrangement, and sizing of steel exoskeletons for seismic retrofitting by minimizing weight and ensuring structural elasticity. Performance was evaluated through finite-element analysis and case studies, demonstrating significant reductions in weight and cost [38]. A novel inverse method utilizes metaheuristic algorithms to optimize strain data from distributed fiber optic sensors [39]. This method helps in determining real strain distributions and solving interfacial mechanics, which in turn improves the understanding of strain transfer, debonding behavior, and structural health monitoring [40]. The greater flexibility of gradient-free methods in global optimization compared to gradient-based ones is, however, usually counterbalanced by the greater number of function evaluations in the former, especially when a closed-form gradient is available. Moreover, gradient-free methods scale very poorly with the dimensionality of the design space.

This work considers prescribed strains as actuation systems [41]. These are often achieved mechanically. However, it is worth noting that more complex actuation systems, such as temperature change [42, 43], and illumination [44], can be also modeled as assigned strains. Similar considerations apply to biological and bio-hybrid structures, where electrophysiological stimuli are converted into mechanical actions based on the active strain approach [45]. Recent works also demonstrate that small beneficial geometric deviations can nudge structures onto favorable equilibrium paths [46], and they can be modeled as equivalent prescribed strains, as shown in [47,48].

This research presents a gradient-based optimization technique integrated with the nonlinear finite element method to minimize actuation for controlling both the deformed shape and stress in geometrically nonlinear structures. The study specifically addresses the quasi-static case. The objective function, representing the total actuation, is minimized using a gradient-based Sequential Quadratic Programming algorithm [49]. The constraints include target nodal positions and/or stress in structural elements, as well as actuation bounds in each actuated element. The weak point of standard optimization tools is that the gradient of nonlinear constraints, i.e., deformed shape and stress, is usually computed approximately through numerical differentiation. The aim of this article is to overcome this weak point, demonstrating how the exact gradient of the nonlinear finite element solution can be evaluated within the structural solver robustly and at a negligible computational cost. The theory behind the developments relies on the chain implicit differentiation and the exploitation of the tangent stiffness matrix already factorized in the structural Newton iterations. As a result, the optimal design of the actuation system is made robust and efficient, especially for structures with many degrees of freedom. The procedure is illustrated for 3D trusses, from small to large structures, due to the simplicity of the models and implementation which are easy to reproduce by interested readers. However, the approach is formulated to apply to any geometrically nonlinear finite element model, using a general formal of discrete equations to facilitate the integration within commercial codes.

The paper is organized as follows: Section 2 describes the geometrically nonlinear problem with reference to a finite element model for 3D trusses with actuation modeled as prescribed strains in some actuated elements, introducing the necessary notation. Section 3 is the core of this work, formulating and addressing the nonlinear optimization problem for displacement and stress control using a gradient-based method. Special attention is given to differentiating displacement and stress coming from the nonlinear finite element solution with respect to the actuation parameters. Section 4 demonstrates the gain in robustness and efficiency achieved in the optimization algorithm using the exact closed-form differentiation strategy compared to the standard numerical differentiation. The reduction of computational time is also discussed for large structures. Conclusions are reported in the last section.

2. The 3D truss model with actuators and initial stresses

The optimization process developed in this work can be applied to generic large deformation structural models with elastic materials discretized by the finite element method. The hypotheses of the work are:

- quasi-static problem;
- finite kinematics with nonlinear strain–displacement relationship;
- equilibrium in the deformed configuration;
- hyperelastic materials.

To simplify the exposition as much as possible and make the procedure easy for the reader to understand, 3D truss structures are considered simple examples of applications where each finite element coincides with the physical bar. This also makes the numerical results very easy to reproduce.

This section recalls the nonlinear discrete equations of a large deformation truss model based on the quadratic Green–Lagrange strain measure. The model is introduced, as usual, starting from the variational problem formulated in a displacement-based approach. Thus, the nonlinear equilibrium equations are derived and the incremental-iterative solution scheme is quickly discussed. This information will be of help for the subsequent developments.

It is worth noting that introducing a general finite element context aims to show that the developments are not influenced by the structural

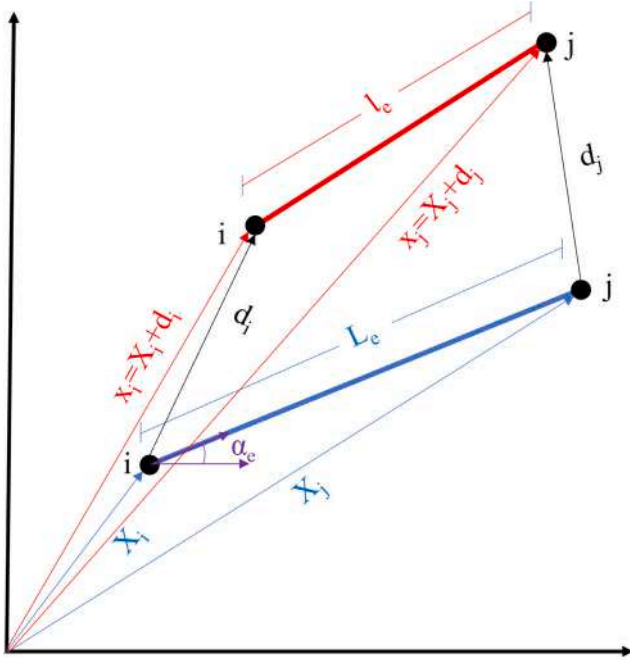


Fig. 1. Kinematics of the bar element.

topology, e.g., geometry, number of nodes, and elements. Conversely, the exact gradient evaluation method developed in the subsequent section can be easily and efficiently integrated within any commercial structural software under the hypotheses listed above. Also, the discrete equations of the finite element model have an identical format as those coming from other structural models (beams/frames, plates/shells, 3D continuum), making the methodology extensible to different structural types.

2.1. The truss element: kinematics and strain measure

The position of the two end-nodes (i and j) of the generic bar (see Fig. 1) in the reference (initial) and current configuration is denoted as $\mathbf{X}_k$ and $\mathbf{x}_k$ respectively, with $k = i, j$. The kinematics of each node can be then expressed as

$$\mathbf{x}_k = \mathbf{X}_k + \mathbf{d}_k$$

where $\mathbf{d}_k$ denotes the displacement of the k th node. For each truss element (bar), it is useful to introduce the following vectors

$$\Delta \mathbf{X}_e \equiv \mathbf{X}_j - \mathbf{X}_i \quad \text{and} \quad \Delta \mathbf{x}_e \equiv \mathbf{x}_j - \mathbf{x}_i = \Delta \mathbf{X}_e + \mathbf{d}_j - \mathbf{d}_i$$

The initial L_e and final ℓ_e lengths of the bar are obtained as

$$\begin{aligned} L_e^2 &= \Delta \mathbf{X}_e^T \Delta \mathbf{X}_e \\ \ell_e^2 &= \Delta \mathbf{x}_e^T \Delta \mathbf{x}_e = L_e^2 + 2\Delta \mathbf{X}_e^T (\mathbf{d}_j - \mathbf{d}_i) + (\mathbf{d}_j - \mathbf{d}_i)^T (\mathbf{d}_j - \mathbf{d}_i) \end{aligned}$$

The Green–Lagrange strain measure expressing the strain–displacement compatibility is

$$\varepsilon_d(\mathbf{d}_e) = \frac{1}{2} \left(\frac{\ell_e^2}{L_e^2} - 1 \right) = \frac{1}{L_e} \alpha_e^T (\mathbf{d}_j - \mathbf{d}_i) + \frac{1}{2L_e^2} (\mathbf{d}_j - \mathbf{d}_i)^T (\mathbf{d}_j - \mathbf{d}_i) \quad (1)$$

where $\alpha_e = \frac{\Delta \mathbf{X}_e}{L_e}$ is the vector containing the cosines directors of the bar generically oriented in a 3D space. Using a standard FE notation, we introduce the element vector $\mathbf{d}_e$

$$\mathbf{d}_e = \begin{bmatrix} \mathbf{d}_i \\ \mathbf{d}_j \end{bmatrix}$$

collecting the 6 discrete displacement DOFs of the bar corresponding to the nodal displacements. The Green–Lagrange strain in Eq. (1) rewrites as

$$\varepsilon_d = \frac{1}{L_e} \left(\mathbf{b}_e^T \mathbf{d}_e + \frac{1}{2} \mathbf{d}_e^T \mathbf{G}_e \mathbf{d}_e \right) \quad (2)$$

where

$$\mathbf{b}_e = \begin{bmatrix} -\alpha_e \\ \alpha_e \end{bmatrix} \quad \text{and} \quad \mathbf{G}_e = \frac{1}{L_e} \begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}$$

and $\mathbf{I}$ is the 3×3 identity matrix. The total strain ε_d coming from the displacement can be decomposed, assuming large deformations with small strains, as a sum of an elastic part ε and inelastic prescribed strains ε_0 and ε_a :

$$\varepsilon_d(\mathbf{d}_e) = \varepsilon + \varepsilon_0 + \varepsilon_a \quad (3)$$

where ε_0 is an initial pre-strain and ε_a is the strain induced by the actuators, which will be the object of the design i.e. the variable of the optimization problem. Consequently, the axial force in the bar can be written as

$$N = EA\varepsilon = EA(\varepsilon_d(\mathbf{d}_e) - \varepsilon_0 - \varepsilon_a) \quad (4)$$

or, alternatively, as

$$N = EA\varepsilon = EA(\varepsilon_d(\mathbf{d}_e) - \varepsilon_a) + N_0 \quad \text{with} \quad N_0 = -EA\varepsilon_0 \quad (5)$$

where N_0 represents a possible axial force imposed as pre-stress before the actuation.

2.2. The equilibrium condition: internal force vector and stiffness matrix

The strain energy of the structures is obtained by summing the contribution, denoted as Φ_e , of each bar finite element:

$$\Phi = \sum_e \Phi_e \quad \text{where} \quad \Phi_e = \frac{1}{2} EAL_e(\varepsilon - \varepsilon_0 - \varepsilon_a)^2 \quad (6)$$

The internal force vector of the element is evaluated as the gradient of the element strain energy

$$\mathbf{s}_e = \frac{\partial \Phi_e}{\partial \mathbf{d}_e} = N \mathbf{q}_e \quad \text{with} \quad \mathbf{q}_e = \mathbf{b}_e + \mathbf{G}_e \mathbf{d}_e \quad (7)$$

with $N = EA(\varepsilon - \varepsilon_0 - \varepsilon_a)$. The Hessian of Φ_e furnishes the tangent stiffness matrix of the element

$$\mathbf{K}_e = \frac{\partial^2 \Phi_e}{\partial \mathbf{d}_e^2} = \frac{EA}{L_e} \mathbf{q}_e \mathbf{q}_e^T + N_e \mathbf{G}_e \quad (8)$$

Letting $\mathbf{p}_k$ represent the forces acting at the k th node of the entire structure, the external work is the sum of the nodal contributions:

$$\mathcal{L}_{ext} \equiv \sum_{k=1}^{N_n} \mathbf{p}_k^T \mathbf{d}_k = \mathbf{p}^T \mathbf{d}$$

where $\mathbf{d}$ is the global vector collecting all the nodal displacement DOFs of the structure. The total potential energy is the sum of the strain energy of all elements and the external work:

$$\Pi(\mathbf{d}) = \sum_e \Phi_e - \mathbf{p}^T \mathbf{d}. \quad (9)$$

The nonlinear equilibrium equations are obtained from the stationary condition of Π :

$$\frac{\partial \Pi}{\partial \mathbf{d}} = \mathbf{s}(\mathbf{d}) - \mathbf{p} = \mathbf{0} \quad \text{with} \quad \mathbf{s}(\mathbf{d}) = \sum_e \mathbf{T}_e^T \mathbf{s}_e(\mathbf{d}_e) \quad (10)$$

where $\mathbf{T}_e$ is the incidence matrix used for the global assembly of the element contributions, so that $\mathbf{d}_e = \mathbf{T}_e \mathbf{d}$. In a similar fashion, we obtain the global tangent matrix that will be used in the iterative solution method

$$\mathbf{K} = \sum_e \mathbf{T}_e^T \mathbf{K}_e \mathbf{T}_e.$$

2.3. Incremental-iterative solution

The equilibrium of slender elastic structures subject to conservative loads amplified by a proportionality factor λ is expressed by Eq. (10)

$$\mathbf{r}(\mathbf{d}, \lambda) \equiv \mathbf{s}(\mathbf{d}) - \lambda \mathbf{p} = \mathbf{0} \quad (11)$$

where $\mathbf{r} : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ is a nonlinear vectorial function of the vector $(\mathbf{d}, \lambda)$, collecting the configuration $\mathbf{d} \in \mathbb{R}^n$ and the load multiplier $\lambda \in \mathbb{R}$, $\mathbf{s}(\mathbf{d})$ is the global internal force vector and $\mathbf{p}$ the reference load vector. Eq. (11) represents a system of equations and unknowns and its solutions define the equilibrium path as a load factor-displacement curve in $\mathbb{R}^{n+1}$. An incremental-iterative solution scheme can be adopted to trace this curve in a step-by-step manner from a known initial configuration corresponding to $\lambda = 0$. The condition $\lambda = 1$ corresponds to the actual reference load of interest. There are two main reasons for introducing the load factor λ : (i) the equilibrium curve can exhibit a limit point in λ , meaning that there may be no close equilibrated solution above a certain load level; (ii) even if an equilibrium point exists for all λ in the range of interest, it is not assured that we are actually able to compute it directly for an assigned load. Regarding the first issue, the Riks arc-length method [50] can be used for dealing with limit points. Instead, for simplicity of exposition, we focus here on the second case.

An incremental solution scheme can be applied by defining a sequence of points (steps) $\mathbf{z}^{(k)} \equiv (\mathbf{d}^{(k)}, \lambda^{(k)})$ belonging to the equilibrium path. Starting from a known equilibrium point $\mathbf{z}_0 \equiv \mathbf{z}^{(k)}$, the new one $\mathbf{z}^{(k+1)}$ is evaluated correcting a first prediction of the displacement DOFs $\mathbf{d}_1$ corresponding to $\lambda^{(k+1)}$ by a sequence of estimates $\mathbf{d}^j$ by Newton iterations

$$\mathbf{d}_{j+1} = \mathbf{d}_j + \bar{\mathbf{d}} \quad \text{with} \quad \bar{\mathbf{d}} = -\mathbf{K}(\mathbf{d}_j)^{-1} \mathbf{r}(\mathbf{d}_j, \lambda^{(k+1)}) \quad (12)$$

At each Newton iteration of each load step the global tangent stiffness matrix, obtained by assembling the element ones in Eq. (8), is computed and factorized for solving the linear systems. This operation is usually the costliest one in the analysis of large structures. The Newton iteration terminates when the norm of the residual vector $\mathbf{r}$ falls below a small predefined threshold.

The convergence of the Newton scheme is local, i.e. it converges when a good initial prediction of the unknowns (the displacement DOFs) is available. Clearly, this is not guaranteed for large load steps. Thus, the stepping procedure is useful for problems with a significant nonlinearity. Reaching the maximum load level in multiple steps allows one to take advantage of the previous equilibrium points in estimating the new one. A typical choice is to extrapolate linearly the two previous points for estimating the newly searched one. The number of load steps can be selected by the user, but can be adjusted automatically following, for example, the adaptive stepping reported in [51]. Clearly, in the present case of an elastic structure under a conservative load, the solution for the final load is unaffected by the stepping scheme (loading path). A mixed solution scheme involving also local stress variables to improve the robustness of the Newton scheme and to reduce the number of iterations [51,52].

3. Optimization: problem statement, solver and gradient evaluation

This section is dedicated to formulating the optimization problem for displacement and stress control in the context of geometrically nonlinear problems, along with its solution strategy. The emphasis is placed on differentiating the nonlinear finite element solution with respect to the actuation inputs to achieve an efficient and robust gradient-based optimization. Sequential Quadratic Programming is selected as the optimization algorithm.

3.1. Problem statement

To compare pure results with what is typically done in the literature, we consider as optimization variables the actuation in terms of prescribed elongation a instead of the axial strain ϵ_a :

$$a = \epsilon_a L \quad (13)$$

where L is the length of the generic actuated element. The optimization problem for displacement and stress control of a geometrically nonlinear structure can be formulated as the minimization of the total actuation inputs (prescribed elongations) subject to the following constraints:

- upper and lower bound for the actuation a in each actuated bar;
- upper and lower bound for the stress in controlled elements coming from the nonlinear finite element model with the prescribed actuation;
- upper and lower bound for the displacement of controlled nodes coming from the nonlinear finite element model with the prescribed actuation.

This can be summarized as the following optimization problem:

$$\begin{aligned} &\text{minimize} \quad \sum_{i \in \mathcal{U}} |a_i| \\ &\text{subject to} \quad d_k^{(\min)} \leq d_k(\mathbf{a}) \leq d_k^{(\max)} \quad k \in \mathcal{K} \\ &\quad N_e^{(\min)} \leq N_e(\mathbf{a}) \leq N_e^{(\max)} \quad e \in \mathcal{E} \\ &\quad a_i^{(\min)} \leq a_i \leq a_i^{(\max)} \quad i \in \mathcal{U} \end{aligned} \quad (14)$$

where $\mathbf{a}$ collects all the actuation inputs. $\mathbf{d}(\mathbf{a})$ and $N_e(\mathbf{a})$ are implicitly defined as those which verify the nonlinear structural equilibrium equations in Eq. (10) for given $\mathbf{a}$, this means that each evaluation of the constraints requires a nonlinear structural analysis according to Section 2.3 for the given load and actuation. Subsets $\mathcal{U}$, $\mathcal{E}$ and $\mathcal{K}$ denote, respectively, the actuated elements, the elements where we want to control the axial force N_e (stress resultant) and the controlled nodal displacement components d_k chosen among those the global vector $\mathbf{d}$ of displacement DOFs.

The number of optimization variables is the size of vector $\mathbf{a}$ corresponding to the cardinality of $\mathcal{U}$. The same value is also the number of linear constraints. Instead, the number of nonlinear constraints depends on the number of elements and nodal displacement components to control, i.e. the sum of the cardinality of $\mathcal{E}$ and $\mathcal{K}$.

It is worth remembering that, for generic statically indeterminate structures, even changing one of the actuation inputs in an element produces different displacements and axial forces in all the structural elements.

3.2. Optimization algorithm

The solution of the optimization problem (Eq. (14)) is tackled by using the Sequential Quadratic Programming method [49] implemented in *MATLAB* in the function named *fmincon* and activated by setting SQP as the solution algorithm. This is the state-of-art iterative method for constrained nonlinear optimization which may be seen as a quasi-Newton method. The main idea behind SQP is to approximate the problem (Eq. (14)) at each iterate with a local quadratic model of the objective function subject to a linearization of the constraints, i.e. a simpler Quadratic Programming. Importantly, the method requires an accurate gradient of the Lagrangian function, namely the gradient of objective function and constraints, while a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is typically employed.

In our optimization problem, the analytical gradient of the objective function is derived very easily. Instead, the gradient of the nonlinear constraints is more complicated since it involves the differentiation of the nonlinear structural response (finite element solution) in terms of stress and displacement with respect to the actuation inputs. The

standard approach uses a numerical differentiation to approximate these derivatives with either forward (default in *fmincon*) or central difference formulas. This means that the gradient evaluation requires a huge number of nonlinear structural analyses, at least one for each of the optimization variables. For structures with a large number of DOFs, this part is the bottleneck of the algorithm, mainly dominating the overall computational time with the repeated evaluation and factorization of the tangent stiffness matrix in the Newton iterations for imposing the structural equilibrium. In addition, it is necessary to consider the well-known trade-off of finite differences: the increment must be small enough for approximating the derivatives, but not too small to avoid round-off errors due to the limited machine precision. Since the nonlinear equilibrium equations are solved iteratively, the accuracy of the numerical differentiation can also be affected by the threshold of the structural iterative solver. As a consequence, the overall numerical approximation in the gradient evaluation can affect negatively the convergence of the optimization process, producing an increment of iterations of the optimization algorithm. Ultimately, both evaluation cost and accuracy of the gradient are relevant factors affecting the design procedure. The next subsection represents the main novelty point of this work, where an alternative closed-form differentiation of both displacement and stress of the nonlinear finite element model with respect to the actuation inputs is derived. The result provides a significant speed-up of the optimization process.

3.3. Gradient of the nonlinear finite element solution with respect to the actuation inputs

The goal is to calculate the gradient of the global vector $\mathbf{d}(\mathbf{a})$, which collects the displacement components of all nodes of the structures at their final equilibrium configuration for assigned final loads and actuation. This gradient is computed with respect to the optimization variables, namely the actuation inputs represented by the elongation values in the actuated elements collected in vector $\mathbf{a}$. Subsequently, these derivatives, denoted as $\nabla_{\mathbf{a}}\mathbf{d}$, can be utilized to derive $\nabla_{\mathbf{a}}N$, enabling the differentiation of the final axial force in each element e .

The difficulty of an analytical differentiation relies on the fact that no explicit expression for $\mathbf{d}(\mathbf{a})$ and $N(\mathbf{a})$ is available. All we know is that $\mathbf{d}(\mathbf{a})$ and $N(\mathbf{a})$ are the values of $\mathbf{d}$ and N satisfying the nonlinear equilibrium condition Eq. (11) for the final load ($\lambda = 1$) and the assigned actuation $\mathbf{a}$. Thus, $\mathbf{d}(\mathbf{a})$ and $N(\mathbf{a})$ are only implicitly defined by the nonlinear equilibrium condition.

Our novel methodology for a closed-form gradient evaluation is based on the so-called implicit differentiation, a strategy in calculus that makes use of the chain rule to differentiate implicitly defined functions. Accordingly, we can differentiate the residual equilibrium equations as

$$\frac{\partial \mathbf{r}(\mathbf{d}(\mathbf{a}), \mathbf{a})}{\partial \mathbf{a}} = \underbrace{\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{d}}}_{\mathbf{K}(\mathbf{d}(\mathbf{a}))} \nabla_{\mathbf{a}}\mathbf{d} + \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}} \bigg|_{\mathbf{d}(\mathbf{a})} = \mathbf{0} \quad (15)$$

where $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{d}} \bigg|_{\mathbf{d}(\mathbf{a})}$ coincides with the tangent stiffness matrix evaluated at the final iteration of the structural equilibrium Newton solver, while $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}} \bigg|_{\mathbf{d}(\mathbf{a})}$ collects the partial derivatives of the internal forces at the final equilibrium Newton iteration with respect to the actuation inputs considering $\mathbf{d}$ independent from $\mathbf{a}$ in the differentiation. This last ingredient can be evaluated by assembling the contribution of each finite element, actuated or not, to the global $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}}$ for assigned displacements. This element contribution can be computed as

$$\frac{\partial \mathbf{s}_e(\mathbf{d}_e, a_e)}{\partial a_e} = \frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial a_e} \mathbf{q}_e(\mathbf{d}_e) \quad \text{with} \quad \frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial a_e} = \begin{cases} -\frac{EA}{L_e} & \text{if } a_e \neq 0 \\ 0 & \text{if } a_e = 0 \end{cases} \quad (16)$$

Eq. (15) provides a way to compute directly and in closed form $\nabla_{\mathbf{a}}\mathbf{d}$ by solving the linear system:

$$\mathbf{K}(\mathbf{d}(\mathbf{a})) \nabla_{\mathbf{a}}\mathbf{d} = - \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}} \bigg|_{\mathbf{d}(\mathbf{a})} \quad (17)$$

This strategy is computationally very interesting since it requires:

- the computation of vector $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}}$ that has the same cost of evaluating the internal force vector, even lower because the equilibrium operator $\mathbf{q}_e$ is already available at the last iteration of the structural Newton solver;
- the solution of a linear system in Eq. (17) with a tangent stiffness matrix $\mathbf{K}(\mathbf{d}(\mathbf{a}))$ already available and factorized at the last iteration of the structural Newton solver.

Compared to the numerical gradient evaluation requiring multiple nonlinear structural analyses, this strategy gives an exact algorithmic gradient at practically no cost.

It is interesting to note, as a mechanical interpretation, that $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{a})}{\partial \mathbf{a}}$ involves the derivative of the axial force of the generic element for assigned displacement with respect to the actuation of the element itself. The actuation inputs in the other elements do not contribute to this term. The fact that varying the actuation in one element can affect the displacements in all structural nodes comes from the coupling in the tangent stiffness matrix.

Once the gradient of the displacements is computed, the gradient of the axial force N_e in the generic bar e can be then evaluated by the chain rule as

$$\nabla_{\mathbf{a}}N_e = \frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial \mathbf{d}_e} \bigg|_{\mathbf{d}_e(\mathbf{a})} \nabla_{\mathbf{a}}\mathbf{d}_e + \frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial a_e} \bigg|_{\mathbf{d}_e(\mathbf{a})} \quad (18)$$

where $\frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial \mathbf{d}_e} = \frac{EA}{L_e} \mathbf{q}_e(\mathbf{d}_e)^T$ and $\nabla_{\mathbf{a}}\mathbf{d}_e$ is the element version of $\nabla_{\mathbf{a}}\mathbf{d}$ obtained by extracting the values in the positions corresponding to the nodal DOFs of the generic element e . By substituting, we have explicitly

$$\nabla_{\mathbf{a}}N_e = \frac{EA}{L_e} \mathbf{q}_e(\mathbf{d}_e(\mathbf{a}))^T \nabla_{\mathbf{a}}\mathbf{d}_e + \frac{\partial N_e(\mathbf{d}_e, a_e)}{\partial a_e} \bigg|_{\mathbf{d}_e(\mathbf{a})}. \quad (19)$$

Again, considering the right hand of Eq. (19), we can observe that the second term takes into account the variation of the axial force of each element with respect to its actuation, while the fact that varying the actuation in one element can affect the stress in all the others is recovered by the first term due to the displacement gradient.

3.4. Remarks

It is worth highlighting that all discrete operators needed in the proposed gradient evaluation strategy are already implemented in structural finite element codes. This means that the exact gradient can be computed with minimal implementation changes directly at the final step of the incremental nonlinear analysis. Indeed, the same routines already available for computing the internal force vector and the element stress can be directly reused for evaluating the partial derivatives of the internal force vector and element stress in Eq. (16) by simply giving different input data. Furthermore, It is interesting to note that multiple load cases can be directly managed in the proposed framework by simply duplicating displacement and stress constraints for each.

4. Results and discussion

In this section, we have implemented the methodology developed in the previous section on various examples, considering displacement and axial force control both independently and in conjunction. The validity of the presented technique has been verified by comparing the obtained results with those described in the literature.

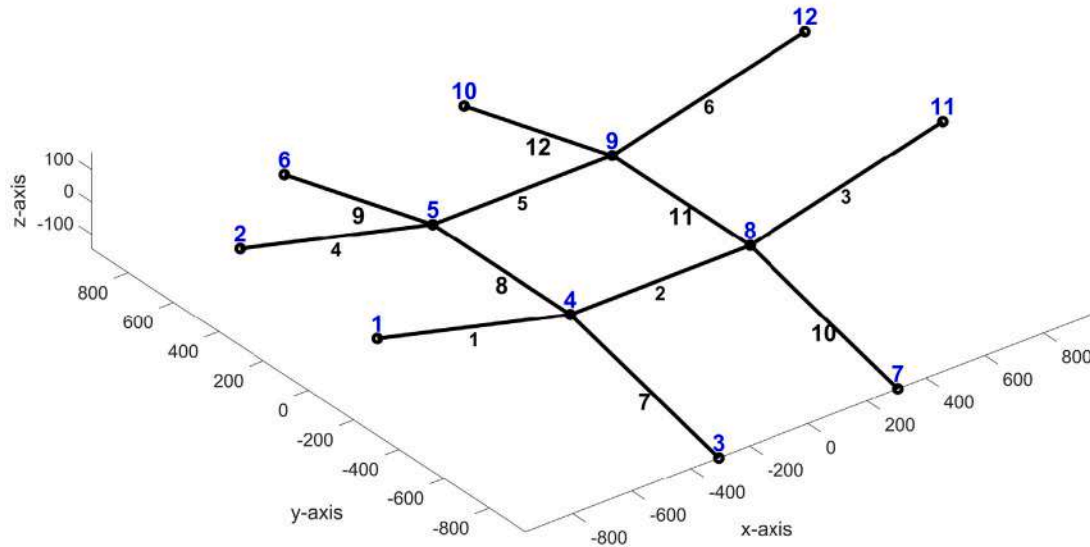


Fig. 2. Geometry of a saddle-shaped cable net structure [55].

Table 1
Displacement control of a saddle-shaped cable net structure: pre (p) and post(a) adjustment.

Joint	Dir.	d_p (mm)	d_a (mm)	Cable	a (mm)	N_p (N)	N_a (N)
4	x	-0.62	-0.03	1	4.73	143.9	85.6
	y	0.59	1.27	2	0.00	139.6	83.3
	z	-8.84	-20.00	3	4.73	143.9	85.6
5	x	-0.62	-0.03	4	4.73	143.9	85.6
	y	-0.59	-1.27	5	0.00	139.6	83.3
	z	-8.84	-20.00	6	4.73	143.9	85.6
8	x	0.62	0.03	7	0.00	24.4	-41.5
	y	0.59	1.27	8	0.00	24.0	-40.6
	z	-8.84	-20.00	9	0.00	24.4	-41.5
9	x	0.62	0.03	10	0.00	24.4	-41.5
	y	-0.59	-1.27	11	0.00	24.0	-40.6
	z	-8.84	-20.00	12	0.00	24.4	-41.5
$\sum a_i = 18.92$							

4.1. Saddle-shaped cable net structure

Fig. 2 illustrates a 3-D cable net featuring twelve nodes and cables made of piano wire (42 mm diameter, 210 MPa modulus of elasticity, and 2000 MPa tensile strength). The cable net is restrained at the circumference joints, while the mid joints remain unrestrained. This structure, subjected to testing by [53] and analyzed numerically by [54], is both statically and kinematically indeterminate. Initial prestressing, following Pellegrino's recommendation [53], is applied by actuating cables 1, 3, 4, 6, 7, 9, 10, and 12 with a displacement of -1.85 mm, and cables 2, 5, 8, and 11 with a displacement of -1.68 mm. Additionally, the free nodes are loaded with 30 N in the downward direction.

Table 1 presents the nodal displacements resulting from external loading, with the objective of positioning the free nodes at a displacement of -20 mm. The table indicates the successful achievement of this goal through the actuation of only four cables. Since the primary focus in this phase is on nodal positions, axial forces were intentionally disregarded, resulting in slack in certain cables.

In the second step, the saddle cable shown in Fig. 2 and the same initial prestress and external loading were examined to control the axial forces in the cables within $10 \text{ N} \leq N_e \leq 125 \text{ N}$ to keep them from slacking and high stress. The target axial forces N_a and the actuation values are reported in Table 2. Since the nodal position of the joints was not considered, the vertical displacement of the free joints after cable actuation became -17.12 mm each.

Table 2
Stress control of a saddle-shaped cable net structure: pre (p) and post(a) adjustment.

Cable	a (mm)	N_p (N)	N_a (N)
1	2.687	143.9	125.0
2	0.000	139.6	121.4
3	2.687	143.9	125.0
4	2.687	143.9	125.0
5	0.000	139.6	121.4
6	2.687	143.9	125.0
7	-0.881	24.4	10.0
8	-0.575	24.0	10.0
9	-0.881	24.4	10.0
10	-0.881	24.4	10.0
11	-0.575	24.0	10.0
12	-0.881	24.4	10.0
$\sum a_i = 15.422$			

For the current example, when considering only the nodal positions, some cables experience slackness, rendering the structure unstable as cables can only carry tension [56]. Conversely, when focusing solely on axial forces, the nodes assume arbitrary positions, as demonstrated earlier. Now, the structures (Fig. 2) is examined to control nodal positions and axial forces in cables simultaneously. Sometimes, it is essential to maintain both shape and stress simultaneously.

In this case, both shape and stress are taken into account: the objective is to set the free nodes to a -20 mm vertical displacement while keeping the axial force on the elements N_e within the range of

Table 3
Displacement and axial force control of a saddle-shaped cable net structure: pre (p) and post (a) adjustment.

Joint	Direction	d_p (mm)	d_a (mm)	Cable	a (mm)	N_p (N)	N_a (N)
4	X	-0.62	-0.03	1	3.50	143.9	122.8
	Y	0.59	1.27	2	0.00	139.6	119.3
	Z	-8.84	-20.00	3	3.50	143.9	122.8
5	X	-0.62	-0.03	4	3.50	143.9	122.8
	Y	-0.59	-1.27	5	0.00	139.6	119.3
	Z	-8.84	-20.00	6	3.50	143.9	122.8
8	X	0.62	0.03	7	-0.93	24.4	10.0
	Y	0.59	1.27	8	-1.59	24.0	10.1
	Z	-8.84	-20.00	9	-0.93	24.4	10.0
9	X	0.62	0.03	10	-0.93	24.4	10.0
	Y	-0.59	-1.27	11	-1.59	24.0	10.1
	Z	-8.84	-20.00	12	-0.93	24.4	10.0
$\sum a =$					20.91		

Table 4
Reduction of the computational cost by analytical gradient for displacement control.

Gradient	Iterations		Structural analyses	
	Numerical	Analytical	Numerical	Analytical
Displacement control	4	4	29	9
Stress control	15	15	440	265
Displacement and stress control	16	13	522	251

10 N $\leq N_e \leq$ 125 N. Table 3 displays the displacements and axial forces in cables before and after applying the set of actuations obtained from the current technique. It is evident that the targets were precisely achieved by actuating ten cables with 20.91 mm.

4.1.1. Reduction of the computational cost

To achieve a reduction in computational cost by leveraging analytical gradients for nonlinear constraint functions, Table 4 provides a comparative analysis of the number of iterations and function/constraint evaluations (structural analyses) for the saddle-shaped cable net structure, considering displacement, stress, and coupled control. Significantly, utilizing the present analytical gradients of the nonlinear finite element solution results in substantially fewer iterations and structural analyses compared to numerical gradients.

Specifically, for displacement control alone, the numerical data reveals that the number of structural analyses using analytical gradients are lower by 68.9%, compared to those using numerical gradients. This indicates that the proposal not only avoids the need for multiple nonlinear structural analyses to compute the gradient but also enhances the accuracy of the gradient itself, leading to improved convergence of the optimization algorithm.

4.2. Space cable structure

Fig. 3 presents the numerical example which was previously investigated by Yuan et al. [21] and which is used in this study to validate the effectiveness of the present method in handling geometrically nonlinear structures. The structure comprises of four restrained and two free joints, a vertical strut with a hollow circular section ($\varnothing 50 \times 4$ mm, elastic modulus of 206 GPa) connecting the top and bottom free joints. Additionally, eight cables with a diameter of 8 mm and elastic modulus of 185 GPa are incorporated. The initial axial force in the strut is of -28,140 N, while for the upper and lower cables, it is set as equal to 19,01 N. Application of a 20,00 N downward force at the top free joint results in a -9.00 mm displacement in the Z direction and changing the axial force in the strut, upper and lower cables to -38 249, 19 140, and 38 775 N respectively.

This case aims to nullify the top free joint's X, Y, and Z displacements. Yuan, Liang [21] addressed this by extending the upper cables by 4.56 mm to control the nodal displacement of the top free joint. A comparable result was achieved to validate the accuracy of the current

Table 5
Axial force (N) control of a space cable structure.

Members	Initial stage	After loading	After actuation			
			Case 1		Case 2	
			[21]	Present	[21]	Present
Strut	-28 140	-38 250	-47 260	-47 225	-48 150	-48 152
Upper cable	29 010	19 140	29 010	29 010	29 010	29 010
Lower cable	29 010	38 780	47 240	47 134	49 660	49 642

approach, the target was achieved by actuating the upper cables by 5.57 mm, which demonstrating a mere 0.2% dissimilarity with the previous study.

The same structural configuration (Fig. 3) undergoes testing to validate the developed technique emphasizing axial force control regardless of displacement. Table 5 overviews the induced axial forces in members after the initial stage and applied loads. The objective is to maintain the internal force in the upper cables at their initial stage (29 010 N), employing two distinct scenarios of cable actuation. In Case 1, only the top cables are actuated, while in Case 2, solely the bottom cables are actuated to sustain the axial force in the top cables. A comparative analysis between the study's results by Yuan et al. [21] and the current investigation reveals that the targets were successfully achieved in both cases, with a slight dissimilarity in the amount of actuation. It should be noted that, for Case 1, the previous study requires shortening the upper cables by 4.03 mm, while the current research asks to shorten the cables by 3.96 mm. This indicates that the current approach achieves the target with 1.7% less actuation effort compared to the previous work, highlighting improved efficiency. Moreover, this reduction in actuation was achieved without compromising the cost-effectiveness of identifying the optimal solution.

4.2.1. Reduction of the computational cost

The efficiency of the advanced optimization, based on analytical gradients for nonlinear constraint functions (displacement and stress), is assessed in Table 6. This table presents the number of iterations and function/constraint evaluations (structural analyses) for the space cable structure, considering both displacement and stress control. The utilization of the present analytical gradients for the nonlinear finite element solution results in significantly fewer iterations and structural analyses compared to numerical gradients. Specifically, for displacement control, the algorithm requires a reduction of 82.9% in the number of function/constraint evaluations using the new analytical gradients compared to those using numerical gradients.

4.3. A 2D cable net

This example validates the present approach for controlling axial force and large displacement in a 2D cable net structure (depicted in Fig. 4). The cable structure underwent experimental testing by

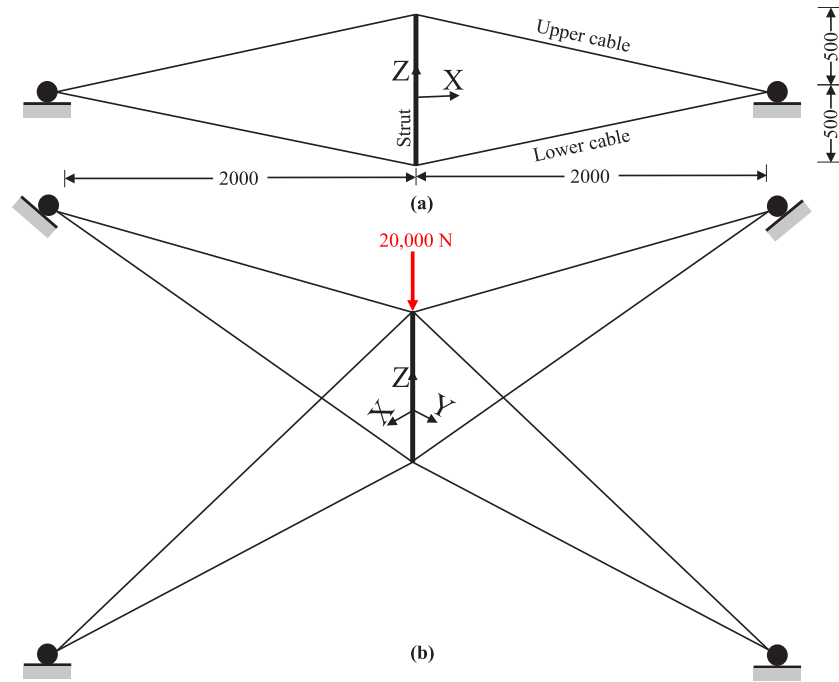


Fig. 3. Space cable structure (a) Front view (b) 3D view [21].

Table 6

Reduction of the computational cost by analytical gradient for controlling the space cable structure.

Gradient	Iterations		Structural analyses	
	Numerical	Analytical	Numerical	Analytical
Displacement control	4	3	41	7
Stress control	4	3	47	21

You [16] for displacement control in prestressed systems. Subsequent investigations by Xu and Luo [57] as well as Saeed [58] delved into controlling shape and stress in nonlinear cable configurations.

The structure consists of three free and four restrained joints, with nine cables featuring $EA = 43160$ N. The initial axial force in cables N_0 is specified in Table 7.

The primary objective is to impart $[-20, 20]$ mm displacement to the X and Y coordinates, respectively, of Joint 6 while maintaining the stress in the cables as close as possible to their initial levels. In particular, we impose that the axial forces of the cables are exactly the target ones N_0 . The problem so formulated could be not feasible. In this case, the optimization algorithm stops when the feasibility error stop decreasing. This solution is then accepted as the closest one to the target problem. Xu and Luo [57] successfully achieved the displacement requirement; however, the axial force in some members deviated slightly from the initial values, as outlined in Table 7. Noteworthy differences between the target and induced axial forces after actuation in the study by Xu and Luo [57] are observed in cables iii, iv, and vii, with discrepancies of 21.8%, 19.4%, and 19.1%, respectively.

In the current study, despite precisely achieving the target displacement, the maximum discrepancy between the target axial force and the post-actuation induced force occurred in Cable v, amounting to 11.5%. The current approach uses fewer actuations by 28.7% compared to the previous method.

The same model was investigated with varying control requirements. The cable net was prestressed by applying elongation of $[-5.02, 4.49, -5.02]$ mm to Cables vii, viii, and ix, respectively. This resulted in nodal displacements and axial forces, as detailed in Table 8. The objective was to eliminate the X and Y displacements of Joint 6,

maintain stress levels above the initial threshold, and utilize only four actuators. All previous studies as well as the present one successfully achieved the specified targets. Shen et al. [59], Saeed et al. [17] and Saeed [58] required 18.6, 18.3, and 18.1 mm of actuation, respectively, to attain the targeted displacements and maintain stress above the initial level. The present approach accomplished the goal using 17.9 mm of actuation as detailed in Table 8.

Table 9 displays the iteration counts and the number of structural analyses for two cases of displacement and stress control employing numerical and analytical gradients of the nonlinear constraints. An outstanding reduction in iterations and function evaluations is evident when utilizing the new closed-form gradients compared to numerical ones. This shows the accuracy of the gradient affects significantly also the iterations of the optimization algorithm. The results of this study demonstrate that the proposed method offers significant advantages over previous approaches. Specifically, it minimizes the objective function while strictly adhering to the imposed constraints. Moreover, the method is computationally efficient, making it a more cost-effective solution than existing alternatives which use numerical gradients.

4.4. Deployable space truss

Fig. 5 illustrates a fully opened deployable truss comprising 25 nodes and 60 bars. Nodes 1, 5, 9, 16, 20, 24, and 25 are restrained in all directions. Cables 1–33 have $EA = 98$ kN, while struts 34–60 have $EA = 350$ kN, with horizontal and vertical elements measuring 500 mm long. Additionally, Fig. 6 displays the self-stress state of a unit.

The objective is to achieve a prestressing level of 2 N for Cables 3, 5, 7, 8, 11, and 1 N for the remaining cables, along with -1 N for the struts, as outlined in Table 10. This is to be accomplished by actuating only Cables 1, 3, 4, 7, 8, and 9.

In a study by Kwan and Pellegrino [53], the goal was achieved with a total actuation of 0.4861 mm, while Abdulkarim and Saeed [20] accomplished it with a total actuation of 0.4755 mm. However, with the current approach, the target can be met with 0.4754 mm, representing a 3.23% and 1.07% reduction compared to the studies by Kwan and Pellegrino [53] and by Abdulkarim and Saeed [20], respectively.

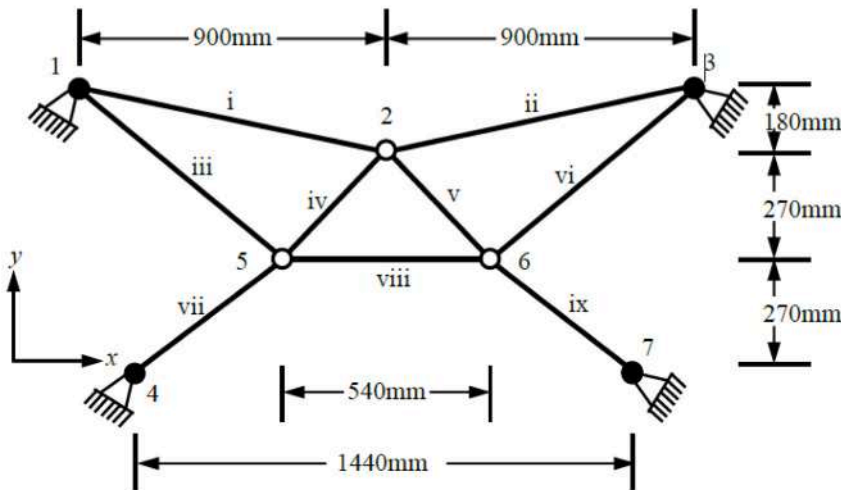


Fig. 4. A 2D cable net.

Table 7
Imparting [−20 20] mm displacement to the X and Y coordinates, respectively, of Joint 6 of the 2D cable net while maintaining the stress in the cables as close as possible to their initial levels N_0 .

Cable	N_0 (N)	[57]			Current study		
		N_a (N)	Error (%)	a (mm)	N_a (N)	Error (%)	a (mm)
i	61.4	63.9	4.1	−14.360	61.6	0.4	0.02
ii	61.4	59.7	2.7	11.130	61.6	0.3	−0.02
iii	23.6	18.4	21.8	9.520	23.5	0.0	−9.80
iv	17.0	13.7	19.4	11.800	17.0	0.1	0.16
v	17.0	18.3	7.7	−13.300	19.0	11.5	−27.28
vi	23.6	28.0	19.1	4.940	24.7	5.0	5.14
vii	50.0	51.0	2.1	−10.620	50.0	0.0	−2.34
viii	50.0	50.4	0.8	−16.910	50.0	0.0	−12.61
ix	50.0	46.9	6.1	27.560	45.2	9.7	28.26
Total actuation (mm)				120.1	85.6		

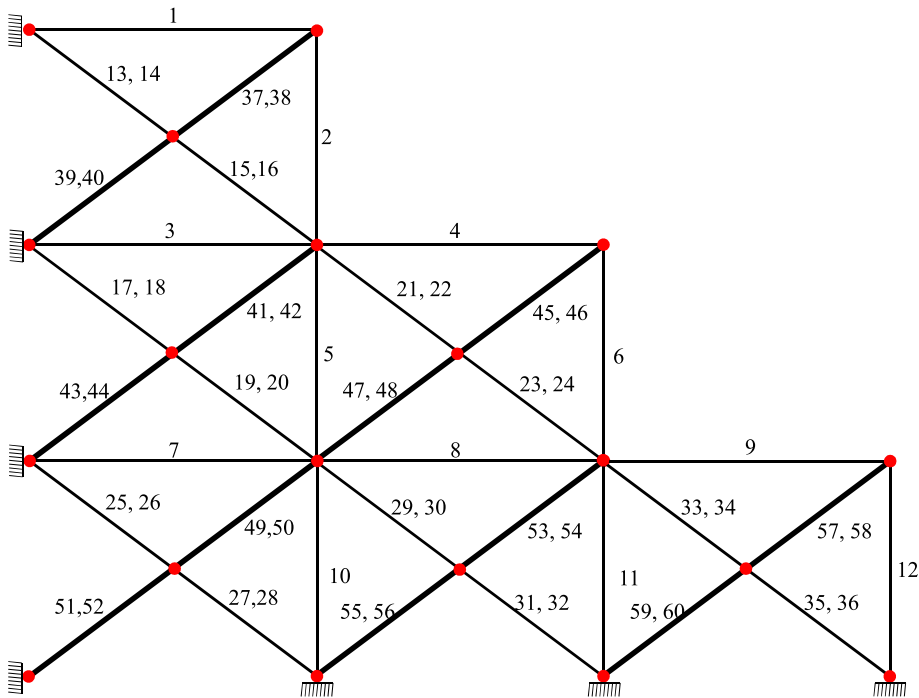


Fig. 5. Deployable space truss.

Table 8

Eliminating nodal displacements of Joint 6 and keeping the stress above the initial level of the 2D cable net with four actuators.

Cable	N_p (N)	[58]		Current study	
		N_a (N)	a (mm)	N_a (N)	a (mm)
i	61	120	0	125	0
ii	61	119	0	117	0
iii	23	23.6	0	23.6	0
iv	17	33.8	0	31.7	0
v	17	34.9	−9.2	43.2	−9.24
vi	23	26.6	−0.4	23.6	−0.42
vii	50	73.1	−4.3	73.6	−4.21
viii	50	57.9	0	58.0	0
ix	50	74.1	4.2	81.5	4.03
Total actuation (mm)			18.1		17.9

Table 9

Reduction of the computational cost by analytical gradient for simultaneous displacement and stress control of the 2D cable net.

Gradient	Iterations		Structural analyses	
	Numerical	Analytical	Numerical	Analytical
Control case 1	156	10	3017	60
Control case 2	23	5	221	11

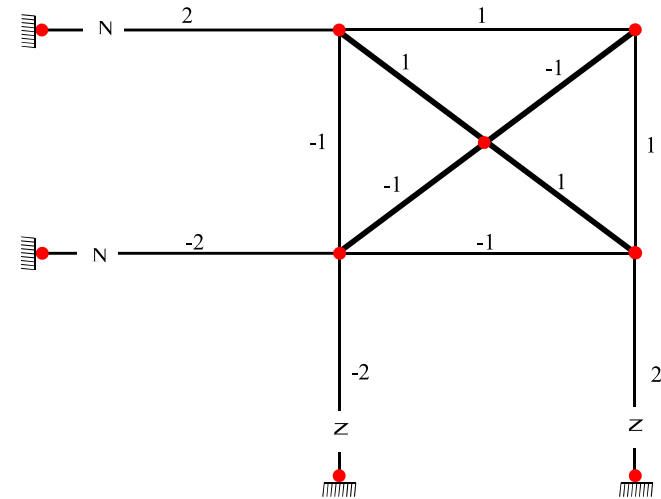


Fig. 6. Self stress state of a unit of the deployable space truss.

Table 10

Prestressing the deployable space truss.

Members	N_{target} (N)	[53]	[20]	Current
1	1	−0.1550	−0.1513	−0.1547
3	2	−0.0931	−0.0867	−0.0930
4	1	0.1238	0.1270	0.1236
7	2	−0.0415	−0.0336	−0.0414
8	2	0.0312	0.0391	0.0312
9	1	−0.0415	−0.0378	−0.0265
5,10,11	2	−	−	−
2,6,12–36	1	−	−	−
37–60	−1	−	−	−
Total actuation (mm)		0.4861	0.4755	0.4704

In this test, the closed-form differentiation of the structural response with respect to the actuation inputs notably reduces the number of structural analyses required to evaluate the nonlinear constraints, such as axial forces, as presented in Table 11. The table illustrates that employing the analytical gradient for both the objective and constraint functions reduces up to 85% in computational effort compared to using a numerical gradient. These findings indicate that the proposed

Table 11

Reduction of the computational cost by analytical gradient for stress control of the deployable space truss.

Gradient	Iterations		Structural analyses	
	Numerical	Analytical	Numerical	Analytical
Stress control	3	3	40	6

approach achieves a lower objective function value than previous methods and meets the target goals more computationally efficient and cost-effective manner.

4.5. Large plate-like truss

This final test involves a large plate-like statically indeterminate truss, depicted in Fig. 7. The plate is divided into 10 modules along each edge, each having a rectangular cuboid shape delimited by horizontal and vertical bars along the 12 edges, with additional diagonal bars in the faces. The members are made of steel with a modulus of elasticity of 210 MPa and an area of 500 mm². The structure comprises 1401 bars and 242 nodes, resulting in a total of 726 nodal displacement degrees of freedom (DOFs). All displacement components are restrained along the top external perimeter of the plate. Additionally, all nodes on the bottom surface are loaded with a vertical downward force of 3 kN. In its initial, unactuated configuration, the structure exhibits a maximum axial force of 112.75 kN. The optimization goal is to minimize the total actuation in the 28 bars highlighted in red in Fig. 7, subject to the following constraints: (i) the absolute value of actuation in each bar is limited to 1.5 mm; (ii) the axial force in each bar is limited to 80 kN in absolute value. These nonlinear constraints involve the axial forces of all 1401 structural elements. The optimal actuation, found by the algorithm, is symmetric (up to machine precision) with respect to the center of the plate and is depicted in Fig. 8. As expected, this actuation reduces the maximum axial force to 80 kN. Conducting the analysis with the same actuation but neglecting the geometric nonlinearity results in a maximum axial force of 102.98 kN, confirming the actual nonlinear behavior of this structure. For this test, the closed-form differentiation strategy allows us to obtain the optimum in just 21 s, compared to the 298 s required by default numerical differentiation. The ratio of the computational time is similar to the ratio of structural analyses (see Table 12), demonstrating how the cost of analytical differentiation is negligible for large structures compared to the structural analysis itself, thanks to the exploitation of the already factorized tangent stiffness matrix.

5. Final discussion, conclusions and outlook

This paper introduced a gradient-based optimization approach for designing actuation systems capable of achieving desired shape and stress control in geometrically nonlinear structures. The actuation is modeled as prescribed deformations in the actuated structural elements, with 3D trusses serving as examples in the application. In typical gradient-based nonlinear optimization, numerical gradient approximation is needed, which can significantly contribute to the overall computational cost. In contrast, this work presents an innovative approach by developing analytical differentiation of the nonlinear finite element solution, considering both displacement and stress with respect to the actuation input parameters. This approach allows for the computation of an exact and cost-effective gradient within the structural analysis. The proposal was presented in a general finite element setting, showing how straightforward its integration into commercial structural software is. Indeed, all ingredients needed by the present exact gradient evaluation strategy are already available in all finite element codes, so only minimal changes are required. The cost-effectiveness is achieved

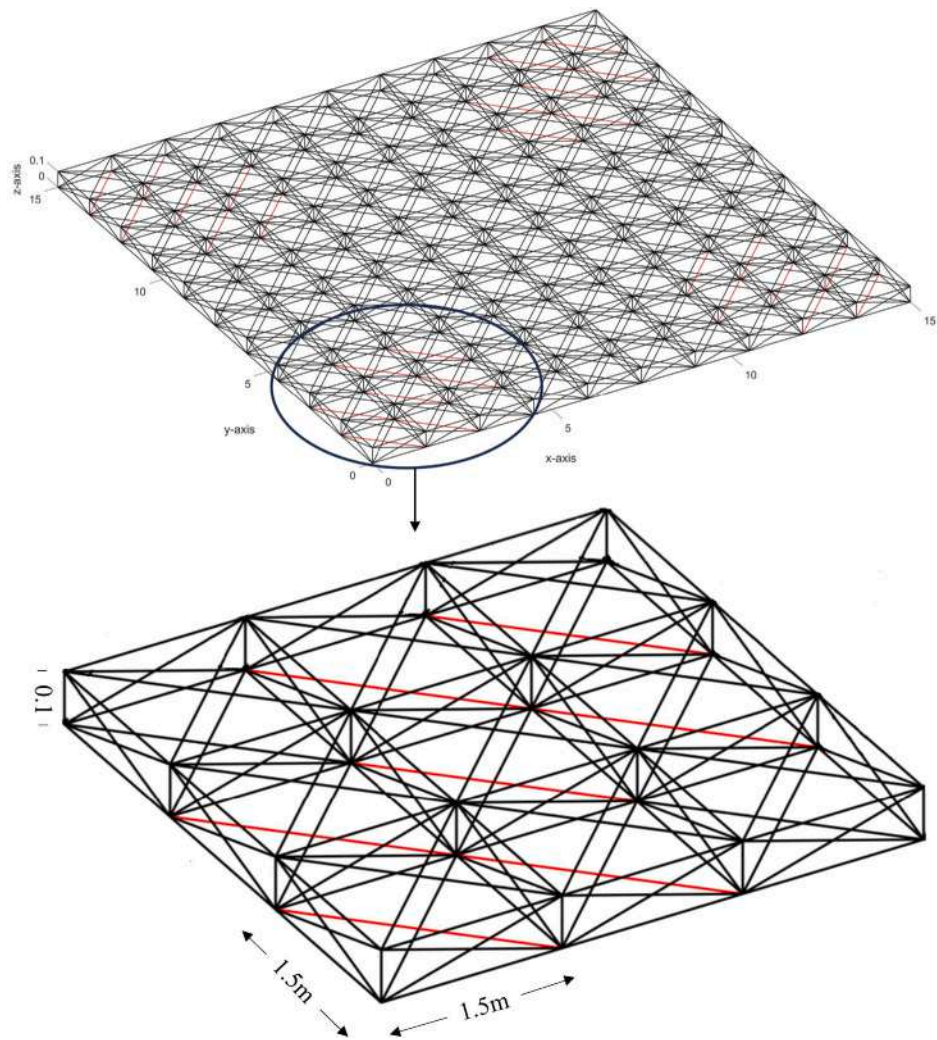


Fig. 7. Large plate-like statically indeterminate truss (lengths in m). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 12
Reduction of the computational cost by analytical gradient for stress control of the large plate-like structure.

Gradient	Iterations		Structural analyses		Computational time	
	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical
Stress control	4	4	149	9	298 s	21 s

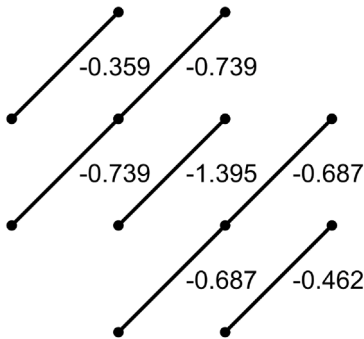


Fig. 8. Optimized actuation in the bars in mm. The solution is symmetric with the plate center located at the bottom left point of this picture.

through a gradient evaluation strategy that leverages the already factorized tangent stiffness matrix during Newton equilibrium iterations. This strategy alone significantly reduces the computational time. Furthermore, the accuracy of the gradient enhances the robustness of the optimization process, reducing, in some cases, the number of iterations required. It is interesting to note the final large structure case, for which the proposed methodology reduced the computational time by about 14 times. The current limitations of this work regard the structural type and the assumption of elastic material. A future development concerns validating the proposal for different structural types, such as beams and shells, for which the present gradient computation is already suitable thanks to the general finite element formalism. Moreover, it would be interesting to extend the exact algorithmic gradient evaluation to non-linear material, such as plasticity. This seems possible by considering the additional state variables and the loading history through a more involved implicit chain differentiation.

CRediT authorship contribution statement

Ahmed Manguri: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Domenico Magisano:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Robert Jankowski:** Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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Paper VII

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RESEARCH ARTICLE OPEN ACCESS

Gradient-Based Weight Minimization of Nonlinear Truss Structures With Displacement, Stress, and Stability Constraints

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ABSTRACT

This paper presents an effective and robust computational method of gradient-based methodology for weight minimization of geometrically nonlinear structures, considering 3D trusses as exemplary case study. The optimization framework can accommodate multiple different constraints: (i) bounds on the cross-sectional area of each design element, (ii) prescribed ranges for displacements and stresses, and (iii) nonlinear stability for geometries such as arches and domes. For large structures, this results in numerous optimization variables and constraints, including the highly nonlinear (ii) and (iii). Such constraints are evaluated consistently and simultaneously by combining path-following finite element analysis and null vector method. Typically, the gradient of the nonlinear structural response is approximated numerically, which is computationally intensive and can introduce inaccuracies deteriorating the optimization process. In contrast, this work derives a fully analytical gradient evaluation for nonlinear deformation, stress, and stability constraints. This is implemented directly within the finite element solver, enhancing robustness and computational efficiency of the optimization. Validation examples range from simple benchmarks to large structures.

1 | Introduction

Geometrically nonlinear structures undergo finite deformations, with their altered geometry greatly impacting their stiffness and strength. These structures are increasingly common in various engineering disciplines, including aerospace, civil, naval, and mechanical engineering.

For structures that support sensitive equipment for communication and scientific purposes, limiting deformations is critical [1, 2]. On the other hand, structures like cable bridges and

tensegrities focus more on regulating internal forces within their components rather than just managing deformations. This is especially important for configurations with cable elements, which can slacken under certain loads, necessitating adjustments to maintain structural integrity [3], and for truss and frame structures to avoid local buckling of the members. In real-world applications, adjusting one variable without affecting others is often challenging. Additionally, a common failure mode for slender structures is also global buckling. Linear eigenvalue analysis is typically used to evaluate the system stability, but it may not be suitable for some structures due to nonlinear pre-buckling

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deformations followed by a limit point, which can result from structural geometry or geometric imperfections. Examples of structures dominated by their geometry include arch-like and dome structures, which provide a completely free interior space without any internal columns and are widely used in arenas, exhibition pavilions, theaters, and terminals. In such cases, it is essential to prevent the possible snap phenomenon.

Over the past three decades, structural optimization has advanced rapidly, but mainly focusing on linear problems [4–6]. Much of the research in this area employs the finite element method (FEM) to evaluate structural responses and their sensitivities. One of the most intriguing problems in many engineering fields is weight minimization, where optimization variables typically include the cross-sectional areas of rods and the thicknesses of plate and shell elements. The typical approach to solving optimization problems involves an iterative process, where structural analysis equations are repeatedly solved until the algorithm achieves no further improvements. This approach can be highly resource-intensive, especially when handling a large number of variables and relying on FEM for structural analysis. The cost of each analysis depends on the type of analysis required to determine the desired responses. Nonlinear analyses are particularly demanding and can render optimization nearly impractical for large-scale structures. Currently, few studies on weight optimization frameworks of geometrically nonlinear structures are available in the literature and even fewer are able to coherently consider the three essential constraints in the design process: stability, deformation, and stress.

Optimization in structures or structural elements is generally applied to the topology, size or shape. Optimization algorithms can be categorized into gradient-based, gradient-free metaheuristic, and Machine learning methods. Gradient-based methods [7, 8], exploiting the gradient of the objective function and constraints, converges usually more rapidly [9] than the gradient-free ones. However, computing gradients, typically done through numerical differentiation, represents a bottleneck in nonlinear design optimization, as discussed in [10], which utilizes parallel computing to address this issue. A reduction in computational cost in large-scale optimization of linear trusses was reported by exploiting sparsity in the gradient and Hessian matrices [11]. A multi-start approach can be used to deal with multiple optima with gradient-based tools. Gradient-free metaheuristic methods, like the genetic algorithm, are also viable options. While they are more specifically designed for global optimization compared to gradient-based ones, they usually require significantly more function evaluations, especially when a closed-form gradient is accessible [12]. Furthermore, gradient-free methods usually perform very poorly as the dimensionality of the design space increases. A two-phase genetic algorithm approach for weight optimization is presented in [13] for reducing the computational time. Hybrid methods combining metaheuristic searches and gradient information were also proposed [14]. The studies based on Machine learning rely on the quality of selected training data, usually necessitating extra effort for model construction and validation [15, 16].

Regarding the optimization constraints, there are studies considering stress, displacement and buckling [17, 18], although many of them are based on linear structural analyses. Among the works

dealing with weight optimization with nonlinear stability constraint, it is worth noting the effort of some authors to select an explicit optimality criterion. For example, in [19], the criterion that the nonlinear strain energy density be equal in all the members at optimum is used to develop an optimization algorithm for nonlinear trusses. The method is subsequently implemented in other contributions, for example, in [20], where it is coupled with an arc-length sparse method. The weak point of the approach is that such a criterion is only heuristic, missing a rigorous theoretical proof. Also, displacement and stress constraints are not considered in the problem. An alternative displacement-based optimization is reported in [21], consisting of two nested problems: (i) cross-sectional areas are evaluated for a given displacement field so that the structural weight is minimized and (ii) optimal displacement field is searched in order to satisfy the constraints of the optimization problem. These includes equilibrium, displacement and stress, beside area bounds. Limit point instability is prevented as violation of the equilibrium constraint. However, this can make the method sensitive to the starting displacement field. Among recent proposals on Machine learning optimization, we can find [15, 16].

A rigorous approach concerning sizing or shape optimization of shells is presented in [22], where the nonlinear critical load computed by the null vector method [23–25], that is, solving iteratively the nonlinear equilibrium of the generic discretized structure along with the singularity of the tangent stiffness matrix, is maximized. The implicit differentiation is used to compute efficiently the objective function gradient, although numerical differentiation is still adopted for the residual equations. The approach does not consider buckle-free designs. The authors of [26] propose a recurrence strategy to take into account the worst imperfection shape in structural optimization of plates prone to buckling: first design optimization, then imperfection optimization, then design optimization with the previous imperfection, and so forth.

This research continues the line initiated in [27] dealing with actuation optimization for shape and stress control with exact gradients. The strategy in [27] is, however, not applicable to structures which may exhibit global instability and limit points in their equilibrium path, as common in arch- and dome-like structures.

This paper presents an efficient and robust gradient-based methodology for weight minimization in geometrically nonlinear elastic structures, exemplified through 3D trusses for clarity. The cross-section area along the bars is constant for each member, but they are the variables to be determined for each member. The optimization framework is able to accommodate coherently various constraints: (i) limits on the cross-sectional area of each design element, (ii) prescribed ranges for displacements and stresses, and (iii) nonlinear stability for geometries such as arches and domes. For large structures, this results in numerous optimization variables and constraints, including the highly nonlinear (ii) and (iii). Such constraints are evaluated combining path-following finite element analysis and null vector method. Typically, the gradient of the nonlinear structural response is approximated numerically, which is computationally intensive and can introduce inaccuracies deteriorating the optimization process. In contrast, this work derives a

fully analytical gradient evaluation for nonlinear deformation, stress, and stability constraints, enhancing robustness and efficiency of the optimal design. The consistent consideration of all these constraints on the final structural performance in the same numerical framework and the efficient fully analytical differentiation distinguishes the present work from previous proposals on weight minimization.

A unified optimization framework that consistently accounts for nonlinear deformations, stress constraints, and nonlinear stability conditions, even for non-monotonic load-displacement responses like limit points and snap-through, is obtained without modifying standard structural analysis routines. The gradients of these constraints are derived analytically and computed at no additional cost using the Jacobian of the structural Newton method. The literature review confirms that this has not been done before. Prior studies either considered deformation and stress constraints under predefined loads, unsuitable for limit points, or addressed stability constraints separately, either approximately [20] or exactly [22, 26] without integrating stress and deformation constraints. While [22] employs a similar approach, it still relies on numerical differentiation for certain terms. In contrast, this work achieves a fully analytical gradient computation.

The article is organized as follows: Section 2 describes the geometrically nonlinear problem with reference to a finite element model for 3D trusses and including the path-following solution and the nonlinear buckling condition. Section 3 formulates the nonlinear weight minimization problem with displacement, stress and stability constraints. Focus is given to differentiating such constraints coming from the nonlinear finite element solution with respect to the design parameters. Section 4 demonstrates the gain in robustness and efficiency achieved with the proposal compared to standard optimization based on numerical gradients. The reduction of computational time is also discussed for large structures. Conclusions are reported in the last section.

2 | 3D Nonlinear Truss Model: Equilibrium Path and Nonlinear Buckling

The optimization technique introduced in this study is applicable to general large deformation structural models with elastic materials discretized using the finite element method. The assumptions of the study are:

- quasi-static problem;
- finite kinematics with nonlinear strain-displacement relationship;
- equilibrium in the deformed configuration;
- hyperelastic materials.

To keep the explanation as easy as possible, 3D truss structures are considered. In this context, each finite element corresponds to a physical bar, simplifying the implementation and reproduction of numerical results. For the same reason, only global instability is considered, while local buckling in individual bars is neglected, although it can be controlled also using the stress constraint. The

model could be improved by directly accounting for local buckling and its interaction with global modes, using a beam finite element model with end releases instead of truss finite elements.

This section reviews the nonlinear discrete equations for a large deformation truss model based on the quadratic Green-Lagrange strain measure. The model is developed, as is customary, from a variational problem using a displacement-based approach. Consequently, the nonlinear equilibrium equations are derived, and a brief discussion of the incremental-iterative solution scheme is provided including also the nonlinear buckling condition. This discussion also serves to introduce the notation used in later sections.

2.1 | The Truss Element: Kinematics and Strain Measure

The spatial coordinates of the two endpoints (i and j) defining the generic bar are represented by $\mathbf{X}_k$ and $\mathbf{x}_k$ in the initial and deformed configuration respectively, with $k = i, j$, as illustrated in Figure 1. The kinematic behavior of each node can subsequently be described as follows:

$$\mathbf{x}_k = \mathbf{X}_k + \mathbf{d}_k$$

with $\mathbf{d}_k$ the displacement of the k th node. For each truss element (bar), it is useful to introduce the following vectors

$$\Delta \mathbf{X}_e \equiv \mathbf{X}_j - \mathbf{X}_i \quad \text{and} \quad \Delta \mathbf{x}_e \equiv \mathbf{x}_j - \mathbf{x}_i = \Delta \mathbf{X}_e + \mathbf{d}_j - \mathbf{d}_i$$

The initial L_e and deformed ℓ_e lengths of the bar are determined as follows:

$$L_e^2 = \Delta \mathbf{X}_e^T \Delta \mathbf{X}_e$$

$$\ell_e^2 = \Delta \mathbf{x}_e^T \Delta \mathbf{x}_e = L_e^2 + 2\Delta \mathbf{X}_e^T (\mathbf{d}_j - \mathbf{d}_i) + (\mathbf{d}_j - \mathbf{d}_i)^T (\mathbf{d}_j - \mathbf{d}_i)$$

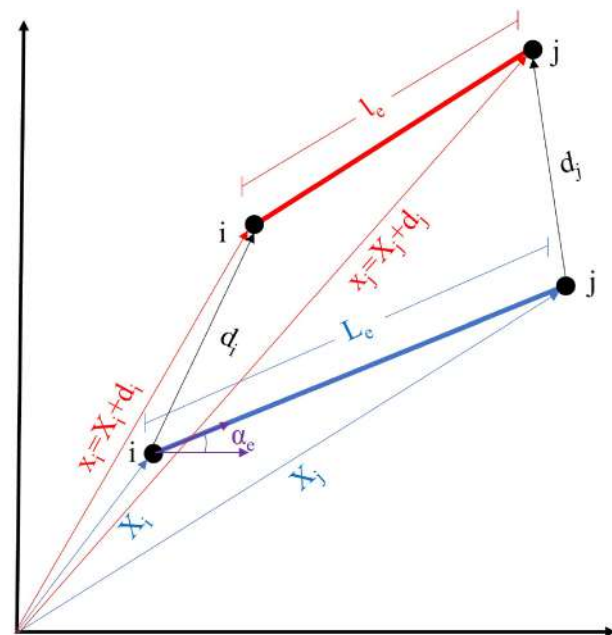


FIGURE 1 | Kinematics of the bar element.

The Green-Lagrange strain measure, quantifying the strain-displacement relationship, is expressed as follows:

$$\begin{aligned}\varepsilon_e(\mathbf{d}_e) &= \frac{1}{2} \left(\frac{\ell_e^2}{L_e^2} - 1 \right) = \frac{1}{L_e} \boldsymbol{\alpha}_e^T (\mathbf{d}_j - \mathbf{d}_i) \\ &+ \frac{1}{2L_e^2} (\mathbf{d}_j - \mathbf{d}_i)^T (\mathbf{d}_j - \mathbf{d}_i)\end{aligned}\quad (1)$$

Vector $\boldsymbol{\alpha}_e = \frac{\Delta \mathbf{X}_e}{L_e}$ contains the cosines directors of the bar with a generic orientation in the three-dimensional space. Employing standard finite element notation, the vector $\mathbf{d}_e$ collecting the 6 discrete displacement DOFs of the bar (nodal displacements) is introduced as follows:

$$\mathbf{d}_e = \begin{bmatrix} \mathbf{d}_i \\ \mathbf{d}_j \end{bmatrix}$$

The Green-Lagrange strain in Equation (1) can be expressed in compact form as

$$\varepsilon_e(\mathbf{d}_e) = \frac{1}{L_e} \left(\mathbf{b}_e^T \mathbf{d}_e + \frac{1}{2} \mathbf{d}_e^T \mathbf{G}_e \mathbf{d}_e \right) \quad (2)$$

where

$$\mathbf{b}_e = \begin{bmatrix} -\boldsymbol{\alpha}_e \\ \boldsymbol{\alpha}_e \end{bmatrix} \quad \text{and} \quad \mathbf{G}_e = \frac{1}{L_e} \begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix}$$

and $\mathbf{I}$ is the 3×3 identity matrix. Consequently, the axial stress in the bar is

$$\sigma_e(\mathbf{d}_e) = E \varepsilon_e(\mathbf{d}_e) \quad (3)$$

2.2 | Internal Force Vector, Equilibrium Condition and Stiffness Matrix

The total strain energy of the structure is calculated by summing the element contributions Φ_e :

$$\Phi = \sum_e \Phi_e \quad \text{where} \quad \Phi_e = \frac{1}{2} E A_e L_e \varepsilon_e \quad (4)$$

The strain energy gradient with respect to $\mathbf{d}_e$ furnishes the element internal force vector:

$$\mathbf{s}_e = \frac{\partial \Phi_e}{\partial \mathbf{d}_e} = \sigma_e(\mathbf{d}_e) \mathbf{q}_e A_e \quad \text{with} \quad \mathbf{q}_e = \mathbf{b}_e + \mathbf{G}_e \mathbf{d}_e \quad (5)$$

with $\sigma_e(\mathbf{d}_e) = E \varepsilon_e(\mathbf{d}_e)$. A further differentiation provides the element tangent stiffness matrix:

$$\mathbf{K}_e = \frac{\partial^2 \Phi_e}{\partial \mathbf{d}_e^2} = \left(\frac{E}{L_e} \mathbf{q}_e \mathbf{q}_e^T + \sigma_e \mathbf{G}_e A_e \right) \quad (6)$$

Letting $\mathbf{p}_k$ represent the forces acting at the k -th node of the structure, the external work is the sum of the nodal contributions:

$$\mathcal{L}_{ext} \equiv \sum_{k=1}^{N_n} \mathbf{p}_k^T \mathbf{d}_k = \mathbf{p}^T \mathbf{d}$$

where $\mathbf{d}$ is the global vector collecting all the nodal displacement DOFs of the structure. The total potential energy is computed as

the summation of strain energy across all elements augmented by the external work:

$$\Pi(\mathbf{d}) = \sum_e \Phi_e - \mathbf{p}^T \mathbf{d} \quad (7)$$

The stationary condition of Π gives the nonlinear discrete equilibrium equations of the structure:

$$\frac{\partial \Pi}{\partial \mathbf{d}} = \mathbf{s}(\mathbf{d}) - \mathbf{p} = \mathbf{0} \quad \text{with} \quad \mathbf{s}(\mathbf{d}) = \sum_e \mathbf{T}_e^T \mathbf{s}_e(\mathbf{d}_e) \quad (8)$$

where $\mathbf{T}_e$ is the matrix used for the global assembly of the element contributions, such that $\mathbf{d}_e = \mathbf{T}_e \mathbf{d}$. Analogously, the global tangent matrix is obtained as

$$\mathbf{K} = \sum_e \mathbf{T}_e^T \mathbf{K}_e \mathbf{T}_e$$

2.3 | Incremental-Iterative Path-Following Analysis

The equilibrium of slender elastic structures subject to conservative loads amplified by a proportionality factor λ is expressed by Equation (8)

$$\mathbf{r}(\mathbf{d}, \lambda) \equiv \mathbf{s}(\mathbf{d}) - \lambda \mathbf{p} = \mathbf{0} \quad (9)$$

Vector $\mathbf{r} : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$ is a nonlinear function of vector $(\mathbf{d}, \lambda)$, collecting the discrete $\mathbf{d} \in \mathbb{R}^n$ variables and the load multiplier $\lambda \in \mathbb{R}$. $\mathbf{s}(\mathbf{d})$ is the global internal force vector and $\mathbf{p}$ the reference load vector. Equation (9) is a system of nonlinear equations defining the equilibrium path as a load factor-displacement curve in $\mathbb{R}^{n+1}$. An incremental-iterative solution scheme is generally needed to trace this curve step-by-step from a known initial point corresponding to $\lambda = 0$. The condition $\lambda = 1$ represents the applied reference load. There are two main reasons for introducing the load factor λ : (i) the equilibrium curve can exhibit a limit point in λ , meaning that there may be no close equilibrated solution above a certain load level; (ii) even if an equilibrium point exists for all λ in the range of interest, it is not assured that we are actually able to compute it directly for an assigned load because of the local convergence of the Newton iterative solution. The Riks arc-length method [28] with multiple steps is a general solution method for the equilibrium curve evaluation, allowing also the analysis of structures with limit and turning points. An incremental solution scheme is applied by defining a sequence of points (steps) $\mathbf{z}^{(k)} \equiv (\mathbf{d}^{(k)}, \lambda^{(k)})$ belonging to the equilibrium curve. Starting from a known equilibrium point $\mathbf{z}_0 \equiv \mathbf{z}^{(k)}$, the next one $\mathbf{z} \equiv \mathbf{z}^{(k+1)}$ is the solution of the extended system

$$\begin{cases} \mathbf{r}(\mathbf{d}, \lambda) \equiv \mathbf{s}(\mathbf{d}) - \lambda \mathbf{p} = \mathbf{0} \\ c(\mathbf{d}, \lambda) \equiv \Delta \mathbf{d}_0^T \Delta \mathbf{d} + \Delta \lambda_0^T \Delta \lambda - \Delta s^2 \end{cases} \quad (10)$$

where $c = 0$ imposes that the estimated increment of arc-length of each step of the incremental analysis is equal to the desired size of the increment Δs , generally changed adaptively [29]. In Equation (10), $\Delta \mathbf{d} = \mathbf{d} - \mathbf{d}_0$ and $\Delta \lambda = \lambda - \lambda_0$ represent the current increments of displacements and load factor with respect to previously evaluated equilibrium point, while $\Delta \mathbf{d}_0$ and $\Delta \lambda_0$ are

the initial estimates of the same quantities along the path tangent at each step. The nonlinear system (10) is solved iteratively using a Newton scheme.

$$\begin{cases} \mathbf{d}_{j+1} = \mathbf{d}_j + \dot{\mathbf{d}} \\ \lambda_{j+1} = \lambda_j + \dot{\lambda} \end{cases} \quad (11)$$

where the subscript denotes the iteration number, while $\dot{\mathbf{d}}$ and $\dot{\lambda}$ are the corrections to displacements and load factors obtained from the linearization of (10):

$$\begin{cases} \mathbf{K}(\mathbf{d}_j)\dot{\mathbf{d}} - \dot{\lambda}\mathbf{p} = -\mathbf{r}_j \\ \Delta\mathbf{d}_0^T\dot{\mathbf{d}} + \Delta\lambda_0\dot{\lambda} = -c_j \end{cases} \Rightarrow \begin{cases} \dot{\mathbf{d}} = \dot{\lambda}\mathbf{K}_j^{-1}\mathbf{p} - \mathbf{K}_j^{-1}\mathbf{r}_j \\ \dot{\lambda} = \frac{\Delta\mathbf{d}_0^T\mathbf{K}_j^{-1}\mathbf{r}_j - c_j}{\Delta\mathbf{d}_0^T\mathbf{K}_j^{-1}\mathbf{p} + \Delta\lambda_0} \end{cases} \quad (12)$$

The linear system resulting from each iteration (left-hand side of Equation (12)) is conveniently solved in block form to exploit the efficient factorization of the symmetric tangent stiffness matrix, as shown in the right-hand side of Equation (12). First, $\dot{\mathbf{d}}$ is explicitly expressed as a function of $\dot{\lambda}$ using the linearized equilibrium equations. This expression is then substituted into the linearized arc-length constraint, allowing the computation of $\dot{\lambda}$ and, by back-substitution, $\dot{\mathbf{d}}$.

If the displacement solution for an exact load factor $\bar{\lambda}$ is desired, one can switch to a load-controlled scheme when $\lambda^{(k)}$ is sufficiently close to $\bar{\lambda}$. In this case, the Newton scheme simplifies as

$$\mathbf{d}_{j+1} = \mathbf{d}_j + \dot{\mathbf{d}} \quad \text{with} \quad \dot{\mathbf{d}} = -\mathbf{K}(\mathbf{d}_j)^{-1}\mathbf{r}(\mathbf{d}_j, \bar{\lambda}) \quad (13)$$

which corresponds to redefine the constraint as $c(\mathbf{d}, \lambda) \equiv \lambda^{(k+1)} - \bar{\lambda}$.

At each Newton iteration within each load increment, the global tangent stiffness matrix is computed by assembling the element matrices as described in Equation (6). This matrix is then factorized to solve the linear systems, a process typically constituting the most computationally intensive part when analyzing large structures. The Newton iteration terminates when the norm of the residuals falls below a predefined threshold.

The convergence of the Newton scheme is local, that is, it converges when a good initial prediction of the unknowns (the displacement DOFs) is available. Clearly, this is not guaranteed for large load steps. Thus, the stepping procedure is useful for problems with a significant nonlinearity. Reaching the maximum load level in multiple steps allows one to take advantage of the previous equilibrium points in estimating the new one. A typical choice is to extrapolate linearly the two previous points for estimating the newly searched one. The number of load steps can be selected by the user, but can be adjusted automatically following, for example, the adaptive stepping reported in [29]. Notably, in the case of an elastic structure subjected to a conservative load, the final solution is independent of the stepping scheme. Additionally, a mixed solution scheme that incorporates local stress variables can enhance the robustness of the Newton method and reduce the number of iterations [29, 30]. A Koiter-Newton method can be also considered to reduce the computational time [31].

2.4 | Nonlinear Buckling Problem

The exact critical condition along the equilibrium path can be defined by an extended system

$$\begin{cases} \mathbf{r} \equiv \mathbf{s}(\mathbf{d}) - \lambda\mathbf{p} = \mathbf{0} \\ \mathbf{g} \equiv \mathbf{K}(\mathbf{d})\mathbf{v} = \mathbf{0} \\ n \equiv \mathbf{v}^T\mathbf{v} - 1 = 0 \end{cases} \quad (14)$$

imposing equilibrium, singularity of the tangent stiffness matrix and norm of the eigenvector.

Letting $\mathbf{q} = \begin{bmatrix} \mathbf{d} \\ \mathbf{v} \\ \lambda \end{bmatrix}$ and the residual as $\mathbf{R} = \begin{bmatrix} \mathbf{r} \\ \mathbf{g} \\ n \end{bmatrix}$, the Newton iterative solution becomes

$$\begin{cases} \mathbf{q} = \mathbf{q}_j + \dot{\mathbf{q}} \\ \mathbf{H}_j\dot{\mathbf{q}} = -\mathbf{R}_j \end{cases} \quad \text{with} \quad \mathbf{H} \equiv \frac{\partial \mathbf{R}}{\partial \mathbf{q}} = \begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{p} \\ \mathbf{\Gamma} & \mathbf{K} & \mathbf{0} \\ \mathbf{0} & 2\mathbf{v}^T & \mathbf{0} \end{bmatrix} \quad (15)$$

This means that the corrections $\dot{\mathbf{d}}$, $\dot{\mathbf{v}}$ and $\dot{\lambda}$ at each Newton iteration are obtained as solution of the following linear system:

$$\begin{bmatrix} \mathbf{K} & \mathbf{0} & -\mathbf{p} \\ \mathbf{\Gamma} & \mathbf{K} & \mathbf{0} \\ \mathbf{0} & 2\mathbf{v}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{v}} \\ \dot{\lambda} \end{bmatrix} = - \begin{bmatrix} \mathbf{r}_j \\ \mathbf{g}_j \\ n_j \end{bmatrix} \quad (16)$$

Matrix $\mathbf{\Gamma} \equiv \frac{\partial(\mathbf{K}(\mathbf{d})\mathbf{v})}{\partial \mathbf{d}}$ can be evaluated analytically as assemblage of element contributions $\mathbf{\Gamma}_e$

$$\mathbf{\Gamma}_e = \frac{EA}{L_e} (\mathbf{G}_e \mathbf{v}_e \mathbf{d}_e^T + \mathbf{d}_e^T \mathbf{v}_e \mathbf{G}_e + \mathbf{d}_e \mathbf{v}_e^T \mathbf{G}_e)$$

where $\mathbf{v}_e$ is the element vector extracted from vector $\mathbf{v}$.

The iterative scheme, in explicit form, then become

$$\begin{cases} \mathbf{K}\dot{\mathbf{d}} - \dot{\lambda}\mathbf{p} = -\mathbf{r}_j \\ \mathbf{\Gamma}\dot{\mathbf{d}} + \mathbf{K}\dot{\mathbf{v}} = -\mathbf{g}_j \\ 2\mathbf{v}^T\dot{\mathbf{v}} = -n_j \end{cases} \Rightarrow \begin{cases} \dot{\mathbf{d}} = \dot{\lambda} \mathbf{K}^{-1}\mathbf{p} - \mathbf{K}^{-1}\mathbf{r}_j = \dot{\lambda}\dot{\mathbf{d}} + \bar{\mathbf{d}} \\ \dot{\mathbf{v}} = -\dot{\lambda} \mathbf{K}^{-1}\mathbf{\Gamma}\dot{\mathbf{d}} - \mathbf{K}^{-1}(\mathbf{g}_j + \mathbf{\Gamma}\bar{\mathbf{d}}) = \bar{\mathbf{v}} - \dot{\lambda}\dot{\mathbf{v}} \\ \dot{\lambda} = \frac{n_j/2 + \mathbf{v}^T\bar{\mathbf{v}}}{\mathbf{v}^T\dot{\mathbf{v}}} \end{cases} \quad (17)$$

where vectors $\mathbf{\Gamma}\dot{\mathbf{d}}$ and $\mathbf{\Gamma}\bar{\mathbf{d}}$ can be conveniently evaluated by assembling their element contributions to avoid the global matrix-vector products. In detail, $\dot{\mathbf{d}}$ is explicitly expressed in terms of $\dot{\lambda}$ by evaluating the vectors $\dot{\mathbf{d}}$ and $\bar{\mathbf{d}}$. Substituting $\dot{\mathbf{d}}$ into the linearized critical condition allows expressing $\dot{\mathbf{v}}$ in terms of $\dot{\lambda}$ through $\bar{\mathbf{v}}$ and $\dot{\mathbf{v}}$. The linearized arc-length equation then provides $\dot{\lambda}$, which, by back-substitution, yields $\dot{\mathbf{v}}$ and $\dot{\mathbf{d}}$.

It is worth noting that, although the tangent stiffness matrix tends to be singular approaching the critical point, all the linear systems in (17) are solvable. However, to avoid numerical inaccuracies, it is preferable to avoid the exact singularity. To this aim, there are very valid proposals in the literature [22, 23] introducing a modification of the nonlinear buckling equations (14). Conversely, in this study, we adopt a different strategy:

1. At each iteration (15), the tangent stiffness matrix is factorized at $0.999\mathbf{d}$, with $\mathbf{d}$ the current displacement DOFs estimate.
2. All the linear systems in (17) are solved iteratively by a modified Newton with this constant iteration matrix, which on average provides the desired solution in 2–3 iterations. Due to the constant factorized matrix, a negligible additional cost is required compared to a direct solution.

3 | Weight Minimization: Problem Statement and Gradient Evaluation

This section is dedicated to formulating the optimization problem for displacement, stress and stability control in the context of geometrically nonlinear problems, along with its solution strategy. The emphasis is placed on differentiating the nonlinear finite element solution with respect to the design parameters to achieve an efficient and robust gradient-based optimization. For weight optimization, the areas of the structural members are typically chosen as optimization variables. We select the Sequential Quadratic Programming as the optimization algorithm, but the present framework remains valid for all the other gradient-based methods.

3.1 | Problem Statement

The optimization of a geometrically nonlinear structure with displacement and stress control can be formulated as the minimization of the total weight subject to the following constraints:

- upper and lower bound for the areas A_e in each structural element;
- upper and lower bound for the displacement of controlled nodes (all displacement DOFs or a subset of them) coming from the nonlinear finite element model with the prescribed areas;
- upper and lower bound for the stress in controlled elements (all structural elements or a subset of them) coming from the nonlinear finite element model with the prescribed areas;
- nonlinear stability condition.

The optimization problem can be stated as the follows:

$$\begin{aligned}
 &\text{minimize} && W(\mathbf{A}) = \sum_{e \in \mathcal{U}'} W_e \equiv \rho A_e L_e \\
 &\text{subject to} && A_e^{(\min)} \leq A_e \leq A_e^{(\max)} && e \in \mathcal{U}' \\
 & && \sigma_i^{(\min)} \leq \sigma_i(\mathbf{A}) \leq \sigma_i^{(\max)} && i \in \mathcal{I} \\
 & && d_k^{(\min)} \leq d_k(\mathbf{A}) \leq d_k^{(\max)} && k \in \mathcal{K} \\
 & && \lambda_c(\mathbf{A}) \geq 1 && (18)
 \end{aligned}$$

where $\mathbf{A}$ collects the areas A_e of all the structural elements, $\mathbf{d}(\mathbf{A})$ and $\sigma_i(\mathbf{A})$ are implicitly defined as those which verify the nonlinear structural equilibrium equations in Equation (8) for a given $\mathbf{A}$. Note that in the objective function the areas are weighted by the initial lengths L_i , which however remains constant in the design

problem. Subsets $\mathcal{U}'$, $\mathcal{I}$ and $\mathcal{K}$ denote, respectively, the design elements, the elements where we want to control the axial stress σ_i and the controlled nodal displacement components d_k chosen among those of the global DOFs vector $\mathbf{d}$. In case of structures prone to instabilities, such as arches and domes characterized by a limit point and snap-through as failure mode, a stability constraint is included in the optimization. $\lambda_c(\mathbf{A})$ is the nonlinear critical load factor of the structure, defined as the lowest load factor satisfying the nonlinear equations (14) for the assigned areas. The nonlinear buckling condition is considered instead of a linearized one, because for structures like arches and domes buckling can occur after finite displacements (nonlinear pre-buckling path). Due to the local convergence of the Newton method, the iterative solution (Equations (15)) of the nonlinear critical condition along the equilibrium path is possible only with a good initial estimate, otherwise the iteration can diverge or converge to a secondary critical load. To achieve this, the following algorithm can be utilized for computing $\lambda_c(\mathbf{A})$:

1. A Riks analysis is run from the unloaded configuration. At each step, the LDL-factorization of the tangent stiffness matrix is adopted to solve the linear systems of the structural Newton iterations. At each converged equilibrium point, the smallest coefficient of the diagonal matrix $\mathbf{D}$ is monitored.
2. When the smallest coefficient of $\mathbf{D}$ at a converged point becomes negative, the extended nonlinear system with the critical condition (Equation (14)) is solved with the Newton scheme using as initial estimates displacement DOFs $\mathbf{d}$ and load factor λ of the previous equilibrium point, while the singularity direction $\mathbf{v}$ is initialized as the eigenvector of the tangent stiffness matrix associated to the smallest eigenvalue.

The number of optimization variables is the size of vector $\mathbf{A}$ corresponding to the cardinality of $\mathcal{U}'$. This is also the number of linear constraints. Instead, the number of nonlinear constraints depends on the number of elements and nodal displacement components to control, that is, the sum of the cardinality of $\mathcal{I}$ and $\mathcal{K}$.

It is worth noting that, for generic statically indeterminate structures, even varying the area in one element produces different displacements and axial forces in all the structural elements.

3.2 | Optimization Algorithm

The optimization problem (Equation (18)) is addressed using the Sequential Quadratic Programming (SQP) method, implemented in MATLAB's *fmincon* function with the SQP algorithm option. This method is a state-of-the-art iterative technique for constrained nonlinear optimization and is akin to a quasi-Newton method. The core concept of SQP involves approximating the optimization problem (Equation (18)) at each iteration with a local quadratic model of the objective function, constrained by a linearization of the original constraints, resulting in a simpler Quadratic Programming problem. A critical requirement of this method is an accurate gradient of the Lagrangian function, which encompasses the gradients of both the objective function and the constraints. A positive definite quasi-Newton approximation

of the Hessian matrix of the Lagrangian function is also typically used.

In our specific optimization scenario, deriving the analytical gradient of the objective function is relatively simple. However, determining the gradient of the nonlinear constraints is more complex, as it requires differentiating the nonlinear finite element solution in terms of stress and displacement relative to the actuation inputs. Normally, these derivatives are approximated through numerical differentiation methods, such as forward (the default in *fmincon*) or central difference techniques. This process demands numerous nonlinear structural analyses, generally requiring at least one analysis per optimization variable. For systems with many degrees of freedom, this step becomes a major bottleneck, due primarily to the repeated calculation and factorization of the tangent stiffness matrix during the Newton iterations used to maintain structural equilibrium. Additionally, a careful balance must be struck with finite differences: the increment must be small enough to accurately approximate the derivatives, yet not so small as to introduce round-off errors. Furthermore, because the nonlinear equilibrium equations are solved iteratively, the accuracy of numerical differentiation can be influenced by the convergence criteria of the iterative solver. Consequently, any numerical approximation in gradient evaluation can make the optimization process less robust, potentially deteriorating its convergence speed.

The following subsection presents a more efficient approach: a closed-form differentiation of both displacement and stress at a specific equilibrium point of the nonlinear finite element model with respect to the design variables. Additionally, it includes an analytical differentiation of the nonlinear critical load. These advancements greatly enhance the speed and efficiency of the optimization process.

3.3 | Gradient of the Nonlinear Displacement and Stress With Respect to the Areas

The first objective is to determine the gradient of the global displacement vector $\mathbf{d}(\mathbf{A})$, encompassing the displacement components of all structural nodes at the equilibrium state corresponding to prescribed final loads and cross-sectional areas. This gradient is calculated with respect to the design variables, represented by the cross-sectional areas of the bars in vector $\mathbf{A}$. The resulting derivatives, denoted as $\mathbf{d}_{,\mathbf{A}}$, are instrumental in obtaining $\sigma_{i,\mathbf{A}}$, representing the derivatives of the final axial stress in each controlled element i .

The challenge of performing an analytical differentiation lies in the absence of explicit expressions for $\mathbf{d}(\mathbf{A})$ and $\sigma(\mathbf{A})$, identified are the values of $\mathbf{d}$ and σ satisfying the nonlinear equilibrium equations (9) for the applied load ($\lambda = 1$) and the assigned areas $\mathbf{A}$. Thus, $\mathbf{d}(\mathbf{A})$ and $\sigma(\mathbf{A})$ are implicit functions of $\mathbf{A}$ through the nonlinear equilibrium condition.

We introduced a method for calculating gradients in closed form based on implicit differentiation, a calculus approach that applies the chain rule to implicitly defined functions. We can, therefore, differentiate the residual equilibrium equations as follows

$$\frac{\partial \mathbf{r}(\mathbf{d}(\mathbf{A}), \mathbf{A})}{\partial \mathbf{A}} = \underbrace{\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{d}}}_{\mathbf{K}(\mathbf{d}(\mathbf{A}))} \mathbf{d}_{,\mathbf{A}} + \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})} = \mathbf{0} \quad (19)$$

In Equation (19), $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{d}} \bigg|_{\mathbf{d}(\mathbf{A})}$ is the Jacobian of the internal force vector with respect to the displacements maintaining the areas constant and evaluated at the displacements $\mathbf{d}(\mathbf{A})$ of the final equilibrium point corresponding to the current areas, that is, the tangent stiffness matrix at the final iteration of the structural equilibrium Newton solver. Instead, $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})}$ collects the partial derivatives of the internal forces at the final equilibrium point with respect to the areas, maintaining $\mathbf{d}$ constant in the differentiation, evaluated at $\mathbf{d}(\mathbf{A})$. This last ingredient can be computed by assembling the contribution of each finite element to the global $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}}$ for fixed displacements:

$$\frac{\partial \mathbf{s}_e(\mathbf{d}_e, A_e)}{\partial A_e} = \sigma_e(\mathbf{d}_e) \mathbf{q}_e \quad (20)$$

Equation (19) provides a direct and closed-form method for computing $\mathbf{d}_{,\mathbf{A}}$ by solving the linear system

$$\mathbf{K}(\mathbf{d}(\mathbf{A})) \mathbf{d}_{,\mathbf{A}} = - \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})} \quad (21)$$

This approach offers significant computational advantages:

- the calculation of vector $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}}$ has fewer cost than evaluating the internal force vector due to the availability of the equilibrium operator $\mathbf{q}_e$ and the stress value at the converged equilibrium point, which were previously utilized for structural Newton scheme convergence assessment.
- the solution of a linear system in Equation (21) can leverage the factorization of the tangent stiffness matrix $\mathbf{K}(\mathbf{d}(\mathbf{A}))$ already available at the last iteration of the structural Newton solution.

In contrast to the computationally expensive numerical gradient evaluation, which necessitates multiple nonlinear structural analyses, the proposed approach yields an exact algorithmic gradient at negligible cost.

It is worth noting that the change of area in one element generally affects the displacements in all structural nodes due to the coupling in the tangent stiffness matrix.

Subsequent to determining the gradient of displacements, the gradient of the axial stress σ_i in the generic controlled bar i can be then evaluated by the chain rule as

$$\sigma_{i,\mathbf{A}} = \underbrace{\left(\frac{\sigma_i(\mathbf{d}_i, A_i)}{\partial \mathbf{d}_i} \bigg|_{\mathbf{d}_i(\mathbf{A})} \right)^T}_{E \mathbf{q}_i(\mathbf{d}_i)^T} \mathbf{d}_{i,\mathbf{A}} + \underbrace{\frac{\partial \sigma_i(\mathbf{d}_i, A_i)}{\partial A_i} \bigg|_{\mathbf{d}_i(\mathbf{A})}}_{=0} = E \mathbf{q}_i(\mathbf{d}_i)^T \mathbf{d}_{i,\mathbf{A}} \quad (22)$$

where $\mathbf{d}_{i,\mathbf{A}}$ is the element version of $\mathbf{d}_{,\mathbf{A}}$ obtained by extracting the values in the positions corresponding to the nodal DOFs of the generic controlled element i . Focusing on Equation (22), it can be observed that, when the displacements are kept constant, the axial stress in the member does not depend on the areas according

to Equation (3), which explains why $\frac{\partial \sigma_i(\mathbf{d}_i, \mathbf{A})}{\partial \mathbf{A}} = \mathbf{0}$. The fact that varying the area of one element can affect the displacements and, consequently, the stress in all other elements is accounted for by the term involving the displacement gradient $\mathbf{d}_{i,A}$ according to the chain rule.

3.4 | Gradient of the Nonlinear Stability Constraint

The same methodology used for the gradient of displacement and stress constraints can be adopted to compute in closed-form the derivatives of the critical load factor with respect to the areas. To this aim, we exploit the implicit differentiation of the nonlinear extended system (14) (equilibrium, critical condition and eigenvector norm) assuming the all equations are verified:

$$\begin{cases} \mathbf{s}(\mathbf{d}, \mathbf{A}) - \lambda_c \mathbf{p} = \mathbf{0} \\ \mathbf{K}(\mathbf{d}, \mathbf{A}) \mathbf{v} = \mathbf{0} \\ \mathbf{v}(\mathbf{A})^T \mathbf{v}(\mathbf{A}) = 1 \end{cases} \Rightarrow \begin{cases} \mathbf{K}(\mathbf{d}(\mathbf{A})) \mathbf{d}_{,A} = \mathbf{p} \lambda_{c,A}^T - \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})} \\ \mathbf{K}(\mathbf{d}(\mathbf{A})) \mathbf{v}_{,A} = - \frac{\partial \mathbf{K}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})} \mathbf{v} - \Gamma \mathbf{d}_{,A} \\ \mathbf{v}_{,A}^T \mathbf{v} = \mathbf{0} \end{cases} \quad (23)$$

Solving in block form, we obtain the gradient $\lambda_{c,A}$

$$\begin{cases} \mathbf{d}_{,A} = \underbrace{\mathbf{K}^{-1} \mathbf{p}}_{\hat{\mathbf{d}}} \underbrace{\lambda_{c,A}^T - \mathbf{K}^{-1} \frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})}}_{\bar{\mathbf{d}}} = \hat{\mathbf{d}} \lambda_{c,A}^T + \bar{\mathbf{d}} \\ \mathbf{v}_{,A} = \underbrace{-\mathbf{K}^{-1} \left(\frac{\partial \mathbf{K}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})} \mathbf{v} + \Gamma \bar{\mathbf{d}} \right)}_{\bar{\mathbf{v}}} - \underbrace{\mathbf{K}^{-1} \Gamma \hat{\mathbf{d}} \lambda_{c,A}^T}_{\hat{\mathbf{v}}} \\ = \bar{\mathbf{v}} + \hat{\mathbf{v}} \lambda_{c,A}^T \\ \lambda_{,A} = \frac{\bar{\mathbf{v}}^T \mathbf{v}}{\hat{\mathbf{v}}^T \mathbf{v}} \end{cases} \quad (24)$$

The same solution strategy discussed at the end of Section 2.4 is exploited to avoid the issue of the singular stiffness matrix.

The evaluation of the term $\frac{\partial \mathbf{s}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})}$ by element contributions was already discussed in Equation (20). Instead, the additional term $\frac{\partial \mathbf{K}(\mathbf{d}, \mathbf{A})}{\partial \mathbf{A}} \bigg|_{\mathbf{d}(\mathbf{A})}$ can be also computed as contribution of the generic element i for each component A_e of $\mathbf{A}$, which is non-null only if $i = e$, that is

$$\frac{\partial \mathbf{K}_e(\mathbf{d}_e, A_e)}{\partial A_e} = \frac{E}{L_e} \mathbf{q}_e \mathbf{q}_e^T + \sigma_e \mathbf{G}_e \quad (25)$$

4 | Results and Discussion

Seven benchmark examples are tested to validate the robustness and computational efficiency of the present study on structural weight minimization. These examples are divided into three categories: three examples for nonlinear stability constraint, three for displacement constraints, one with simultaneous displacement and nonlinear stability constraints and one for a large structure with displacement and stress constraints simultaneously. Importantly, all optimizations are carried out considering the full geometrically nonlinear structural response. All computations were performed using MATLAB R2024a on a machine with an Intel

Core i7-10610U CPU@1.8 GHz (8 CPUs) up to 2.3 GHz and 16 GB of RAM. The results obtained from the proposed optimization framework, along with its performance, are compared with those reported in several literature contributions. Regarding the latter, the comparison includes the number of structural analyses as objective data, and computational time, but only for recent proposals. For completeness and future reference, we present a comprehensive set of results. The abbreviation NA (not available) is used when some results are missing in previous works.

4.1 | Stability Constraint

This section's structures are prone to snap-through instability and are designed to achieve optimal weight just before instability occurs. Three benchmark examples are examined to validate the approach's robustness and the results regarding total weight and computational optimization are compared with existing literature studies.

4.1.1 | Two-Leg Truss

The two-legged structure illustrated in Figure 2 represents the most basic form of geometrically nonlinear behavior, culminating in a snap-through phenomenon under quasi-static loading conditions. Snap-through occurs when the structure loses stability due to a mechanical mechanism triggered by the applied load. A Young modulus of 8.0885e6 psi (55.7682 GPa) is adopted. The aim is to find the optimal weight of the structure just before instability. Missoum et al. [21] studied the structure; Table 1 tabulates the results obtained from their and current studies. The table shows that the optimal area of their study was 0.79 in² (5.0968 cm²) obtained in five iterations. In comparison, the current study gives 0.7818 in² (5.0439 cm²) as the optimal area in two iterations. The equilibrium path of the structure for the applied load is shown in Figure 3.

4.1.2 | Four-Bar Truss

In this example, an asymmetric four-bar truss shown in Figure 4 is tested for weight optimization with an stability constraint. Hrinda et al. [20] previously explored this area with a load of 200 lbs (889.644 N) and material properties characterized by $E = 1e7$ psi (68.947 GPa) and a density of 0.1 lb/in³ (0.002768 kg/cm³). The results were compared with those obtained by [20]. Table 2 showcases the outcomes, revealing that both methods converged

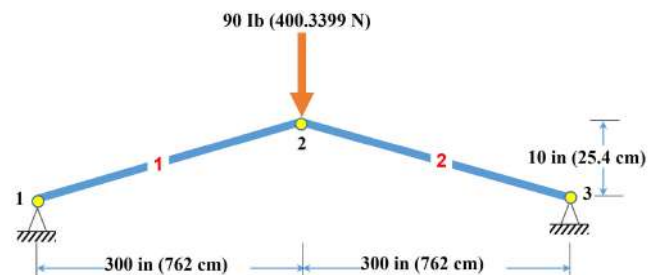


FIGURE 2 | The two-legged symmetrical truss.

to an optimal solution of 115.06 lbs (52.19 kg). Despite this similarity, there were discrepancies in the element areas employed by each method. The current approach reached the optimal solution with 20 structural analyses using analytical gradients, compared to 48 analyses with finite differences, completing the process in 1 s. These findings underscore the efficiency and efficacy of the proposed methodology for weight optimization in structural design. In Figure 5, the equilibrium path of the four-bar truss before, during, and after snap-through is illustrated. It can be seen from the figure that the snap-through occurs when the vertical downward movement of Node 5 is 1.276 in (3.241 cm) for the design area found in this study.

4.1.3 | Thirty-Bar Dome

This example investigates weight optimization with stability constraint in the context of the thirty-bar truss shown in Figure 6, which Khot et al. [19] tested for the first time. The dome members are divided into four groups of design variables. The truss incorporates modules of elasticity and material density set at 1e7 psi

(68.947 GPa) and 0.1 lb/in³ (0.002768 kg/cm³), respectively. The upper and lower bounds of the design variables are constrained within the range of 0.1 in² (0.6452 cm²) and 2 in² (12.903 cm²). The goal is to ascertain the minimum achievable weight of the truss before snap-through phenomena manifest under the designated design downward load of 2000 lbs (8896.4 N) applied in the Z-direction on Joint 1. This examination contributes to the broader understanding of structural optimization methodologies, particularly in scenarios where instability poses a critical design consideration. Table 3 shows the optimal solutions for the 30-bar space dome of five nonlinear methods, including the current technique. One can see that Khot [19], Missoum [21], and Hrinda [20] reached the optimal solution with 766.19, 766.72, and 765.00 lbs (347.54, 347.78, and 347 kg), respectively. On the other hand, FEA-DE, DNN-DE, and the current method's optimal weight are 730.11, 733.63, and 730.16 lbs (331.17, 332.77, and 331.19 kg), respectively. Their corresponding structural analyses are 2821111, 4120, and 13. Similarly, their computational time in seconds is 8959, 3746.936, and 1.52. The computational time of the current approach is more than 99% less than that of the study

TABLE 1 | Comparison of the optimal solutions for the two-legged truss.

Studies	Missoum [21]	Current study without Grad.	Current study with Grad.
Design variables A in ² (cm ²)	0.7900 (5.0968)	0.7818 (5.0439)	0.7818 (5.0439)
Limited displacement in (cm)	-4.226 (-10.734)	-4.226 (-10.734)	-4.226 (-10.734)
Iterations	5	2	2
Structural analyses	NA	8	5

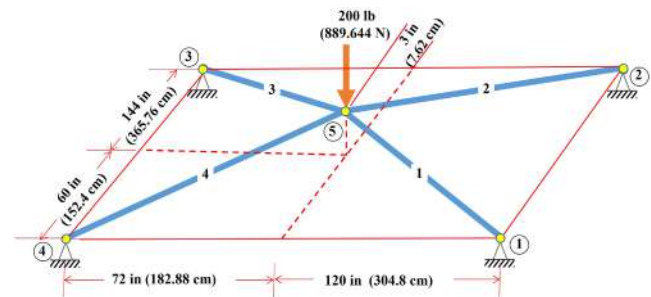


FIGURE 4 | Asymmetric four-bar truss.

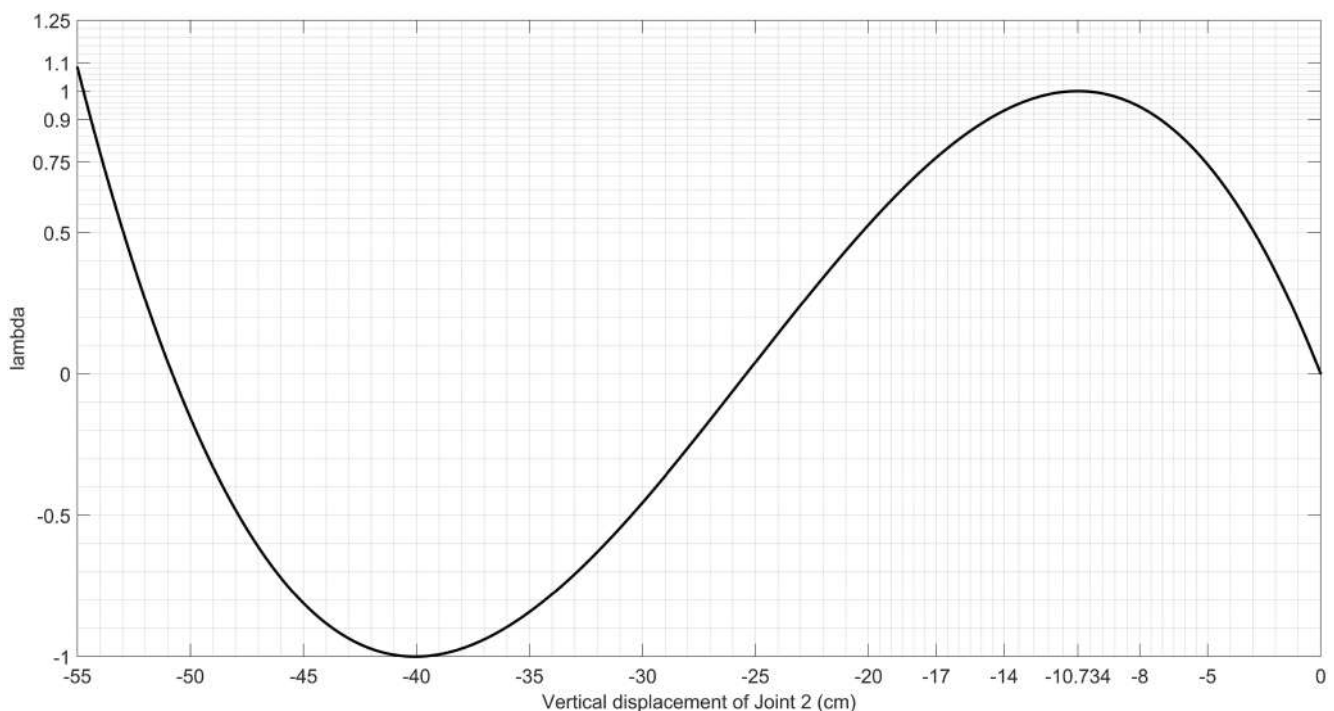
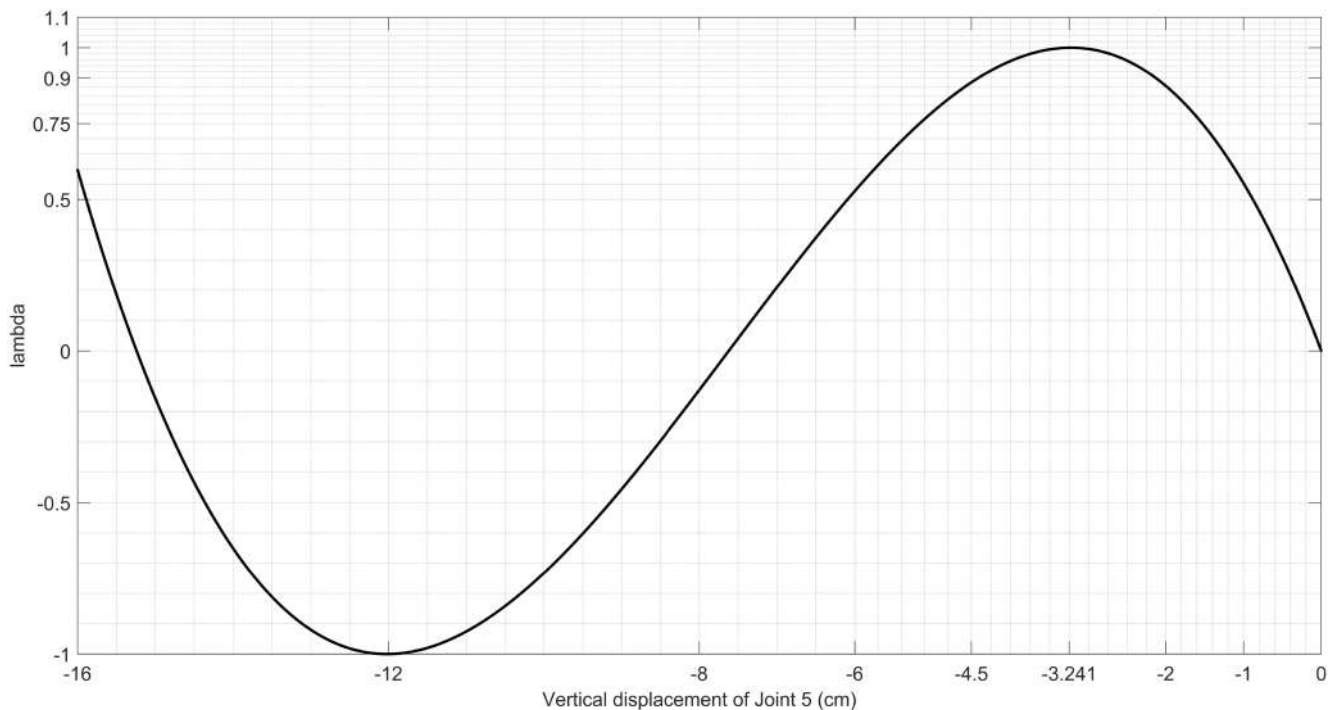


FIGURE 3 | Equilibrium path at the minimum weight of the two-legged symmetrical truss.

TABLE 2 | Comparison of the current and Hrinda et al. [20] study for weight optimization of the four-bar shallow truss.

Members	Length in (cm)	A in ² (cm ²)		
		Hrinda [20]	Current study without Grad.	Current study with Grad.
1	134.1976 (340.8619)	2.1724 (14.0152)	2.1647 (13.9656)	2.1647 (13.9656)
2	187.47 (476.1738)	1.6383 (10.5695)	1.6550 (10.6772)	1.6550 (10.6772)
3	161.02 (408.9908)	1.7255 (11.1688)	1.7312 (11.1688)	1.7312 (11.1688)
4	93.77 (238.1758)	2.9233 (18.8597)	2.8915 (18.6545)	2.8915 (18.6545)
Total weight lb (kg)		115.06 (52.19)	115.06 (52.19)	115.06 (52.19)
Max disp in (cm)		−1.248 (−3.169)	−1.276 (−3.241)	−1.276 (−3.241)
Structural analyses		NA	48	20

**FIGURE 5** | Equilibrium path at the minimum weight of the asymmetric four-bar truss.

by [16]. Figure 7 presents the load-deflection curve of the top free node. It illustrates that the snap-through occurs when the nodal vertical displacement of Node 1 is -9.74 in (-24.74 cm).

4.2 | Displacement Constraint

4.2.1 | The Ten-Bar Truss

The ten-bar truss, a benchmark example in structural optimization depicted in Figure 8, is evaluated for weight optimization under displacement constraints. Nodes 5 and 6 are constrained against translation, while nodes 2 and 4 are subjected to a downward load of $1e7$ lb (44,482.2 kN) each. The material properties are defined with an elasticity of $1e7$ psi (68.947 GPa) and a density of 0.1 lb/in³ (0.002768 kg/cm³). The goal is to minimize the total weight of the structure under the applied load while limiting the vertical displacement of Nodes 1 and 2 to 100 inches (254 cm).

Additionally, the lower bound for the cross-sectional area of the members is set at 0.1 in² (0.6452 cm²), and no upper bound is imposed. Table 4 presents a comparison of the results obtained in this study with those from reference [21]. It is observed that the optimal weight achieved in this study is 9454 lb (4288.3 kg), compared to 9519.5 lb (4318 kg) obtained in the previous study. Consequently, the design achieved in this study is 0.69% lighter than that of the previous research.

4.2.2 | 25-Bar Truss

In this example, a 3D tower truss shown in Figure 9 is analyzed to optimize its weight while meeting a displacement constraint. The truss members are divided into eight design variables with the modulus of elasticity and material density set at 20,700 and 7.85 kN/cm³ respectively. Displacements at Joints 1 and 2 must not exceed 1 cm each under the loading conditions specified in

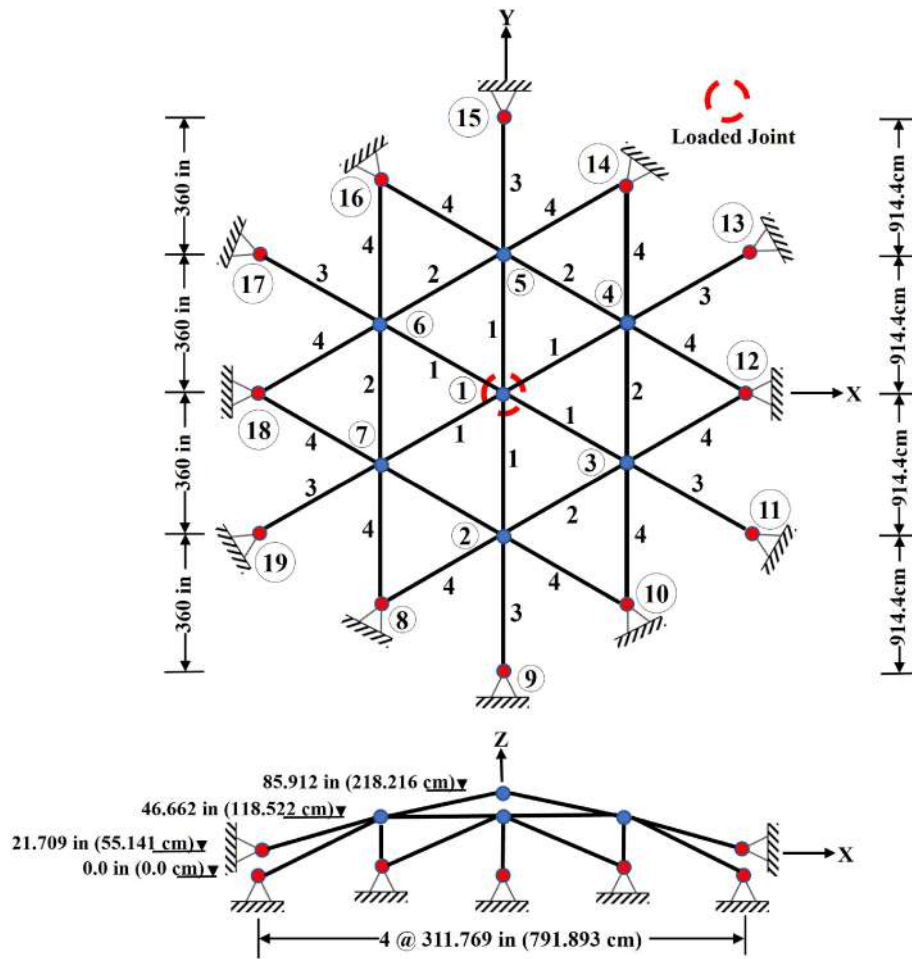


FIGURE 6 | Thirty-bar dome.

TABLE 3 | Weight optimization with stability constraint in thirty-bar truss.

Design vars	A in ² (cm ²)					Current study without Grad.	Current study with Grad.
	Khot [19]	Missoum [21]	Hrinda [20]	Mai FEA-DE [16]	Mai DNN-DE [16]		
1	1.6926 (10.92)	1.6904 (10.91)	1.69407 (10.93)	1.6113 (10.40)	1.51 (9.74)	1.613 (10.41)	1.613 (10.41)
2	1.3754 (8.87)	1.3833 (8.92)	1.3750 (8.87)	1.4629 (9.44)	1.5810 (10.20)	1.4616 (9.43)	1.4616 (9.43)
3	0.2693 (1.74)	0.2674 (1.73)	0.1004 (0.65)	0.1000 (0.65)	0.1000 (0.65)	0.1000 (0.65)	0.1000 (0.65)
4	0.1000 (0.65)	0.1000 (0.65)	0.2635 (1.70)	0.1000 (0.65)	0.1000 (0.65)	0.1000 (0.65)	0.1000 (0.65)
Weight lb	766.19	766.72	765.00	730.11	733.63	730.16	730.16
Weight (kg)	(347.54)	(347.78)	(347.00)	(331.17)	(332.77)	(331.20)	(331.20)
J1 disp in	NA	−10	NA	−9.697	−9.697	−9.7402	−9.7402
J1 disp (cm)	NA	(−25.40)	NA	(−24.63)	(−24.63)	(−24.74)	(−24.74)
Str. analyses	NA	NA	NA	2821111	4120	41	13
Time (s)	NA	NA	NA	8959	3746.94	4.13	1.52

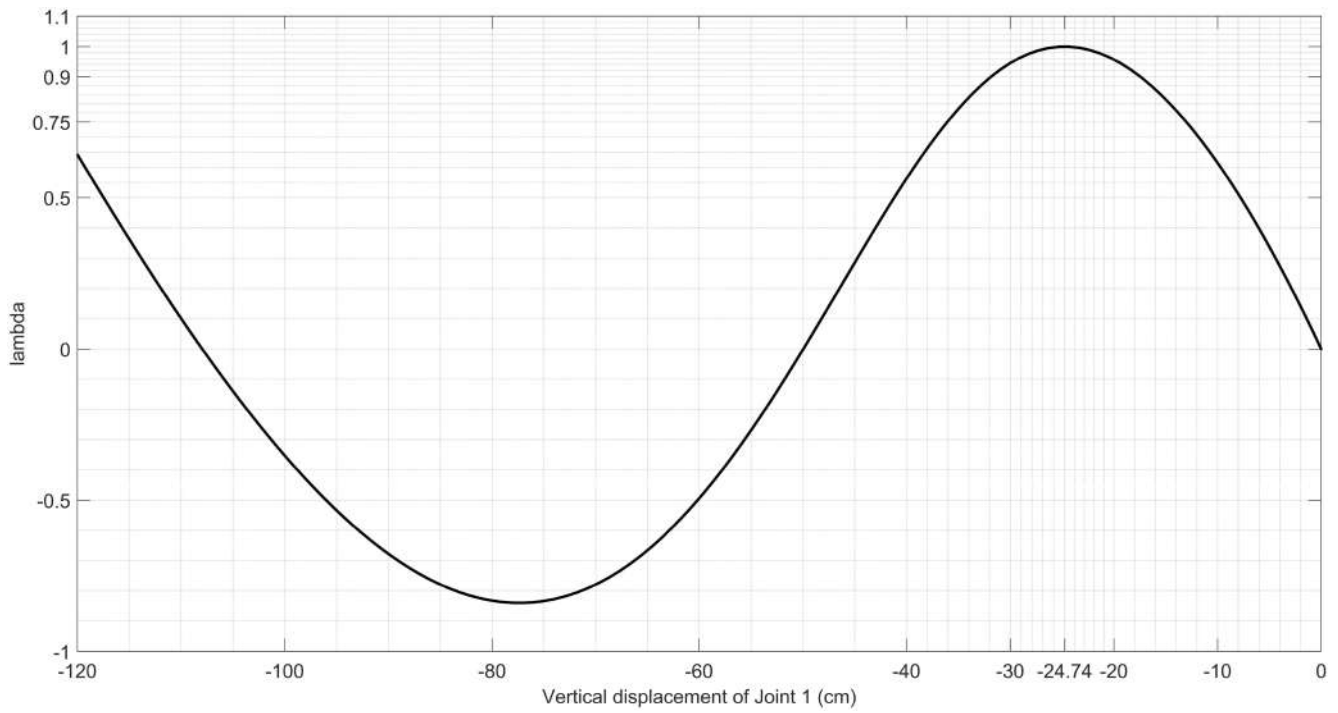


FIGURE 7 | The load-deflection curve of the top free node of the thirty-bar dome.

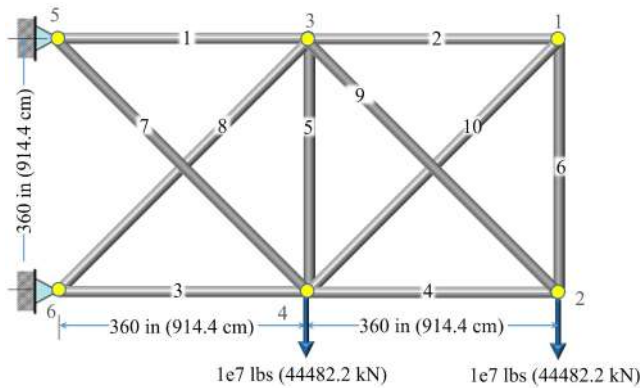


FIGURE 8 | The ten-bar truss.

Table 5. The design variables are restricted to a range between 2 and 20 cm² for their lower and upper bounds respectively.

Table 6 presents the optimal solutions for minimizing the weight of the 25-bar tower truss as determined by three methods including the current technique. The results indicate that the Saka and Mai Kang [16, 32] FEA-DE DNN-DE and the current method achieve optimal weights of 507, 504.215, 505.493, and 501.348 kg respectively. Notably, the current method reduces the total structural weight and computational time. The associated studies reveal that these optimal solutions required 63, 1,490,799, 18,120, and 24 function evaluations. It can be seen that the current method results in a 1.11% reduction in total weight and a 99.99% reduction in function evaluations compared to [16, 32] methods. Regarding time efficiency, Mai et al. [16] reached the optimal solution in 28,179 s while the proposed method found the optimal weight in merely 1.97 s.

TABLE 4 | Comparison of the optimal results for the ten-bar truss.

Member	A in ² (cm ²)		
	Missoum [21]	Current study without Grad.	Current study with Grad.
1	56.160 (362.316)	53.46 (344.897)	53.460 (344.897)
2	0.368 (2.3742)	0.100 (0.6452)	0.100 (0.6452)
3	42.118 (271.7243)	43.003 (277.4339)	43.003 (277.4339)
4	21.5456 (139.0014)	24.787 (159.9133)	24.787 (159.9133)
5	0.100 (0.6452)	0.100 (0.6452)	0.100 (0.6452)
6	0.100 (0.6452)	2.072 (13.3675)	2.072 (13.3675)
7	22.883 (147.6297)	15.541 (100.2628)	15.541 (100.2628)
8	39.858 (257.1439)	43.911 (283.2918)	43.911 (283.2918)
9	38.623 (249.1763)	38.625 (249.1892)	38.625 (249.1892)
10	0.486 (3.1354)	0.284 (1.8322)	0.284 (1.8322)
Weight lb (kg)	9519.5 (4318)	9454.4 (4288.4)	9454.4 (4288.4)
Max. disp Joint			
2 and 4 in (cm)	-100 (-254)	-100 (-254)	-100 (-254)
Structural analyses	NA	412	81

4.2.3 | 52-Bar Dome Truss

The 52-bar space dome shown in Figure 10, previously studied by Saka et al. [32], is evaluated for weight optimization under a displacement constraint. The material properties are specified with a modulus of elasticity of 21,000 kN/cm² and a density of 7.85 N/cm³. The truss members are grouped into eight categories, and the allowable displacement for all free nodes is limited to 1 cm. This test applies a 150 kN downward load to joints 6–13. The results shown in Table 7 indicate that the optimal weights determined by Saka [32], Mai [16], and the current study are 5161, 2142.41, and 2143.10 kg, respectively. Mai and the current research achieve significantly lower weights than Saka's results.

Furthermore, the current study arrived at these results by evaluating only 13 functions in 1.60 s, while Mai's approach required the evaluation of 14,100 functions, taking 3948 s. The computational time needed for the proposed approach is 99.9% less than that required by Mai's study. Furthermore, Table 8 tabulated the nodal displacements of four free joints obtained from the three

compared methods. It can be seen that the study by [32] violated the limits of the displacement of Joint 6, while the study by [16] and the current study kept the limits of the constraints.

4.3 | Displacement and Stability Constraints

4.3.1 | Thirty-Bar Dome

In this subsection, displacement and stability are considered simultaneously. The 30-bar dome truss shown in Figure 6 is examined simultaneously under constraining displacement and

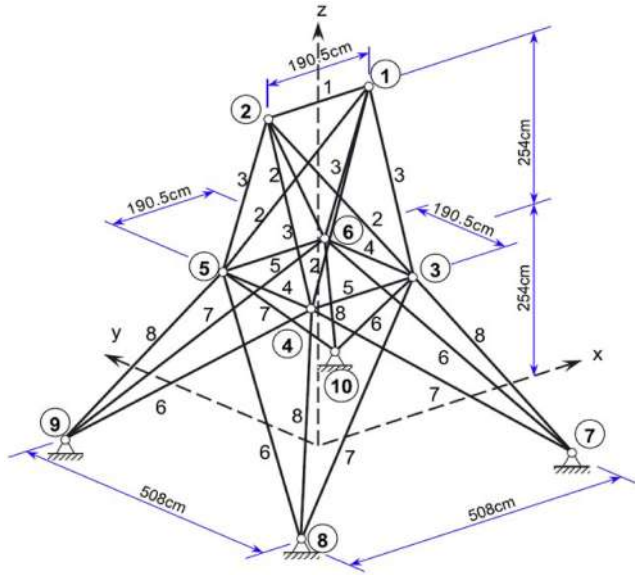


FIGURE 9 | The 25-bar tower truss.

TABLE 5 | Loading condition for the 25-bar tower truss.

Joints	Loading (kN)		
	X	Y	Z
1	-80	-120	30
2	-60	-100	30
3	-30	0	0
6	-30	0	0

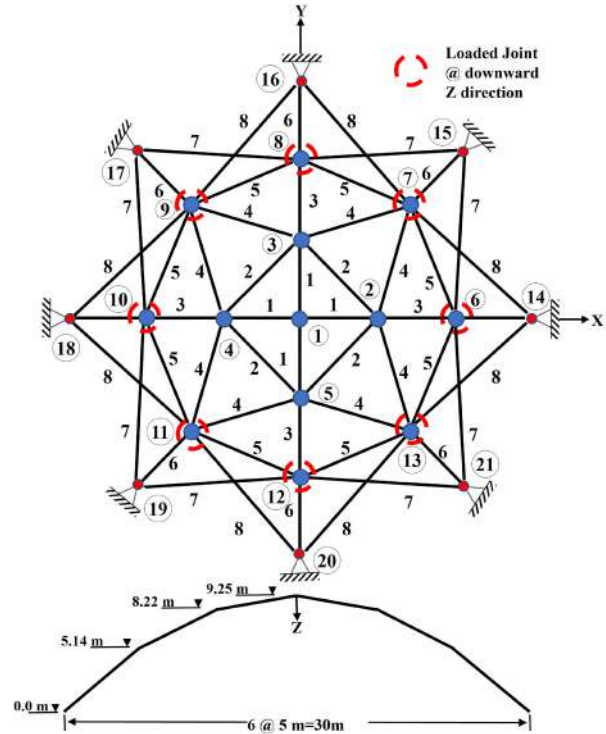


FIGURE 10 | The 52-bar dome truss.

TABLE 6 | Comparison between three methods for the optimal solution for the 25-bar tower truss.

Groups	A (cm ²)				
	Saka [32]	Mai FEA-DE [16]	Mai DNN-DE [16]	Current study without Grad.	Current study with Grad.
1	2.000	2.000	2.000	2.000	2.000
2	7.500	3.098	4.099	3.054	3.054
3	13.120	16.910	15.113	16.942	16.942
4	2.000	2.000	2.000	2.000	2.000
5	4.270	6.566	3.906	6.507	6.507
6	3.800	3.891	4.851	3.902	3.902
7	4.220	3.330	3.744	3.341	3.341
8	17.150	18.320	17.779	18.332	18.332
Weight (kg)	507.000	504.315	505.493	501.348	501.348
Max. disp (cm)	-1.00	-1.00	-1.00	-1.00	-1.00
Structural analyses	63	1,490,799	18,120	120	24
Time (s)	NA	28,179	2748.79	5.04	1.70

TABLE 7 | Comparison between three methods for the optimal solution for the 52-bar space dome truss.

Groups	A (cm ²)				
	Saka [32]	Mai FEA-DE [16]	Mai DNN-DE [16]	Current study without Grad.	Current study with Grad.
1	81.820	2.000	2.000	2.000	2.000
2	22.410	2.000	2.000	2.000	2.000
3	33.580	2.000	2.000	2.000	2.000
4	14.450	2.000	2.000	2.000	2.000
5	10.640	16.672	16.753	16.586	16.586
6	25.1606	17.585	17.869	17.699	17.699
7	2.000	2.519	2.301	2.522	2.522
8	2.000	2.000	2.000	2.000	2.000
Total weight (lb)	5161.00	2141.87	2142.41	2143.10	2143.10
Structural analyses	NA	1,276,2048	14,100	79	13
Time (s)	NA	68,408	3948.98	8.60	1.60

TABLE 8 | The displacement of four constraint joints obtained by three different approaches.

Joints	Disp. (cm)				
	Saka [32]	Mai FEA-DE [16]	Mai DNN-DE [16]	Current study without Grad.	Current study with Grad.
Joint 1	−0.277	0.133	0.121	−0.127	−0.127
Joint 2	−0.283	0.072	0.074	−0.066	−0.066
Joint 6	1.304	1.000	1.000	1.000	1.000
Joint 7	0.949	1.000	0.965	1.000	1.000

stability. The structure was tested in Section 4.1.3, constraining only the stability; the optimal weight was 730.16 lbs (331.2 kg), and the displacement at the critical point was 9.74 in (24.74 cm). In this section, the structure with the same geometry and prosperity was tested to find the optimal weight, maintaining the downward displacement of the top node below or at 7 in (17.78 cm) and preventing instability. In this case, the algorithm stops before violating any constraints. The results show that, the top node displaces 7 in (17.78 cm) under the applied load (load factor $\lambda = 1.0$) before instability that now occurs at $\lambda = 1.052$, and the optimal weight found by the algorithm is 785.2 lbs (356.1607 kg). The selected areas for the four groups are 1.751, 1.475, 0.201, and 0.1 in² (11.2966, 9.5160, 1.2968, and 0.6452), respectively. The optimal solution is obtained by evaluating only 44 functions (structural analyses) using the analytical gradient of the objective and constraint functions while evaluating 108 functions using the numerical gradients of the nonlinear constraints. If only stability is considered, the optimal weight is 730.16 lbs (331.2 kg), the limit point is at $\lambda = 1.0$ and the displacement is 9.74 in (17.74 cm); in contrast, if stability and displacement are considered simultaneously, the optimal weight becomes 785.2 lbs (356.1607 kg), that is, greater than the previous value as expected due to the additional displacement constraint. Figure 11 illustrates the equilibrium path for Joint 1 of the thirty-bar dome. The load-deflection curve in the case of constraining both displacement and stability is steeper than that considering only stability.

4.4 | Displacement and Stress Constraints

This subsection considers displacement and stress constraints while optimizing the structure's total weight. A large-scale structure is tested for this purpose.

4.4.1 | A Large Plate-Like Truss Structure

To validate efficiency and low-cost computational complexity of the proposed approach for large-scale structures, the structural weight of the plate-like truss structure depicted in Figure 12 is minimized under the nodal displacement of two joints. The structure is formed by 1401 members that connected 242 nodes, resulting in a total of 726 degrees of freedom for nodal displacement (DOFs). The members are made of steel with a modulus of elasticity of 210 MPa, material density of 7750 kg/m³, and allowable stress of 224 MPa. All displacement components are restrained along the top external perimeter of the plate. Additionally, all nodes on the bottom surface are loaded with a vertical downward force of 3 kN. The plate is divided into 10 modules along each edge, each having a rectangular cuboid shape delimited by horizontal and vertical bars along the 12 edges, with additional diagonal bars in the faces. For the given loading, members with axial force within a range of 12 kN are put in a group as it's preferred in practice: members with axial force from 0 to 12 kN are in group 1, from 13 to 24 kN in group 2 and so on with a total of 10

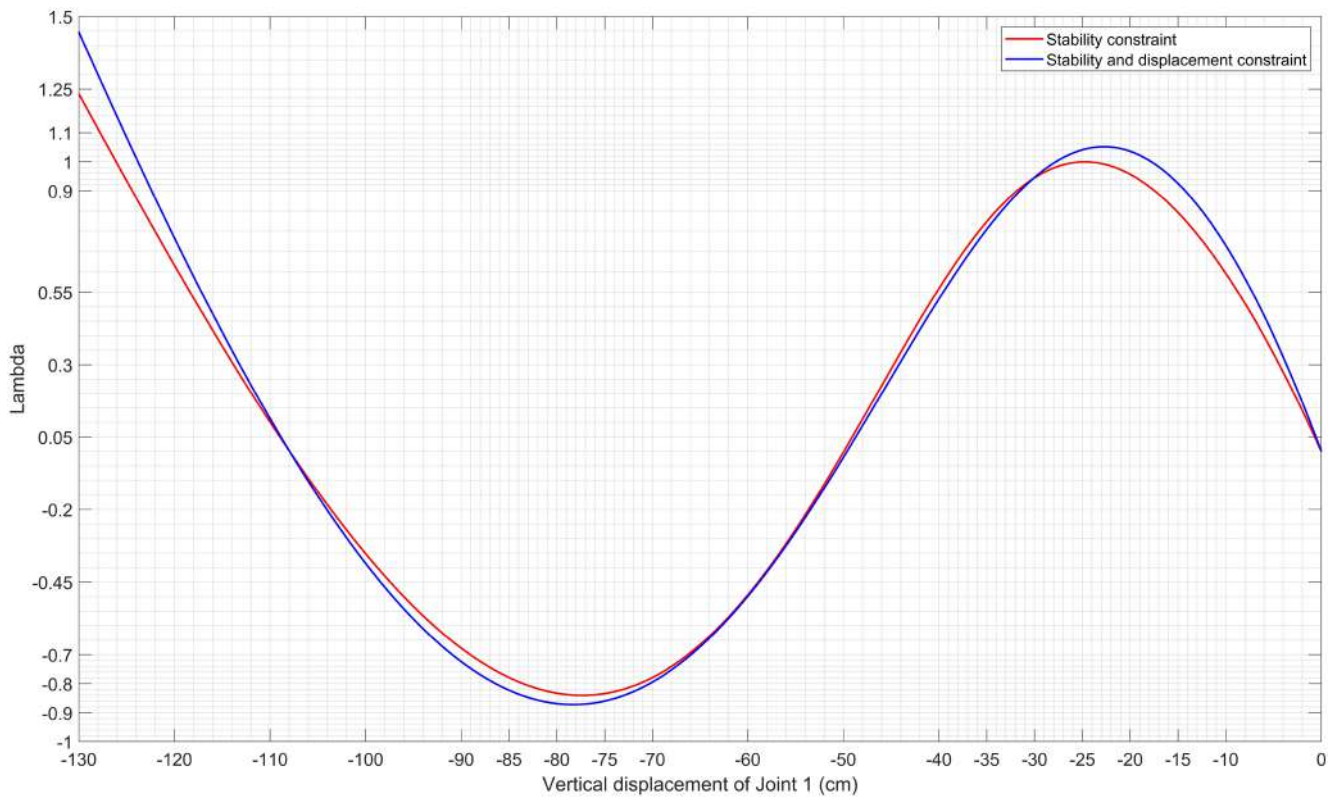


FIGURE 11 | The equilibrium path of the top free node of the thirty-bar dome constraining stability and displacement and stability only.

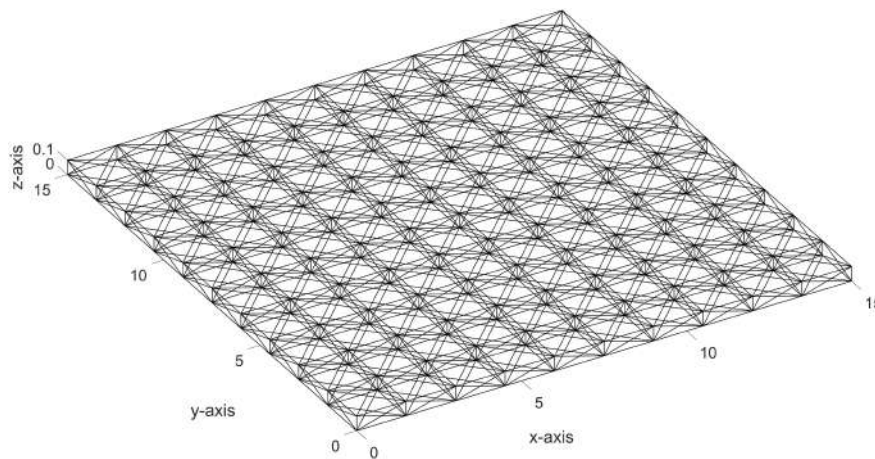


FIGURE 12 | The plate-like truss structure (lengths in m).

groups. The goal is to minimize the total weight of the structure considering nodal vertical displacement of Joints 201 and 221 to not greater than 26 mm and the stress in all members within the allowable limit (224 MPa).

Table 9 shows the optimal solution for the plate-like truss structure using the current method with and without implementation of the gradients of the objective and constraint functions. One can see that the optimal weight is 4725.3 kg. Using gradients, spend 42 iterations, running 264 structural analyses in 513 s to reach the optimal solution. On the other hand, without implementing gradients, the method spends 44 iterations, running 701 structural analyses in 1621 s.

5 | Conclusions

This study regarded a computational strategy for weight minimization of geometrically nonlinear structures. The novelty of the proposal is the possibility to take into account displacement, stress and stability constraints, also simultaneously, within a consistent, robust and efficient formulation. The structural response is evaluated by a path-following finite element analysis that provides the nonlinear load-displacement curve as well as the exact nonlinear critical load using the null vector method. Importantly, this work demonstrated that the gradient of displacement, stress and critical point coming from the nonlinear finite element analysis can be computed in an exact closed form. In addition, it is

TABLE 9 | Optimal solution for the plate-like truss structure.

Groups	Current without Grad.	Current with Grad.
1	100.000	100.000
2	100.000	100.000
3	161.910	161.910
4	248.820	248.820
5	518.190	518.190
6	931.620	931.620
7	1088.200	1088.200
8	1079.500	1079.500
9	1241.300	1241.300
10	1702.300	1702.300
Weight (kg)	4725.3	4725.3
Max. disp. (mm)	26.0	26.0
Iterations	44	42
Structural analyses	701	264
Time (s)	1621	513

worth noting that this gradient computation can be integrated in the structural solver and requires practically no additional cost with respect to the structural analysis itself. The results show that our strategy outperforms the current approach based of numerical gradients in terms of robustness and computational time, particularly for large-scale structures. Although this paper focused on trusses as example of application, the proposed optimization framework can be easily applied to plates and shells as well since it relies on a finite element discretization, with a more involved parametrization of the design (e.g., thicknesses and holes). Reduced-order methods such as the Koiter [33] and Koiter-Newton [34] methods have the potential to achieve a further crucial reduction of computational cost both in function evaluations and gradient computations. This possibility represents one very productive area of research in the future, particularly for including an imperfection sensitivity in the optimization. Furthermore, subsequent studies could enhance the proposal by incorporating dynamic loading circumstances and contemporary presence of geometric and material nonlinearity [35].

Author Contributions

Ahmed Manguri: data curation, conceptualization, methodology, software, visualization, validation, writing – original draft. **Domenico Magisano:** data curation, conceptualization, methodology, software, visualization, validation, writing – original draft, writing – review and editing, supervision, funding acquisition. **Robert Jankowski:** writing – review and editing, supervision, funding acquisition.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request. Data will be made available on request.

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Paper VIII

Paper VIII: Manguri, A., N. Saeed, M. Szczepanski, and R. Jankowski, *Bending moment control and weight optimization in space structures by adding extra members in the optimal locations*. Advances in Science and Technology Research Journal, 2023. **17**(4): p. 313-324. <https://doi.org/10.12913/22998624/169573>

Bending Moment Control and Weight Optimization in Space Structures by Adding Extra Members in the Optimal locations

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ABSTRACT

This paper investigates the reduction of bending moment in critical members by adding some extra members in the optimum location. Instead of enlarging the size of members to resist the moment, eight additional members are added in the optimum location to reduce the bending moment in the critical members. The total weight of the proposed structure with extra members is less than that of the original structure that resists the induced bending moment. Moreover, the location of the additional bars significantly reduces the nodal displacements. This paper investigates the effect of placing extra members on vertically and/or horizontally loaded egg-shaped single-layer frames. An egg-shaped structure is designed based on the maximum induced moment; in such frames, the bending moment is the dominant internal force. Then some extra members are suggested to be added to the structure to reduce the maximum bending moment to the lowest possible value; thus, the designed cross-sectional area is minimized. Furthermore, the optimized structure's total weight and shape deformation is less than the ordinary structure's. The study results show that the extra bars' location depends on the loadings' direction. Moreover, the weight of the horizontally loaded egg-shaped structure can be optimized by up to 28%. The results were verified by MATLAB and SAP2000 software.

Keywords: size optimization, weight optimization, additional bars, bending moment control, egg-shaped structure

INTRODUCTION

In the modern world, space structures have a large free column span and a low weight ratio to the covered area. Other purposes of building such systems are their aesthetic view and use as a storage [1]. Space structures can be found in several tourist cities, such as Pavilion and Science Museum (Nur Alem) in Kazakhstan [2, 3], MSG Sphere in Las Vegas in the US [4], and Ericsson Globe in Stockholm, Sweden [5, 6]. Due to loadings, the structural shape can be disturbed, and the internal bending moment, torsion, and axial forces can pass the allowable limits.

In general, the bending moment is the critical internal force in moment frames that causes noticeable deformation and sometimes failure

in frame structures [7, 8]. Chen and Lui [9] discussed the types of space frames and several analysis methods. Researchers used different methods to analyze space frames, for example, the modified arch length method [10], Euler's finite rotation formula [11], and the stiffness matrix method [12]. Regarding minimizing bending moments in simple frames, Wang [13] suggested a computational technique to reduce flexural moments in frame members. Moreover, the shape disturbance of some types of space structures was reformed after deformation, for instance, double-layer domes [14, 15] and egg-shaped single-layer frames [16]. Moreover, the internal bar force is also reduced in the members with high-stress levels by prestressing some active members [17].

Finding the optimum solution for a given problem is essential for cost-effectiveness. Structural optimization opened a new chapter in structural engineering [18]. Structural optimization is defined as the process of finding the best solution for a particular concern [19-21] in systems. Bar size optimization was performed for truss [22-28], dome [29-31], and frame [32, 33] structures. Furthermore, the optimum location of different types of shear walls for various buildings [34] on several soil types [35] was investigated. The optimal design was done for I- and H-shaped crane bridge girders [36], reinforced concrete sections [37], and steel moment frames [38]. Moreover, the significance of the most active members' location to change the bars' lengths to reshape and redistribute stress in members was also studied [39-43]. So far, studies have not been carried out either to control bending moment in beams or to beam size optimization for space frames.

Moreover, Saeed et al. [16] added extra members to a single-layer egg-shaped frame; they were not interested in finding the best places for the extra bars to reduce the structural deformation and bar internal forces. They arbitrarily put eight members in two different places to change the length of the additional bars to minimize the deformed shape without regard to the bending moments and axial forces. So far, the significance of the location of additional bars on a frame has not been studied. This paper deals with adding extra bars in different places of two numerical models of

a single-layer egg-shaped frame. One of the structures is hinged support, and the other is fixed support. The optimum location of the extra bars is determined based on the shape disturbance and structural stability. Aside from the original shape (Case 0), ten cases are evaluated; for each case, the extra members are added to different frame levels.

METHODS

Numerical modeling

The numerical model is 5000 mm high and 4000 mm wide. The model is formed by eleven circles of eight joints on top of each other. In addition to two joints, one at the very top and the other at the very bottom of the structure (Fig. 1a). Moreover, the numerical model is supported at the nine bottom joints. The ninety joints were connected with 176 bars, as illustrated in Figure 1b. The members are made of steel, with Young's modulus of 200 GPa; the bars' diameter depends on the members' maximum bending moment.

Eight members are attached to different levels of the structure to investigate the effect of the added members on the end moments and nodal displacements. For example, maximum absolute moment about the X-axis ($\max(\text{abs}(\text{mx1}))$, $\max(\text{abs}(\text{mx2}))$), the Y-axis ($\max(\text{abs}(\text{my1}))$, $\max(\text{abs}(\text{my2}))$), maximum absolute nodal displacements in X-direction ($\max(\text{abs}(\text{dx}))$), Y-directions ($\max(\text{abs}(\text{dy}))$), and Z-direction ($\max(\text{abs}(\text{dz}))$).

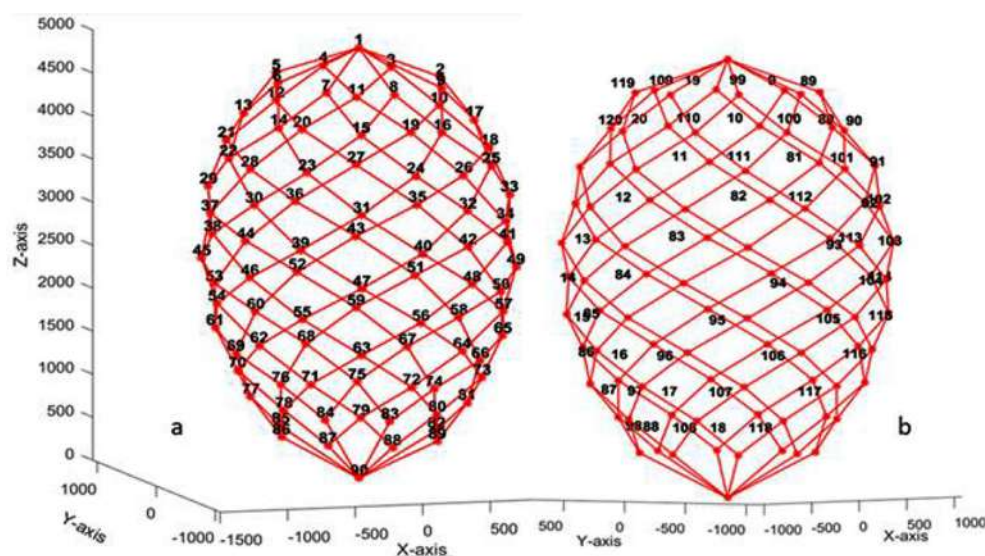


Figure 1. Single-layer egg-shaped frame: (a) joint labeling, and (b) member labeling

Optimization calculation

The area of the bars is selected based on the maximum moment in members, and the shape modulus is found as presented in Equation (1):

$$sx_i = M_{\max i} / f_y \quad (1)$$

where: i is the case number, sx is the required shape modulus, $M_{\max i}$ is the yield moment, and f_y is the materials yield strength.

$$D_i = 2I_y / sx \quad (2)$$

where: D_i is the required bar's diameter to resist the maximum moment, and I_y is the moment of inertia.

The area and the volume for each case is calculated as Equations 3 and 4:

$$A_i = \frac{\pi}{4} D_i^2 \quad (3)$$

where: A_i is the required area for each case based on the maximum bending moment.

$$W_i = A_i * \text{sum}(L) * \gamma \quad (4)$$

where: W_i is the total weight of the structure bars for each case, and $\text{sum}(L)$ is the summation of the length of the bars, and γ is the density of the material which 25 kN/m³.

It should be noted that the $\text{sum}(L)$ of case 0 is less than that of the other cases since, for Case 0, there are no additional members. The optimization

in the cross-sectional area and the total volume is based on Equations 5 and 6:

$$Opt_{area} = \frac{A_0 - A_i}{A_0} * 100 \quad (5)$$

$$Opt_{vol} = \frac{W_0 - W_i}{W_0} * 100 \quad (6)$$

RESULTS AND DISCUSSION

This section studies the significance of extra members' locations. Aside from the original shape (case 0), ten other cases are evaluated. For each case, the additional members are added in different levels of the frame. Since the number of joints in each level is eight, the extra member's number for each case is eight bars. In other words, the additional members are horizontal and connect the eight joints in ten various levels, as presented in Figure 2.

Horizontal loading

Two loading cases are investigated to understand the structure's behavior for different horizontal loadings cases.

Loading Joints 18-21

In this loading case, joints 18-21 are loaded horizontally with 60N. The maximum induced

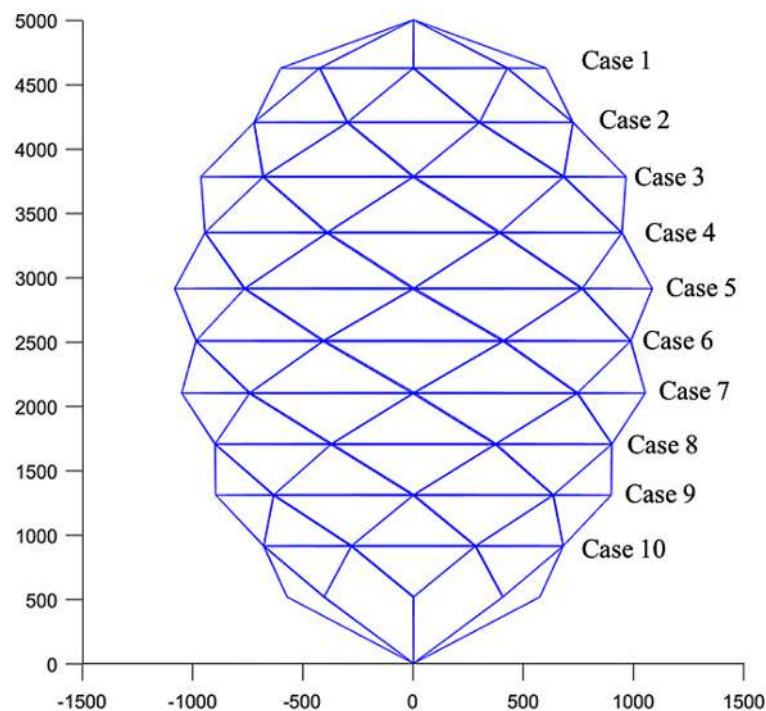


Figure 2. Optimization of cross-sectional area and total volume of the structure in eleven cases

bending moments for the original structure (Case 0) and the ten cases are illustrated in Figure 2. According to the data presented in Figure 3, the optimum location for placing the extra members for the given loading case is Case 9. Numerically speaking, the maximum bending moment for Case 9 is 9077 N·mm, while the maximum induced bending moment for Case 0 is 16249 N·mm. The optimization of beam cross-sectional area and material expenditure is illustrated in Figure 4. In terms of nodal displacements, the joints of the structure of Case 0 face larger movements than that of Case 9 (see Figure 5). This is because the bending moment is the substantial factor of deformation in moment frames. Furthermore, the

bending moment in members for Case 0 is larger than that for Case 9, as presented in Figure 6.

Loading Joints 26-29

In this case, Joints 26-29 are loaded horizontally to see if still Case 9 is the optimum case. By changing the loading position, the amount of bending moment is changed; however, Case 9 is still the best-case scenario. Figure 7 shows the maximum absolute bending moments for eleven cases, including the original case (Case 0). It can be seen that the maximum induced moment for cases 0 and 9 is 13262 N·mm and 5543 N·mm, respectively. Based on the maximum bending moments, the optimization of the cross-sectional

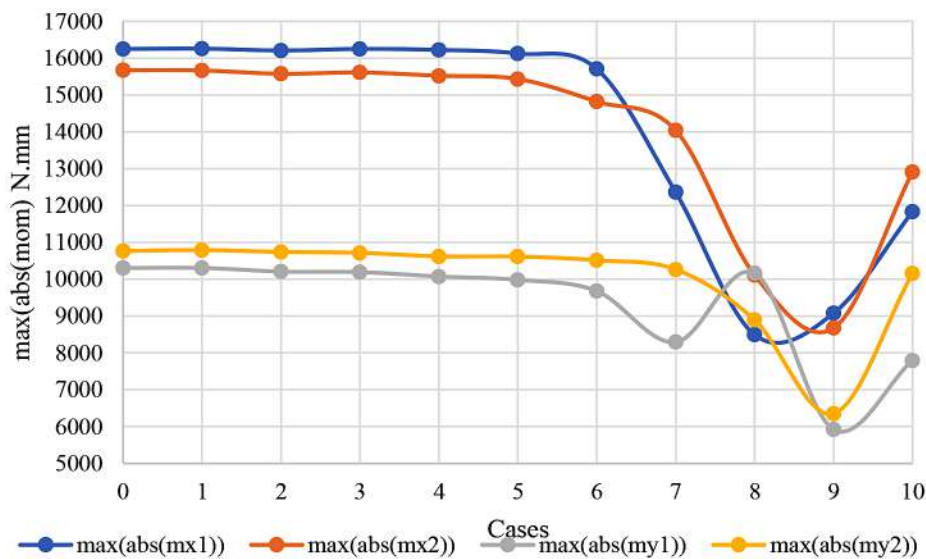


Figure 3. Maximum absolute bending moments for eleven cases

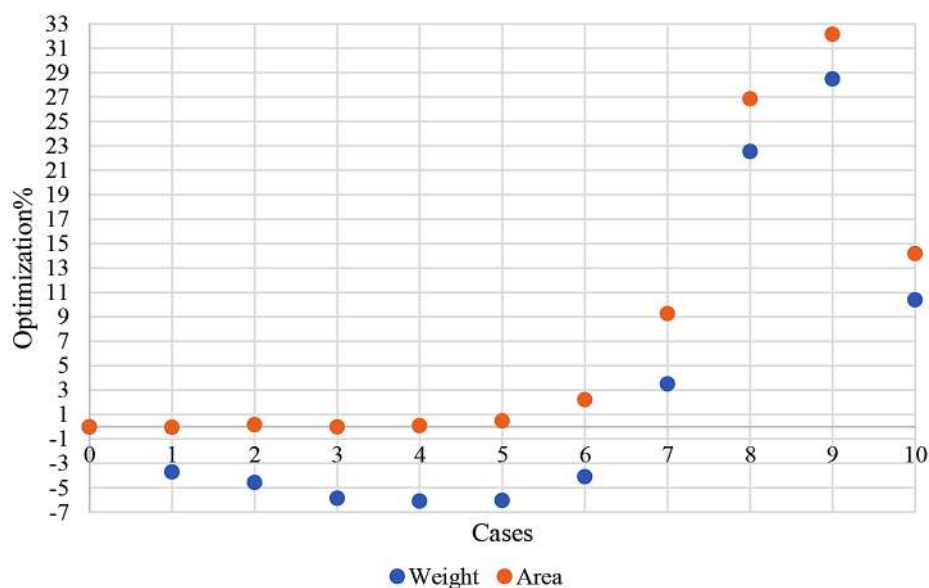


Figure 4. Optimization of cross-sectional area and the total weight of the structure in eleven cases

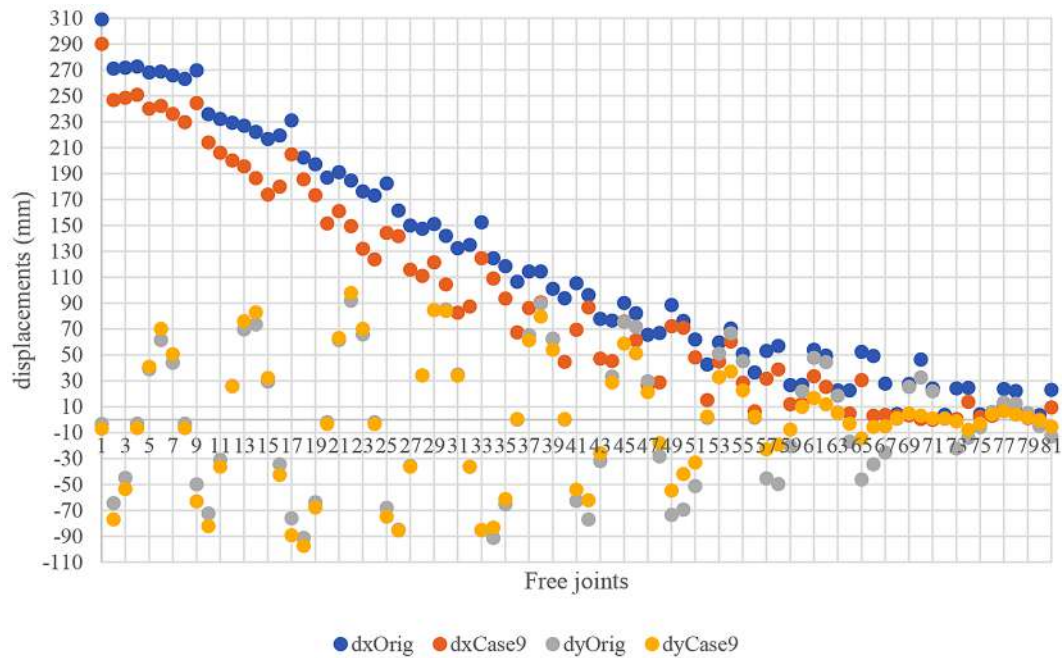


Figure 5. Nodal displacements of Case 0 and 9 in X and Y directions

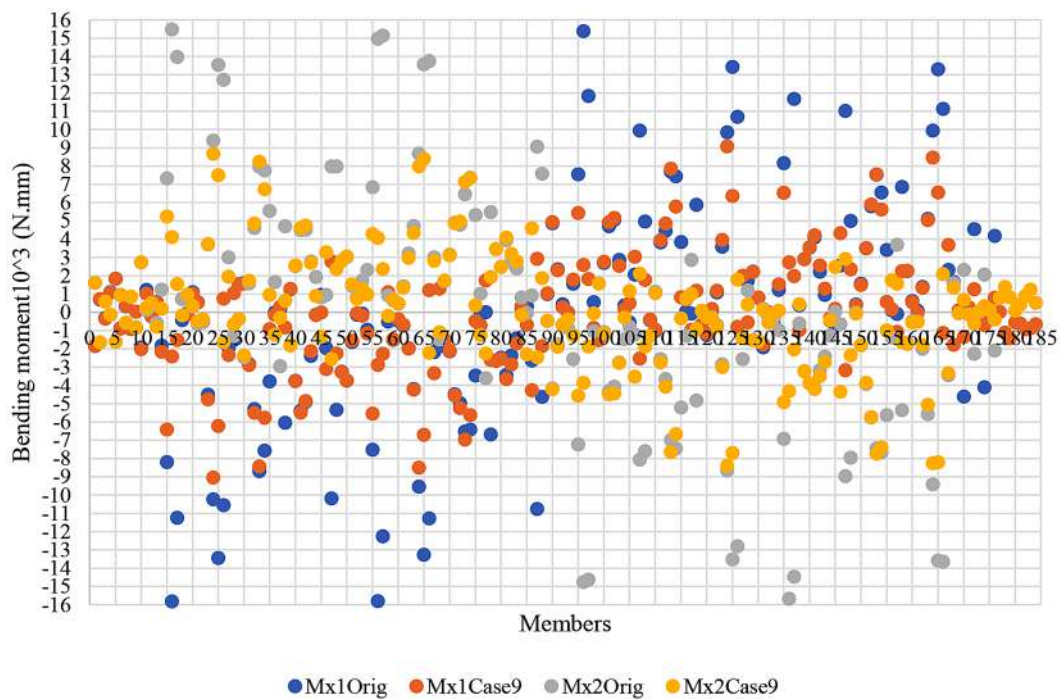


Figure 6. Bending moments in members about the X axis for Cases 0 and 9

bar area and the total weight of the structure is presented in Figure 8. The figure shows that the optimization of the cross-sectional area and the total weight of Case 9 based on the original case is 25% and 20% respectively.

Vertical loading

This section studies two different cases of vertical loadings to investigate the effect of loading

in bending moments and the place of additional members.

Loading Joints 1, 10-17

In this case, joints 1, 10-17 are loaded vertically with 180 N downward. The induced flexural moment in eleven cases is presented in Figure 9. The figure shows that the best-case scenario is Case 6, which provides with least bending moments. Case

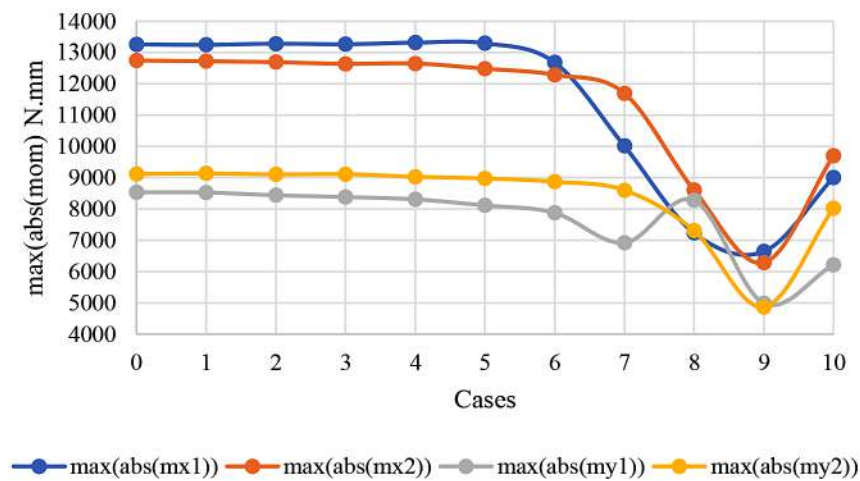


Figure 7. Maximum absolute bending moments for eleven cases

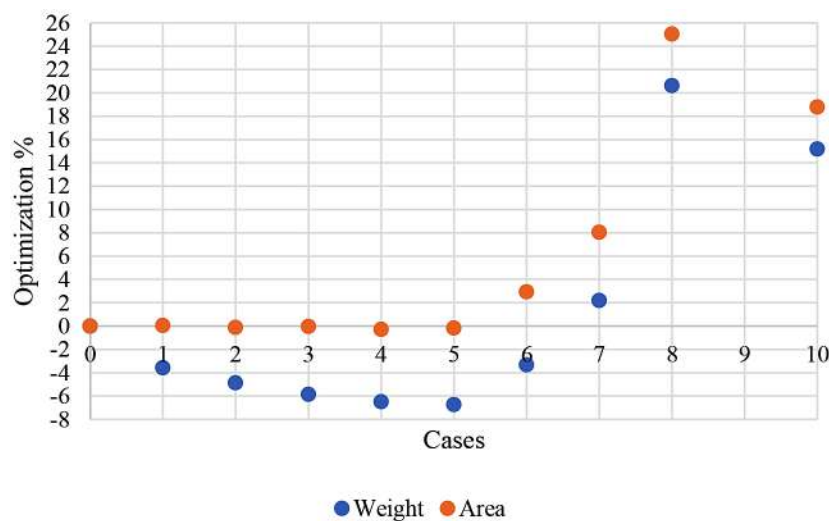


Figure 8. Optimization of cross-sectional area and total weight of the structure in eleven cases

6 provides bar size and structural total weight by 12 and 6%, respectively, as illustrated in Figure 10. Moreover, the states of the nodal displacements in X and Y directions of the optimal case (Case 6) and the original structure (Case 0) is presented in Figure 11. Despite of smaller size of beams in case 6 compared to case 0, the nodal displacements of case 6 are smaller than that of case 0. Similarly, the states of bending moments of all members about the X-axis for the optimal case and the original structure are illustrated in Figure 12. The figure shows that the induced bending moments in members for Case 0 are more prominent than that in case 6.

Loading Joints 1, 18-25

In this case, joints 1, 18-25 are loaded vertically with 180 N downward. The purpose of changing the loading joints is to see the behavior of the

structure in eleven cases. Figure 13 shows the induced bending moments in all cases, and one can see that still Case 6 is the best-case scenario. Figure 14 shows the structure's optimization in area and weight in Cases 1-10 based on Case 0. However, the beam cross-sectional area is minimized in Cases 5, 6, and 7, the total weight of Cases 5 and 7 is greater than that of Case 0. This is because Case 0 has only 176 beams, while other cases have 174 (176+8) beams. Moreover, the total weight optimization of Case 6 is just under 4%.

Horizontal and vertical loading simultaneously

In this loading case, Joints 1 to 9 are loaded vertically with -180 N, and joints 18-21 are loaded horizontally with 60 N. The induced bending moments in eleven cases are illustrated in Figure 15.

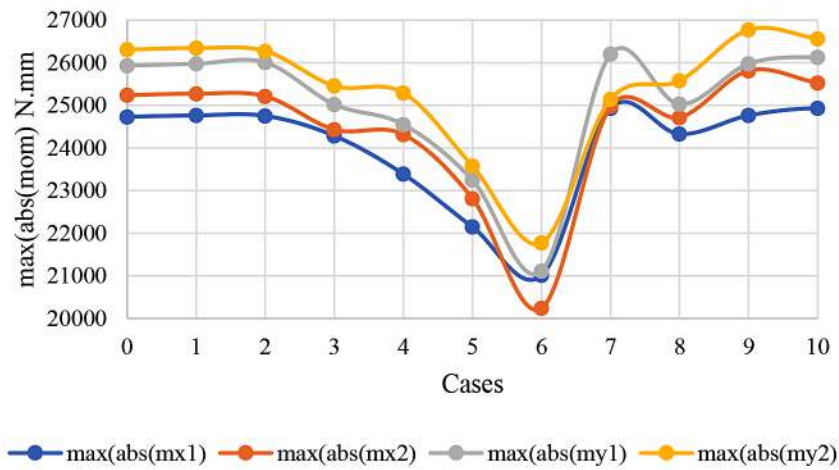


Figure 9. The maximum absolute bending moments of the egg-shaped single layer in eleven cases

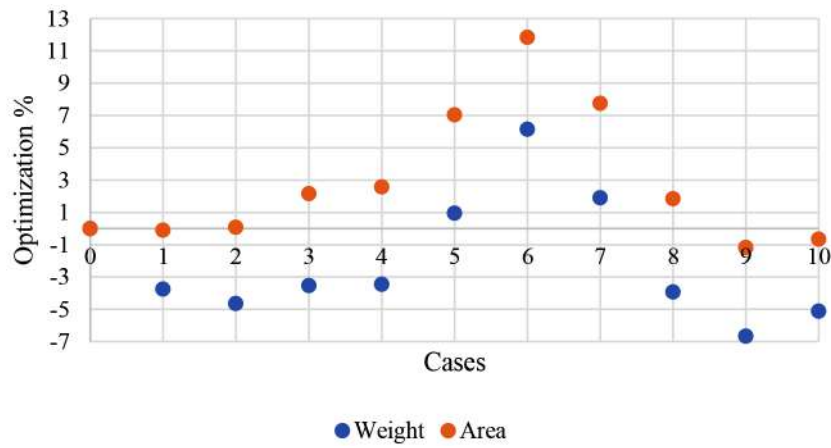


Figure 10. Optimization of bar size and weight in eleven cases

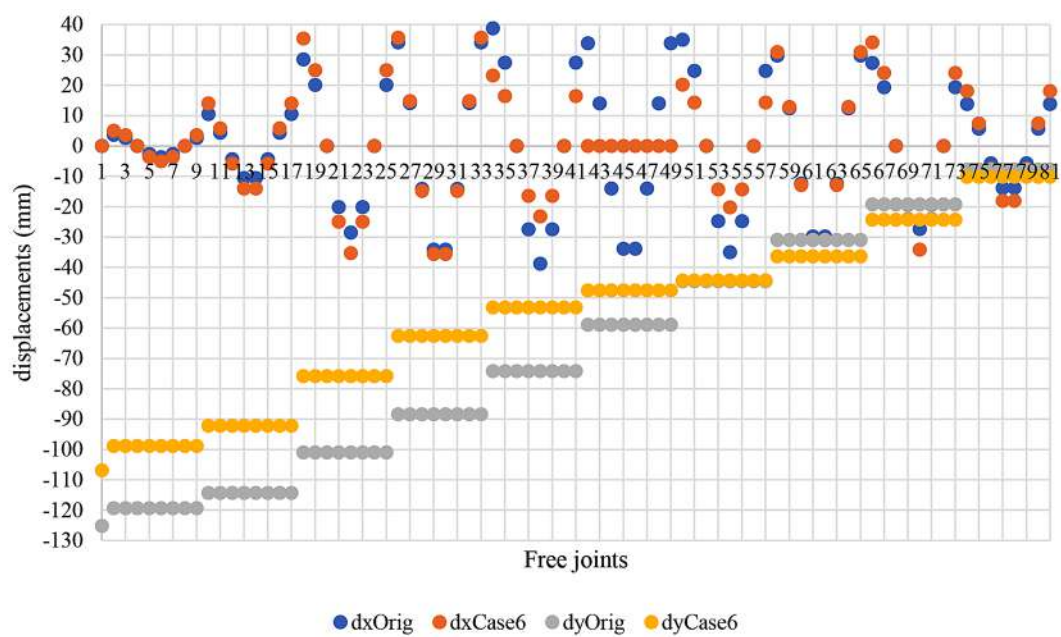


Figure 11. X and Y displacements of nodes for Cases 0 and 6

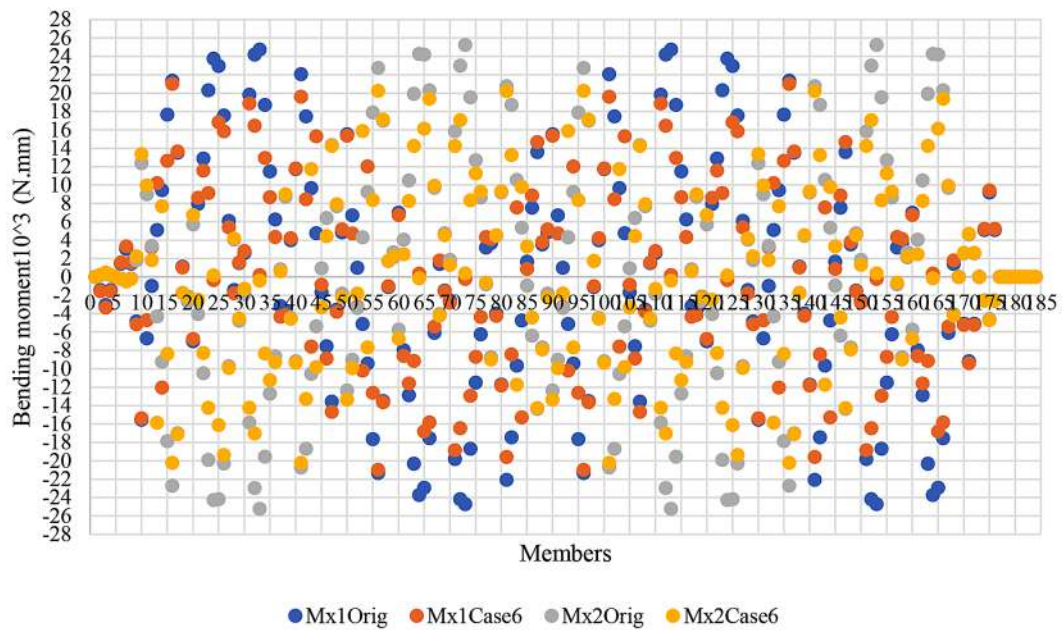


Figure 12. The states of bending moments in members for cases 0 and 6

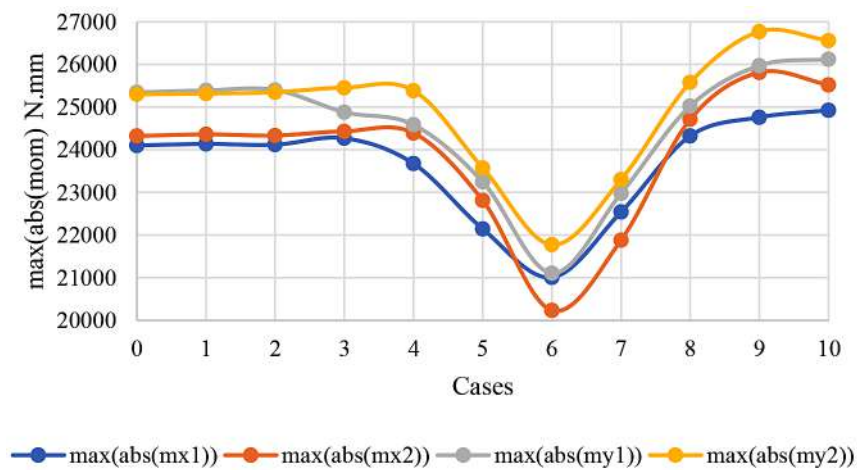


Figure 13. Maximum bending moments in eleven cases

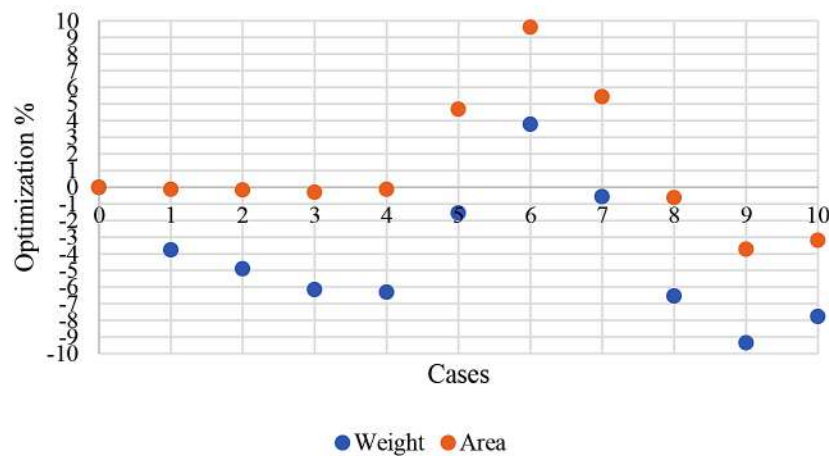


Figure 14. Cross-sectional area and weight optimization of ten cases compared to Case 0

One can see that the best case scenario is case 7, with the maximum bending moment of 30597 N.mm. Regarding the bar size and weight optimization, Figure 16 shows that the beam size and weight optimization of case 7 are 14% and 8.5%, respectively. Regarding nodal displacements, the

movements of the optimum case are smaller than that of the original case (Fig. 17). Furthermore, the states of the bending moments of each member for Cases 0 and 7 are illustrated in Figure 18. One can see that the optimum case's bending moments are more miniature than Case 0.

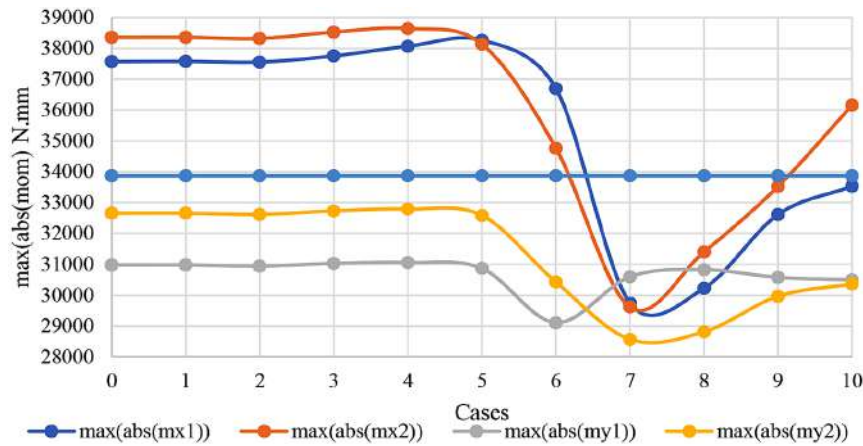


Figure 15. Maximum bending moments for eleven cases

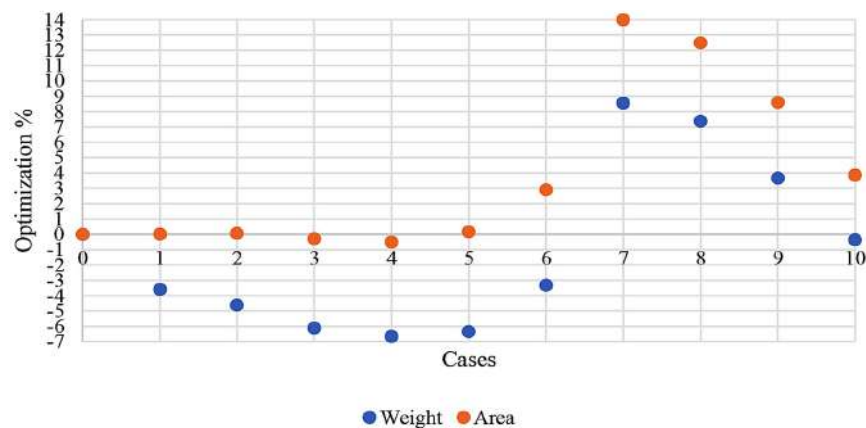


Figure 16. Cross-sectional area and weight optimization for X and Y loading case

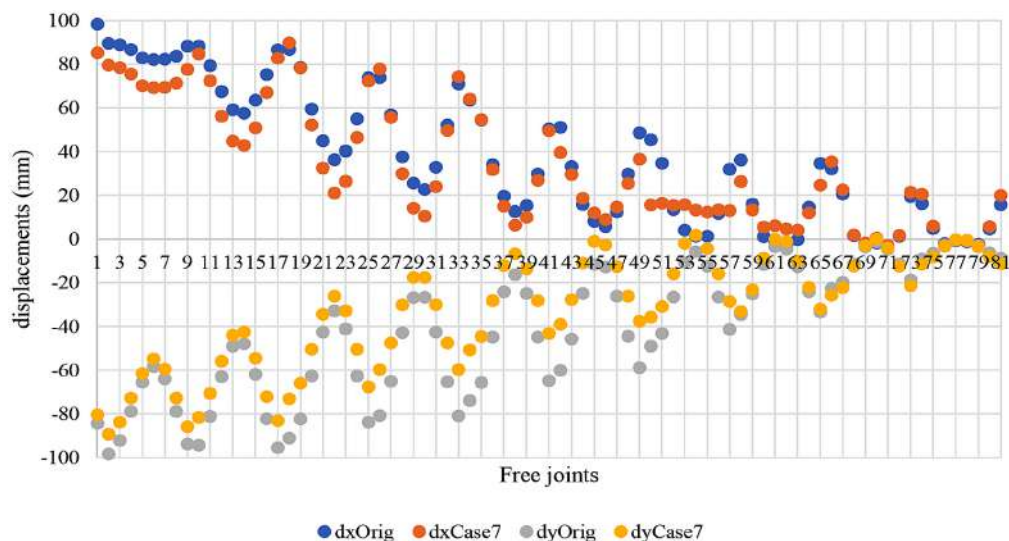


Figure 17. Nodal displacements for the optimum cases and the original structure

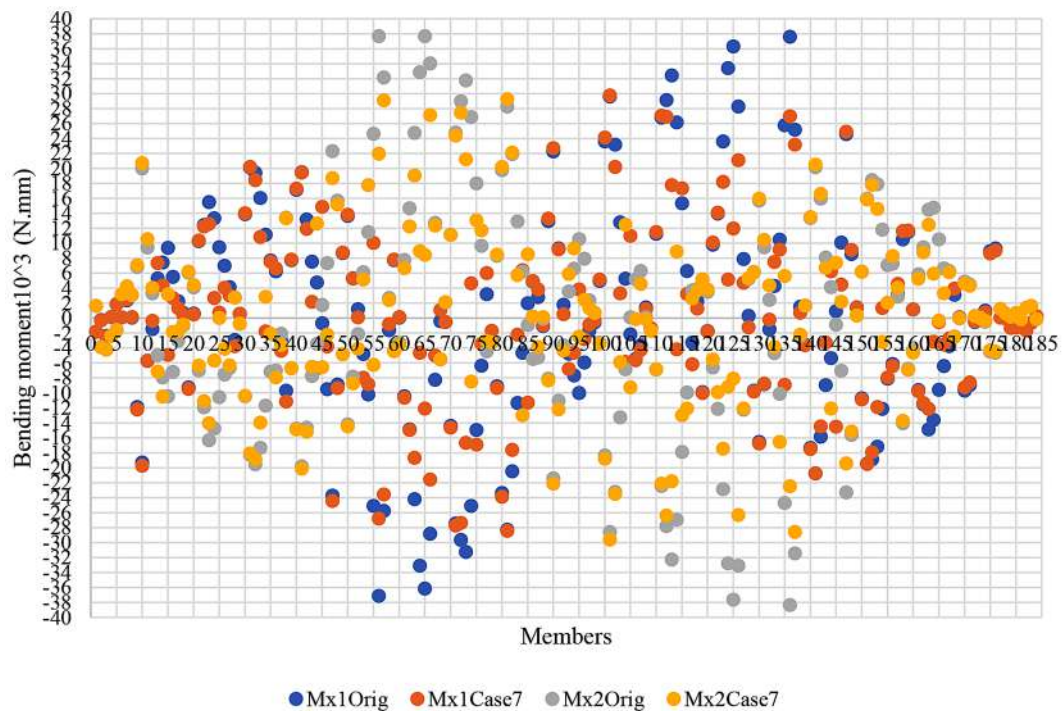


Figure 18. Bending moments in all members for Cases 0 and 7

CONCLUSION

In this study, the total structural weight of a space frame has been minimized; in addition, the significance of the location of the additional bars on the behavior of single-layer egg-shaped frames in several loading cases was investigated. The effectiveness of the location of the redundant on bending moment, beam size and total weight of the structure, and deformation of the structure has been extensively discussed. It was found that the place of the redundant is significant in the behaviors of the structure. The optimum location of the extra members depends on the loadings' direction. The structure's total weight optimization can be obtained up to 28%, 6%, and 8.5% for horizontally, vertically, and simultaneously horizontally and vertically loaded structures, respectively.

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